Designing Cocoa Transport Networks Using a Supply Chain Network Equilibrium Model with the Behaviour of Freight Carriers

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This paper presents a multi-channelled supply chain network equilibrium (SCNE) model with the behaviour of freight carriers, which is developed to represent the trading chain of cocoa in Indonesia. The supply chain of cocoa is presented, taking into account the multiple channels of SCN. Utilising this model as the lower level, a combinatorial optimisation model is also developed in the upper level within the framework of a mathematical program with equilibrium constraints, where a suitable set of transport network improvement actions is selected from a number of possible actions for improving the efficiency of cocoa SCN. The upper level is solved with a modified probability-based discrete binary particle swarm optimisation. Finally, the model is applied to an actual cocoa SCN for investigating the impact of freight-related transport measures to the international trading of cocoa.

**Keywords:** cocoa transport, supply chain network, multiple channels, freight carriers’ behaviour

1. Introduction

Indonesia is the 3rd largest cocoa producers in the world, where accounted for more than 10% of total world production\textsuperscript{1}. The area of cocoa plantations in Indonesia reached 992,448 hectares in which it is mostly planted in Sulawesi Island\textsuperscript{2}. As export commodities, cocoa is pretty valuable for Indonesian economy, where it (with palm oil and rubber) has contributed more than 24 billion USD, and created job opportunities for 10 million people. However, the complex trading chain and the inefficiencies of logistics system (e.g., the poor road infrastructure, insufficient port infrastructure, and complex export procedures) significantly affect the profit margins of Indonesian business owners\textsuperscript{3,4}.

Supply chain network (SCN) represents the linkage among the economic entities and the resulting their behavioural interactions. Nagurney et al.\textsuperscript{5} commenced multi-tiered SCN equilibrium modelling (SCNE), which incorporates decentralised decision-making by multiple agents on an SCN and takes into account their behavioural interactions. In this model, the decision-makers are manufacturers, retailers, and consumers, who compete within a tier but cooperate between tiers. Therefore, the product can flow from the manufacturers to the consumers via transactions. The model can also provide several valuable outputs for the SCN, such as the amount of the products produced and transacted, and the price of the products. Recently, there has been a lot of significant effort to expand the SCNE model (see the details in Yamada et al.\textsuperscript{6}), including Nagurney\textsuperscript{7} showing that the SCNE models can be reformulated and solved as a transport network equilibrium problem. However, this does not explicitly consider the behaviour freight carriers as well as the endogenous transport cost. Hence, the effects of any proposed freight transport measure cannot be assessed over the entire SCNs. Yamada et al.\textsuperscript{9} remarkably initiated a SCNE model, in which the behaviours of freight carriers is encompassed within the model. The model enables to investigate the impacts of freight transport-related policies on the whole supply chain of a product.

Compared with other sectors (e.g., electronic and manufactured goods), the cocoa sector consists of a long and complex trading chain. The chain involves local collectors, local traders, exporters, processors (i.e., local and multinational), local manufacturers and freight companies\textsuperscript{8,10}. This paper formulates a multi-channelled supply chain network equilibrium (SCNE) model with the behaviour of freight carriers. Unlike the existing SCNE-related papers, this model is more dedicated to the supply chain of
cocoa, which might be typical to other agricultural product chains.

After deriving the multi-channelled SCNE with the behaviour of freight carriers, this paper develops a discrete optimisation model within the framework of mathematical programmes with equilibrium constraints (MPEC). In this modelling framework, the SCNE model is incorporated within the lower level, and the upper level corresponds to combinatorial optimisation. The upper level involves binary (0–1) decision variables, representing whether or not a candidate for freight transport improvement is chosen. Nevertheless, the proposed MPEC is an NP-complete problem due to the parameterised variational inequality (VI) constraints. Thus, metaheuristics-based solution procedures, which can handle combinatorial optimisation problems in relatively shorter computational times, are applied to the upper level. Specifically, a modified probability-based discrete binary particle swarm optimisation (MPBPSO) \(^{(11)}\) is applied as a solution technique in the upper level.

The MPEC-based model is then applied to an actual cocoa SCN for analysing the impact of freight transport improvement. In addition, the model is also validated and calibrated using the data actually observed.

2. MPEC-based Model
2.1. Overall Framework and Upper Level

The proposed MPEC model consists of two levels. The lower level involves the multi-channelled SCNE, which is able to estimate the prices of cocoa beans (hereafter, referred to as cocoa) and the amount of cocoa transacted (i.e., those transported or distributed). The upper level optimises the combination of improvement actions for freight transport. The solutions derived in both levels influence with each other.

The model can be formulated as follows, where the term \((\bullet, \bullet)\) represents the inner product in \(N\)-dimensional Euclidean space, and equilibrium solutions are denoted by \(\bullet\).

\[
\begin{align*}
\text{Max} & \quad P(u, Z^*) \\
\text{subject to} & \quad (u, Z) \in K \\
& \quad g(u, Z^*)Z - Z^* \geq 0
\end{align*}
\]

where,

- \(u\) : vector of the set of improvement actions,
- \(Z\) : vector of state variables on a supernetwork,
- \(K\) : non-empty feasible space.

Maximising objective function (1) involves the upper level, which is a combinatorial optimisation problem with 0-1 variables. Constraint (2) corresponds to each variable condition; and constraint (3) to SCNE (specifically, VI (25)), and both makes the lower level.

The efficiency of SCNs is assessed using total surplus being calculated as the sum of producer surplus and consumer surplus. The producer surplus is estimated as the sum of the profits for all local collectors, all local traders, all exporters and all freight carriers, which can be computed with the solutions obtained by solving the SCNE. Therefore, the objective function of the upper level is to maximise the Benefit Cost Ratio (BCR), namely the ratio of the increased total surplus with the transport-related actions implemented as compared to without them to the investment/operational cost required for implementing them. The objective function can be represented as follows:

\[
\varphi (u, Z^*) = \frac{U_0(Z^*)}{\sum_{a \in A_2} \alpha a u_a}
\]

where,

- \(U(\bullet)\) : total surplus obtained in SCNs with actions implemented,
- \(U_0(\bullet)\) : total surplus obtained in SCNs without any action implemented,
- \(\alpha_a\) : investment/operation costs for action \(a\).

PSO is originally designed as optimisation technique for use in real-number spaces\(^{(12)}\). Kennedy and Eberhart\(^{(13)}\) then extended the continuous PSO to deal with discrete optimisation problems, which is also known as the discrete binary PSO (DBPSO). Menhas et al.\(^{(14)}\) proposed the probability-based discrete binary PSO (PBPSO) algorithm. Recently, Zukhruf et al.\(^{(11)}\) proposed the modified PBPSO (MPBPSO) algorithm, where an updating strategy for the change in position is embedded in the existing PBPSO algorithm. Results of the numerical tests show that the MPBPSO could provide better performance as compared to the conventional PBPSO algorithms (See the detailed algorithms in Zukhruf et al.\(^{(11)}\)).

A set of improvement actions, \(u\), is regarded as a particle of PSO, where its dimension represents the total number of possible actions to be implemented. The length of the particle is assumed to be 16. Every position of a particle, namely action implementation indicator \(u_a\), is formed in such a way that it takes a binary value of 1 if the corresponding action is implemented and 0 if it is otherwise. The value of objective function is calculated for each particle, and its fitness is evaluated. The swarm consists of a specific number of sets of actions.

2.2 SCNE at Lower Level

In this section, the multi-channelled cocoa supply
chain network model is developed. The decision-makers on the supply side (i.e., local collectors, local traders, exporters, and freight carriers) are concerned only with profit maximisation. To represent the SCN with multiple channels, the model allows the local collectors not only for transacting with the local traders, but also with the exporters. Let’s consider \( I \) local collectors collecting cocoa from farmers/growers, which is then shipped to \( J \) local traders or \( K \) exporters. The local traders, in turn, ship cocoa to \( K \) exporters. Finally, the exporters offer cocoa to the consumers, which is associated with \( L \) demand markets. Cocoa is transported by \( H \) freight carriers, where the links in the SCN represent those for transport and transaction.

![Figure 1. Description of cocoa SCN](image)

Let’s denote a typical local collector by \( i \), a typical local trader by \( j \), a typical exporter by \( k \), a typical demand market by \( l \), and a typical freight carrier by \( h \). As depicted in Figure 1, local collectors are located at the top tier of the network, the local traders at the second tier, the exporters at the third, and consumers at the bottom tier.

### 2.2.1 The Behaviour of Local Collectors and their Optimality Conditions

The local collectors purchase and collect cocoa beans from farmers or growers, which is then shipped to the local traders or the exporters. They play important roles in cocoa supply chain, since farmers are mostly not able to sell directly to the local traders or the exporters due to insufficient capital. The local collectors are profit-maximisers, and the optimisation problem of local collector \( i \) is given by:

\[
\begin{align*}
\text{Max} & \quad \sum_{j=1}^{J} \sum_{h=1}^{H} \rho_{ij}^1 h_{ij} + \sum_{k=1}^{K} \sum_{l=1}^{L} \rho_{ik}^2 h_{ik} - f_i(q^1, q^2) \\
& - \sum_{k=1}^{K} c_{ik}(q^2) - \sum_{j=1}^{J} g_{ij}(q^1, q^2) - \sum_{j=1}^{J} c_{ij}(q^1) \\
& - \sum_{k=1}^{K} c_{ik}(q^2) - \sum_{h=1}^{H} \sum_{j=1}^{J} \rho_{hij}^1 q_{hij} - \sum_{h=1}^{H} \sum_{k=1}^{K} \rho_{hik}^2 q_{hik}
\end{align*}
\]  

\[\tag{5}\]

subject to \( q_{hij} \geq 0 \quad \forall h, j \)

\( q_{hik} \geq 0 \quad \forall h, k \)  

\[\tag{6}\]

where,

- \( \rho_{ij}^1 \): price charged for cocoa by local collector \( i \) to local trader \( j \),
- \( \rho_{ik}^2 \): price charged for cocoa by local collector \( i \) to exporter \( k \),
- \( q_{hij} \): amount of cocoa transacted/transported from local collector \( i \) to local trader \( j \) by freight carrier \( h \),
- \( q_{hik} \): amount of cocoa transacted/transported from local collector \( i \) to exporter \( k \) by freight carrier \( h \),
- \( f_i(q^1, q^2) \): collection cost to local collector \( i \),
- \( c_{ik}(q^2) \): handling/inventory costs to local collector \( i \),
- \( g_{ij}(q^1, q^2) \): facility cost of local collector \( i \),
- \( c_{ij}(q^1) \): transaction cost incurred between local collector \( i \) and local trader \( j \),
- \( c_{ik}(q^2) \): transaction cost incurred between local collector \( i \) and exporter \( k \),
- \( \rho_{hij}^1 \): carriage charged by freight carrier \( h \) for transporting cocoa between local collector \( i \) and local trader \( j \),
- \( \rho_{hik}^2 \): carriage charged by freight carrier \( h \) for transporting cocoa between local collector \( i \) and exporter \( k \).

\( q_{hij}^1 \): \( HIJ \)-dimensional vector with component \( hij \) denoted by \( q_{hij}^1 \),

\( q_{hik}^2 \): \( HK \)-dimensional vector with component \( hij \) denoted by \( q_{hik}^2 \),

\( Q_{hij}^1 \): \( HIJ \)-dimensional vector with component \( hij \) denoted by \( q_{hij}^1 \),

\( Q_{hik}^2 \): \( HK \)-dimensional vector with component \( hij \) denoted by \( q_{hik}^2 \).

A local collector \( i \) is faced with a handling/inventory cost, which may be associated with the cost of holding cocoa in stock, for example, loading/unloading, sorting, drying, packaging and storage costs. In addition, each local collector is faced with collection cost, which relates to the expenses incurred for collecting cocoa from farmers.

The cocoa shipment between local collector \( i \) and local trader \( j \) is denoted by \( q_{hij} \), where the cocoa shipments between all pairs of local collectors and
local traders are grouped into the column vector $Q^1 \in R^{HI}$. Furthermore, $q_{hik}$ denotes the amount of cocoa transacted from local collector $i$ to exporter $k$, and these cocoa shipments are grouped into the column vector $Q^2 \in R^{HIK}$. Freight carrier $h$, in turn, transports the amount of cocoa from local collector $i$ to local trader $j$, and exporter $k$ as well. Hence, the transport volume is simultaneously determined with the amount of cocoa transacted.

Facility cost, denoted by $g$, includes the expenditure for operating, improving, maintaining the local collector facilities, and its usage costs. Assuming that the collection cost functions, handling/inventory cost functions, facility cost functions and transaction cost functions for each local collector are continuous and convex. The optimality conditions for all local collectors can simultaneously be expressed as the following VI: determine $(Q^1, Q^2) \in R^{HI+HIK}$, which satisfies:

\[
\begin{align*}
\sum_{h=1}^{H} \sum_{i=1}^{I} \sum_{j=1}^{J} \left( c_{hij} \frac{\partial Q^2}{\partial q_{hij}} - c_{hik} \frac{\partial Q^2}{\partial q_{hik}} \right) + \sum_{h=1}^{H} \sum_{i=1}^{I} \sum_{j=1}^{J} \frac{\partial g_{ij}(Q^1)}{\partial q_{hij}} - g_{ij}(Q^1) - c_{hik} \frac{\partial Q^2}{\partial q_{hik}} \\
+ \sum_{h=1}^{H} \sum_{i=1}^{I} \sum_{j=1}^{J} \left[ c_{hij} - c_{hik} \right] q_{hij} - g_{ij}(Q^1) - c_{hik} q_{hik} = 0
\end{align*}
\]

\[\forall (Q^1, Q^2) \in R^{HI+HIK}\]

2.2.2 The Behaviour of Local Traders and their Optimality Conditions.

The local traders, in turn, must determine not only the amount of cocoa that they wish to obtain from the local collectors but also those purchased by the exporters in order to maximise their profits. Assuming that the local traders are also profit-maximisers, the optimisation problem of a local trader $j$ is given by:

\[
\begin{align*}
\text{Max}_{Q^1, Q^2} & \sum_{k=1}^{K} \rho_{jk}^3 \sum_{h=1}^{H} q_{hjk} - c_{j}(Q^1) - g_{j}(Q^1) \\
- & \sum_{k=1}^{K} c_{jk}(Q^1) - \sum_{h=1}^{H} \sum_{k=1}^{K} \rho_{hk} q_{hkj} - \sum_{i=1}^{I} \sum_{j=1}^{J} \rho_{ij}^3 \sum_{h=1}^{H} q_{hij} \sum_{h=1}^{H} \sum_{i=1}^{I} q_{hij} \geq 0 \forall h, i, q_{hij} \geq 0 \forall h, k
\end{align*}
\]

where,

- $\rho_{jk}^3$ : sales price charged by local trader $j$ to exporter $k$.
- $q_{hjk}$ : amount of cocoa transacted/transported from local trader $j$ to exporter $k$ by freight carrier $h$.
- $c_{j}(Q^1)$ : handling/inventory costs to local trader $j$.
- $g_{j}(Q^1)$ : facility cost to local trader $j$.
- $c_{jk}(Q^3)$ : transaction cost incurred between local trader $j$ and exporter $k$.
- $\rho_{hk}$ : carriage charged by freight carrier $h$ for transporting cocoa between local trader $j$ and exporter $k$.
- $Q^1$ : $HI$-dimensional vector with component $hij$ denoted by $q_{hij}$.
- $Q^2$ : $HK$-dimensional vector with component $hjk$ denoted by $q_{hjk}$.
- $Q^3$ : $HIK$-dimensional vector with component $hjk$ denoted by $q_{hjk}$.

Assuming that the handling/inventory cost functions, facility cost functions and transaction cost functions are continuously differentiable and convex, then the optimality conditions for all local traders simultaneously coincide with the solution of the following VI: determine $(Q^1, Q^2, \gamma) \in R^{HI+HIK}$ which satisfies:

\[
\begin{align*}
\sum_{h=1}^{H} \sum_{i=1}^{I} \sum_{j=1}^{J} \left[ c_{hij} \frac{\partial Q^1}{\partial q_{hij}} + c_{hik} \frac{\partial Q^2}{\partial q_{hik}} + g_{ij} - g_{ij}(Q^1) \right] q_{hij} - g_{ij}(Q^1) + c_{hik} q_{hik} = 0 \\
+ \sum_{h=1}^{H} \sum_{i=1}^{I} \sum_{j=1}^{J} \left( c_{j} + c_{jk} Q^3 \right) q_{hjk} + \rho_{hk} q_{hkj} + \gamma_j \sum_{k=1}^{K} \rho_{hk} q_{hkj} = 0 \\
+ \sum_{j=1}^{J} \sum_{k=1}^{K} \left( \sum_{i=1}^{I} q_{hij} - \sum_{k=1}^{K} q_{hjk} \right) \gamma_j - \gamma_j^* \geq 0
\end{align*}
\]

\[\forall (Q^1, Q^2, \gamma) \in R^{HI+HIK}\]

Here, the term $\gamma_j$ is the Lagrange multiplier associated with constraint (9), and $\gamma$ is an $J$-dimensional vector with component $\gamma_j$.

2.2.3 The Behaviour of Exporters and their Optimality Conditions.

The exporters, in turn, conduct transactions with both the local collectors and the local traders, in which they wish to obtain cocoa, as well as with the overseas consumers who are the ultimate purchasers of cocoa. The behaviour of exporter $k$ who seeks a maximum profit can be formulated as follows:

\[
\begin{align*}
\text{Max}_{Q^1, Q^2, Q^3} & \sum_{k=1}^{K} \rho_{jk}^3 \sum_{h=1}^{H} q_{hjk} - c_{k}(Q^2, Q^3) \\
- & \sum_{k=1}^{K} c_{jk}(Q^1) - \sum_{h=1}^{H} \sum_{k=1}^{K} \rho_{hk} q_{hkj} - \sum_{i=1}^{I} \sum_{j=1}^{J} \rho_{ij}^3 \sum_{h=1}^{H} q_{hij} \sum_{h=1}^{H} \sum_{i=1}^{I} q_{hij} \geq 0 \forall h, i, q_{hij} \geq 0 \forall h, k
\end{align*}
\]
- g_k(Q^2, Q^3) - \sum_{i=1}^{l} c_{ik}(Q^i) - \sum_{h=1}^{H} \sum_{l=1}^{L} \rho^*_h q_{hlk} \\
- \sum_{j=1}^{J} \rho^*_j \sum_{h=1}^{H} q_{hjk} - \sum_{k=1}^{K} \rho^*_k \sum_{h=1}^{H} q_{hik} \\
(13)

subject to \sum_{h=1}^{H} \sum_{l=1}^{L} q_{hlk} \leq \sum_{h=1}^{H} \sum_{l=1}^{L} q_{hik} + \sum_{j=1}^{J} q_{hjk} \\
q_{hik} \geq 0 \quad \forall h, i, \quad q_{hjk} \geq 0 \quad \forall h, j, \quad q_{hlk} \geq 0 \quad \forall h, l \\
(14)

where, 
\rho^*_h : sales price charged for cocoa by exporter to demand market l, 
q_{hlk} : amount of cocoa transacted/transported from exporter k to demand market l by freight carrier h, 
c_{k}(Q^2, Q^3) : handling/inventory costs to exporter k, 
g_{k}(Q^2, Q^3) : facility cost to exporter k, 
c_{ik}(Q^i) : transaction cost incurred between exporter k and demand market l, 
\rho^*_hlk : carriage charged by freight carrier h for transporting cocoa between exporter k and demand market l, 
Q^2_k : HI-dimensional vector with component hik denoted by q_{hik}, 
Q^3_k : HI-dimensional vector with component hjk denoted by q_{hjk}, 
Q^4_l : HL-dimensional vector with component hlk denoted by q_{hlk}, 
Q^4 : HKL-dimensional vector with component hkl denoted by q_{hkl}.

Assuming that handling/inventory cost functions, facility cost functions and transaction cost functions are continuously differentiable and convex, the optimality conditions for all exporters can simultaneously be formulated as the following VI: determine (Q^2, Q^3, Q^4, \delta) \in R_{+,+}^{HIK+HIJK+HKL+K} satisfying:
H \sum_{h=1}^{H} \sum_{i=1}^{I} \sum_{k=1}^{K} \left[ c_{ik}(Q^2, Q^3) \frac{\partial c_{ik}}{\partial q_{hik}} + g_{ik}(Q^2, Q^3) \frac{\partial g_{ik}}{\partial q_{hik}} \\
+ \rho^*_i - \delta^*_i \right] \left[ q_{hik} - q_{hlk} \right] \\
+ H \sum_{h=1}^{H} \sum_{j=1}^{J} \sum_{k=1}^{K} \left[ c_{jk}(Q^2, Q^3) \frac{\partial c_{jk}}{\partial q_{hjk}} + g_{jk}(Q^2, Q^3) \frac{\partial g_{jk}}{\partial q_{hjk}} \\
+ \rho^*_j - \delta^*_j \right] \left[ q_{hjk} - q_{hjk} \right] \\
+ H \sum_{h=1}^{H} \sum_{l=1}^{L} \sum_{k=1}^{K} \left[ c_{lk}(Q^2, Q^3) \frac{\partial c_{lk}}{\partial q_{hlk}} + \rho^*_l - \delta^*_l \right] \left[ q_{hlk} - q_{hlk} \right] \\
+ \sum_{k=1}^{K} \left[ \sum_{h=1}^{H} \left( \sum_{i=1}^{I} q_{hik} + \sum_{j=1}^{J} q_{hjk} - \sum_{l=1}^{L} q_{hlk} \right) \right] \left[ \delta^*_k - \delta^*_k \right] \geq 0 \\
\forall (Q^2, Q^3, Q^4, \delta) \in R_{+,+}^{HIK+HIJK+HKL+K}
(15)

Here, the term \delta^*_k is the Lagrange multiplier associated with constraint (13), and \delta is an K-dimensional vector denoted by \delta^*_k.

2.2.4 The Overseas Consumers in Demand Markets and the Equilibrium Conditions.

The overseas consumers located at the demand markets are then discussed. The overseas consumers take into account the prices charged for cocoa by the exporters in making their consumption decisions. Market demand describes the total quantity demanded by all consumers, which depends upon the demand market prices. The demand function is assumed to be continuous. Then, the following complementarity conditions hold for demand market l.

\rho^*_l \begin{cases} = \rho^*_l^* & \text{if } \rho^*_l > 0 \\
\geq \rho^*_l^* & \text{if } \rho^*_l = 0 \\
\geq \sum_{q=1}^{Q} q^*_q & \text{if } \rho^*_l^* > 0 \\
= \sum_{q=1}^{Q} q^*_q & \text{if } \rho^*_l^* = 0 \\
= \rho^*_l^* & \text{if } \rho^*_l < 0 \end{cases}
(16)

\rho^*_l : market price of cocoa at demand market l, 
\rho^*_l^* : L-dimensional vector with component \rho^*_l denoted by \rho^*_l.

d(l\rho^*_l) : demand function at demand market l.

In equilibrium, conditions (16) and (17) will have to hold for all demand markets, and these, in turn, can also be expressed as a VI, and given by:
\begin{align*}
&\sum_{h=1}^{H} \sum_{i=1}^{I} \sum_{k=1}^{K} \left[ c_{ik}(Q^2, Q^3) \frac{\partial c_{ik}}{\partial q_{hlk}} + g_{ik}(Q^2, Q^3) \frac{\partial g_{ik}}{\partial q_{hlk}} \\
+ \rho^*_i - \delta^*_i \right] \left[ q_{hlk} - q_{hlk} \right] \\
+ \sum_{j=1}^{J} \sum_{h=1}^{H} \sum_{k=1}^{K} \left[ c_{jk}(Q^2, Q^3) \frac{\partial c_{jk}}{\partial q_{hlk}} + g_{jk}(Q^2, Q^3) \frac{\partial g_{jk}}{\partial q_{hlk}} \\
+ \rho^*_j - \delta^*_j \right] \left[ q_{hlk} - q_{hlk} \right] \\
+ \sum_{l=1}^{L} \sum_{h=1}^{H} \sum_{k=1}^{K} \left[ c_{lk}(Q^2, Q^3) \frac{\partial c_{lk}}{\partial q_{hlk}} + \rho^*_l - \delta^*_l \right] \left[ q_{hlk} - q_{hlk} \right] \\
+ \sum_{k=1}^{K} \left( \sum_{h=1}^{H} \left( \sum_{i=1}^{I} q_{hlk} + \sum_{j=1}^{J} q_{hlk} - \sum_{l=1}^{L} q_{hlk} \right) \right) \left( \delta^*_k - \delta^*_k \right) \geq 0 \\
\forall (Q^2, Q^3, Q^4) \in R_{+,+}^{HIK+HIJK+HKL+K}
\end{align*}
(18)

2.2.5 The Behaviour of Freight Carriers and their Optimality Conditions.

Assuming that the freight carriers are concerned with profit maximisation, thus optimisation problem for freight carrier h can be formulated as:
\begin{align*}
\text{Max}_{Q^2, Q^3, Q^4} & \sum_{i=1}^{I} \sum_{k=1}^{K} \rho^*_h q_{hik} + \sum_{l=1}^{L} \sum_{k=1}^{K} \rho^*_h q_{hlk}
\end{align*}
(19)
where,

\( \mathbf{Q}_h \) : \( IJ \)-dimensional vector with component \( h_{ij} \) denoted by \( q_{hij} \),

\( \mathbf{Q}_k \) : \( IK \)-dimensional vector with component \( h_{ik} \) denoted by \( q_{hik} \),

\( \mathbf{Q}_l \) : \( JK \)-dimensional vector with component \( h_{jk} \) denoted by \( q_{hjk} \),

\( \mathbf{Q}_k \) : \( KL \)-dimensional vector with component \( h_{kl} \) denoted by \( q_{hkl} \). 

\( g(h,h',Q) \) : facility cost of freight carrier \( h \),

\( w_h(Q^1, Q^2, Q^3, Q^4) \) : operation cost of freight carrier \( h \).

In case where the variability of travel time on the transport network is incorporated within this model, the operation cost is assumed as the sum of average transport cost (i.e., \( w_h(Q^1, Q^2, Q^3, Q^4) \)) and delay penalty cost. It is also assumed that the delay penalty cost is the cost incurred for avoiding substantial delay by the increased number of freight vehicles used and proportional to the amount of cocoa transported. According to existing study\(^5\), the delay penalty cost was then calculated as follow.

Let \( I_{hij} \) denote the maximum allowable travel time of freight carrier \( h \) to transport the product from \( i \) to \( j \), where travel time \( I_{hij} \) is assumed to follow a probability distribution \( \phi_{hij}(t) \). If the travel time is larger than the allowable travel time (i.e., \( t > I_{hij} \)), the delay penalty cost can thus be expressed as \( \Delta_{hij}^+ = t - I_{hij} \). Therefore, the expected delay penalty time is determined as follows:

\[
e^{h}_{hij}(\Delta_{hij}) = E[\Delta_{hij}] = \int_{I_{hij}}^{\infty} t - I_{hij} \phi_{hij}(t) dt \quad (21)
\]

Let's define \( \lambda_{hij}^+ \) as unit cost for delay penalty, and the expected delay penalty cost can be formulated as follows:

\[
E[\lambda_{hij}^+ \Delta_{hij}^+] = \lambda_{hij}^+ E[\Delta_{hij}^+] = \int_{I_{hij}}^{\infty} (t - I_{hij}) \phi_{hij}(t) dt 
\]

2.2.6 The Equilibrium Conditions of the Supply Chain Network.

The equilibrium state of the supply chain network can be characterised as one where the optimality conditions (7), (11), (15), (18) and (24) hold simultaneously. The equilibrium conditions governing the supply chain network model are equivalent to the solution to the VI given by:

\[
H \sum_{h=1}^{H} \sum_{i=1}^{I} \sum_{j=1}^{J} \left[ c_i h_{ij} + \frac{\partial g(h_{ij})}{\partial h_{ij}} - \rho_{h_{ij}}^+ \right] [q_{hij} - q_{hij}^*] + H \sum_{h=1}^{H} \sum_{i=1}^{I} \sum_{k=1}^{K} \left[ c_i h_{ik} + \frac{\partial g(h_{ik})}{\partial h_{ik}} - \rho_{h_{ik}}^+ \right] [q_{hik} - q_{hik}^*] + H \sum_{h=1}^{H} \sum_{j=1}^{J} \sum_{k=1}^{K} \left[ c_i h_{jk} + \frac{\partial g(h_{jk})}{\partial h_{jk}} - \rho_{h_{jk}}^+ \right] [q_{hjk} - q_{hjk}^*] \quad (24)
\]

where \( H \) is the number of freight carriers, \( I \) is the number of origin nodes, \( J \) is the number of destination nodes, \( K \) is the number of facilities, \( I_{hij} \) is the maximum allowable travel time of freight carrier \( h \) between \( i \) and \( j \), and \( \lambda_{hij}^+ \) is the delay penalty cost. The optimal solution to this VI is the equilibrium conditions for the supply chain network.

Assuming that facility cost functions and opportunity cost functions (i.e., average transport cost functions and unit cost functions for delay penalty) are continuously differentiable and convex, the optimality conditions for all freight carriers can simultaneously be formulated as the following VI: determine \( (Q^1, Q^2, Q^3, Q^4) \in R_+^{HIJ+HIK+HKL} \) satisfying:

\[
H \sum_{h=1}^{H} \sum_{i=1}^{I} \sum_{j=1}^{J} \left[ c_i h_{ij} + \frac{\partial g(h_{ij})}{\partial h_{ij}} - \rho_{h_{ij}}^+ \right] [q_{hij} - q_{hij}^*] + H \sum_{h=1}^{H} \sum_{i=1}^{I} \sum_{k=1}^{K} \left[ c_i h_{ik} + \frac{\partial g(h_{ik})}{\partial h_{ik}} - \rho_{h_{ik}}^+ \right] [q_{hik} - q_{hik}^*] + H \sum_{h=1}^{H} \sum_{j=1}^{J} \sum_{k=1}^{K} \left[ c_i h_{jk} + \frac{\partial g(h_{jk})}{\partial h_{jk}} - \rho_{h_{jk}}^+ \right] [q_{hjk} - q_{hjk}^*] \quad (25)
\]
The price variables $\rho^1_{ij}$ and $\rho^2_{ik}$ can be retrieved from the eventual solution by Equations (26) and (27), respectively, setting $q_{hij} > 0$ and $q_{hik} > 0$ in inequality (6).

$$
\rho^1_{ij} = \frac{\partial c_i}{\partial q_{hij}}(Q_{ij}^0, Q_{ij}^{0*}, Q_{ij}^{0**}) + \frac{\partial c_k}{\partial q_{hij}}(Q_{ij}^{0*}) + \frac{\partial c_j}{\partial q_{hij}}(Q_{ij}^{0**}) + \rho_{hij}^1
$$

$$
\rho^2_{ik} = \frac{\partial q_{hik}}{\partial q_{hij}}(Q_{ij}^0, Q_{ij}^{0*}, Q_{ij}^{0**}) + \frac{\partial q_{hik}}{\partial q_{hij}}(Q_{ij}^{0*}) + \frac{\partial q_{hik}}{\partial q_{hij}}(Q_{ij}^{0**}) + \rho_{hik}^2
$$

The equilibrium solutions of $\gamma, \delta$ can be obtained from inequality (25), and the prices $\rho^3_{hj}$ can also be obtained by finding a $q_{hjk} > 0$ in inequality (11) as follows:

$$
\rho^3_{hj} = \frac{\partial c_j}{\partial q_{hjk}}(Q_{ij}^0, Q_{ij}^{0*}, Q_{ij}^{0**}) + \rho_{hjk}^3 + \gamma_j
$$

Also, if $q_{hkl} > 0$ in inequality (14), the $\rho^4_{il}$ can be derived as Equation (29).

$$
\rho^4_{il} = \frac{\partial c_l}{\partial q_{hkl}}(Q_{ij}^0, Q_{ij}^{0*}, Q_{ij}^{0**}) + \rho_{hkl}^4 + \delta_k
$$

Furthermore, in the same way, from Equation (24), the carriage charged by freight carrier can be obtained as follows:

$$
\rho_{hji} = \frac{\partial q_{hj}}{\partial q_{hji}}(Q_{ij}^0, Q_{ij}^{0*}, Q_{ij}^{0**}) + \frac{\partial q_{hj}}{\partial q_{hji}}(Q_{ij}^{0*}) + \frac{\partial q_{hj}}{\partial q_{hji}}(Q_{ij}^{0**})
$$

$$
\rho_{hjk} = \frac{\partial q_{hjk}}{\partial q_{hji}}(Q_{ij}^0, Q_{ij}^{0*}, Q_{ij}^{0**}) + \frac{\partial q_{hjk}}{\partial q_{hji}}(Q_{ij}^{0*}) + \frac{\partial q_{hjk}}{\partial q_{hji}}(Q_{ij}^{0**})
$$

$$
\rho_{hkl} = \frac{\partial q_{hkl}}{\partial q_{hji}}(Q_{ij}^0, Q_{ij}^{0*}, Q_{ij}^{0**}) + \frac{\partial q_{hkl}}{\partial q_{hji}}(Q_{ij}^{0*}) + \frac{\partial q_{hkl}}{\partial q_{hji}}(Q_{ij}^{0**})
$$

From Equations (26)-(33), as different from Nagurney et al.\(^3\), it can be inferred that the sales price charged by SCN entities is not influenced by the transport cost itself but the marginal transport cost.

### 2.2.7 Retrieving the Price Variables

The model developed is then numerically tested to validate its performance using a network as shown in Figure 2. Both the functional forms and parameter values of $f_i, c_i, c_p, c_s, c_h, c_k, c_j, c_l, g_l, g_r, g_s, g_h, w_h, d_l, \theta_{hij}, \theta_{hik}, \theta_{hjk}, \theta_{hkl}$, $\lambda_{hij}, \lambda_{hik}, \lambda_{hjk}, \lambda_{hkl}$ are predetermined on the basis of the existing studies as well as the uniqueness of the solutions is ensured. (Nagurney et al.\(^5\))

Assuming that $\varphi_{hij}$ follows the normal distribution (i.e., $\varphi_{hij} \sim N(\bar{\varphi}_{hij}, \sigma^2_{hij})$), where $\bar{\varphi}_{hij}$ and $\sigma^2_{hij}$ denote the average and standard deviation of the travel time, respectively. In addition, the average travel time is set according to the distance, and the standard deviation is determined based on the actual data, which is surveyed in the Sulawesi Island. These also hold for $\varphi_{hik}$, $\varphi_{hjk}$, and $\varphi_{hkl}$. Let us assume that freight carriers consider the variability of travel time. Furthermore, allowable travel time $\bar{t}_{hij}$ may vary depending on average travel time $\bar{t}_{hij}$. Thus, $\bar{t}_{hij}$ is set as $\bar{t}_{hij} = \bar{t}_{hij} + 2\sigma_{hij}$. These assumptions also hold between $i$ and $k$, $j$ and $k$, and $i$ and $k$ as well.

As described above, the case studies to be carried out in this paper may lead to the unpractical results, since it depends on the configuration of so many functional forms and parameter values, which are given in advance. Therefore, to facilitate the better interpretation of the results, the actual cities are set as can be seen in Figure 2, and the parameter values are set by considering the location of such cities. The actual cities are located in the Sulawesi Island, which is the biggest cocoa producer in Indonesia\(^3\).

The local collectors usually operates in the farmers regions, as indicated by Syahrudin\(^20\). Therefore, we set the local collectors to be located in the sub-district, which is the smallest tier of administration unit in Indonesia. In general, the local traders can be located not only in the sub-district, but also in regency/city, even though the biggest trader is mostly located in the capital of regency/city. Moreover, the exporters are mainly located in the capital of province. The overseas markets are typically set in the European region (i.e., Rotterdam), American region (i.e., New Jersey), and Asian region (i.e., Shanghai). Furthermore, we assume that all exporters are transported cocoa to overseas market through the international seaport, which is located in Makassar.
In order to represent the actual conditions of the cocoa SCN, the functional forms and parameter values are calibrated using the results of the logistics cost survey to export industries and the cocoa stakeholders.

The functions used in the case studies are described as follow:

$$f_i = 1000 \left( \sum_{h=1}^{2} \sum_{j=1}^{4} q_{hij} + \sum_{k=1}^{3} q_{hik} \right)$$  \hspace{1cm} (34)$$

$$c_i = 3.0 \left( \sum_{h=1}^{2} \sum_{j=1}^{4} q_{hij} + \sum_{k=1}^{3} q_{hik} \right)^{1.7}, \ c_j = 1.5 \left( \sum_{h=1}^{2} \sum_{i=1}^{4} q_{hij} \right)^{1.7} .$$  \hspace{1cm} (35)$$

$$c_k = 3.5 \left( \sum_{h=1}^{2} \sum_{j=1}^{4} q_{hjk} + \sum_{k=1}^{3} q_{hik} \right)^{1.8} .$$  \hspace{1cm} (36)$$

$$c_{ij} = 15 \sum_{h=1}^{2} q_{hij}, \ c_{ik} = 15 \sum_{h=1}^{2} q_{hik} .$$  \hspace{1cm} (37)$$

$$c_{jk} = 20 \sum_{h=1}^{2} q_{hjk} .$$  \hspace{1cm} (38)$$

$$g_i = 22 \left( \sum_{h=1}^{2} \sum_{j=1}^{4} q_{hij} + \sum_{k=1}^{3} q_{hik} \right) , \ g_j = 22 \sum_{h=1}^{2} \sum_{i=1}^{4} q_{hij} .$$  \hspace{1cm} (39)$$

$$g_k = 22 \left( \sum_{h=1}^{2} \sum_{j=1}^{4} q_{hjk} + \sum_{k=1}^{3} q_{hik} \right) .$$  \hspace{1cm} (40)$$

$$\bar{w}_h = \sum_{i=1}^{4} \sum_{j=1}^{4} (\beta_{hij} q_{hij}^2 + \alpha_{hij} q_{hij})$$

\[ + \sum_{i=1}^{4} \sum_{k=1}^{3} (\beta_{hik} q_{hik}^2 + \alpha_{hik} q_{hik}) \]

\[ + \sum_{j=1}^{4} \sum_{k=1}^{3} (\beta_{hjk} q_{hjk}^2 + \alpha_{hjk} q_{hjk}) \]

\[ + \sum_{l=1}^{3} \sum_{k=1}^{3} (\beta_{hkl} q_{hkl}^2 + \alpha_{hkl} q_{hkl}) . \]  \hspace{1cm} (41)$$

As shown in Table 1, the values of $\alpha_{hij}$, $\alpha_{hik}$, $\alpha_{hjk}$, $\alpha_{hkl}$ are set based on the average travel times and actual carriage. $\beta_{hij}$, $\beta_{hik}$, $\beta_{hjk}$, $\beta_{hkl}$ are set to 2.5 for the city which located in the Sulawesi Island, and 2.6 for otherwise.

The computational results are to be given during the presentation. Their outline can be seen below.

Prior to full computation, the SCNE model in the lower level is to be first validated. The estimated ratio of logistics-related costs (i.e., facility cost for freight carriers, average logistics cost, and handling cost) to total sales will be compared to the actually observed data\(^{(17)-(19)}\). Furthermore, to represent the market size, the ratio of market shares between cities resulting from the model will also be confirmed with the actual data\(^{(21)}\).

Since the model incorporates the international trading with high transport costs, unrealistic results might be obtained depending on the settings. Therefore, the estimated ratio of transport cost to total sales for distributing cocoa from Makassar to

![Figure 2. Test Network](image-url)
the overseas market is to be compared to the actual observed values. The applicability of the model will then be tested using the validated cocoa SCN and the freight transport improvement actions. Sixteen alternative actions, such as those relating to reduction of transport-related cost on road network are set as candidates. The best combination of actions that maximises the value of objective function (4) is to be selected as a solution.

REFERENCES
2) Indonesian Ministry of Industry: Road map of development cocoa industry, 2010.