Stochastic Modeling on the Relationship between Breakdown and Discharge Flow Rates at Intercity Expressway Bottlenecks

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Breakdown and discharge flow rates are significant indicators to represent breakdown phenomena on intercity expressways. This paper proposes a methodology for stochastic modeling on the relationship between them based on three diverge and three merge bottlenecks on intercity expressways in central Japan. Firstly, breakdown probability models are developed based on breakdown flow rates which are extracted through the lane-based breakdown identification method. Secondly, discharge flow rates are modeled in a stochastic way by considering their relationship with breakdown flow rates. Furthermore, the relationship between breakdown and discharge flow rates is generalized by referring to site-specific geometries which enables the breakdown estimation at potential bottlenecks. Finally the developed models are applied to reproduce breakdown phenomena through Monte Carlo method. The results demonstrate that the developed models can well reproduce breakdown phenomena.

Key Words: *breakdown flow rate, discharge flow rate, intercity expressway,diverge section, merge section*

1. INTRODUCTION

The concept of capacity plays an important role in the planning, design, and operation stages of intercity expressway facilities. Breakdown and discharge flow rates represent two distinct aspects of bottleneck capacity which are significant indicators to represent breakdown phenomena.

With respect to breakdown flow rate (*BDF*), considerable number of studies have adopted breakdown probability model[1), 2), 3)] to describe and quantify it in light of its stochastic nature. As for discharge flow rate (*DCF*), it is typically characterized in a deterministic manner by the existing studies. In other words, after breakdown occurs, it is assumed that the queue discharges at a deterministic flow rate. Based on field data, some studies have indicated that discharge flow rate is also stochastic in nature[4), 5)]. However, few efforts[2)] have been made on modeling discharge flow rate by considering its stochastic characteristic.

In addition, the relationship between breakdown and discharge flow rates has not been well understood, which significantly impacts intercity expressway performance accompanied with breakdown phenomena. Limited studies on the investigation of this relationship is to estimate capacity drop due to breakdown occurrence in a deterministic way[6), 7), 8), 9)]. However, the stochastic nature of discharge flow rate has not been interpreted. Therefore, this paper proposes a methodology for stochastic modeling on the relationship between breakdown and discharge flow rates based on three diverge and three merge bottlenecks on intercity expressways in central Japan.

The rest of this paper is organized as follows. In section 2, the features of the study sites are presented. Lane based breakdown identification method is applied for modeling breakdown probability in section 3. Then section 4 introduces the methodology for stochastic modeling on discharge flow rate by considering its relationship to breakdown flow rate. In section 5, Monte Carlo simulation is carried out to reproduce breakdown phenomena by applying the developed models. Finally, section 6 offers recommendations for practical applications and suggestions for future research.

2. DESCRIPTIONS OF TEST BED

In the intercity expressway network of central Japan, there exist breakdown phenomena at various bottlenecks at sag, diverge and merge sections. This study focuses on diverge sections and merge sections considering lane usage preference by drivers due to the influence of diverge and merge traffic flow there.

As illustrated in **Fig.1**, three diverge bottlenecks and three merge bottlenecks are chosen as test beds. Detailed descriptions of the test bed sites are listed in **Tables 1** and **2**. All the diverge and merge sections have left-side off-ramps. With respect to lane configuration, all of their mainlines have two lanes, namely shoulder lane and median lane (for Toyota diverge and merge sections on Tomei Expressway, their mainlines have been broadened to three lanes by fully utilizing shoulder from 10/21/2011).

Representative geometries, lane configurations and detector locations are presented in **Fig.2** and **4** at Toyota diverge and merge sections. **Fig.3(a)** and **3(b)** present more distinct and direct the overlooks of Toyota diverge section layout. At this section, frequent breakdown events have been observed due to high diverge traffic flow which aims at using Shinmeishin Expressway after its operation in March, 2008.

Along the mainlines and off-ramps of these bottlenecks, double-loop detectors are installed approximately every 2 kilometers. Thanks to Central Nippon Expressway Company Limited (NEXCO), traffic flow data records around these bottlenecks are available. This study investigates recurrent breakdown phenomena during the period from 3/1/2008 to 12/31/2009.

Traffic flow rates and average speeds on each lane were measured and aggregated at a 5-minute interval. Breakdown events due to other non-recurrent causes such as roadway maintenance works and accidents were excluded. Some preliminary studies have been conducted on constructing flow-speed diagrams at each detector, which help identify bottleneck locations.

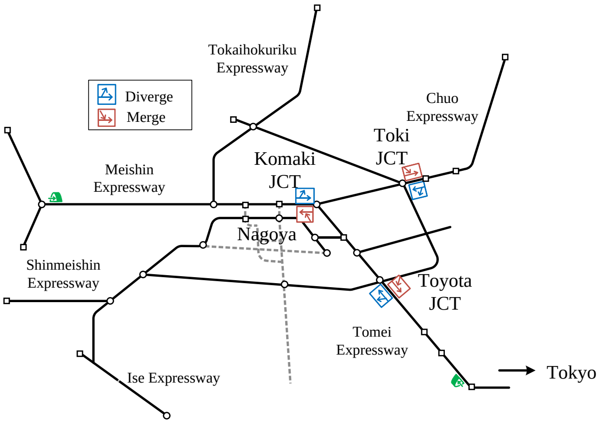


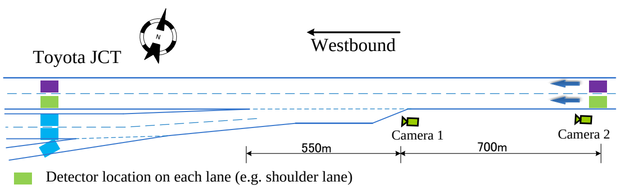
Fig.1 Diverge and merge bottlenecks in the intercity expressway network of central Japan

Table 1 Diverge bottleneck descriptions

Name	Expressway	Direction	Location
Toyota diverge section	Tomei	Westbound	Toyota JCT
Toki diverge section	Chuo	Southbound	Toki JCT
Komaki diverge section	Tomei	Westbound	Komaki JCT

Table 2 Merge bottleneck descriptions

Name	Expressway	Direction	Location
Toyota merge section	Tomei	Eastbound	Toyota JCT
Toki merge section	Chuo	Northbound	Toki JCT
Komaki merge section	Tomei	Eastbound	Komaki JCT



(Note: for Toyota diverge and merge sections, their mainlines have been broadened to three lanes by fully utilizing shoulder from 10/21/2011.)

Fig.2 Toyota diverge section

(b) View from camera 1 **(c)** View from camera 2

Fig.3 Overlooks of Toyota diverge section layout

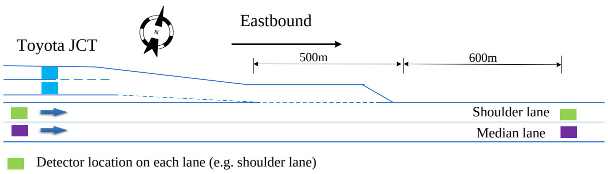


Fig.4 Toyota merge section

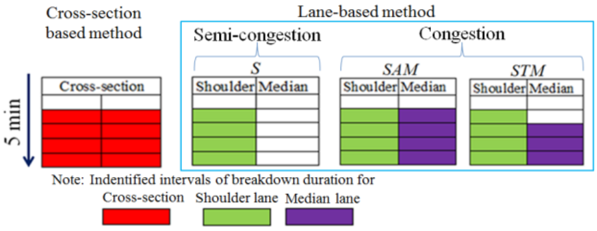


Fig.5 Classification of breakdown types at two-lane diverge and merge sections

3. BREAKDOWN PROBABILITY

(1) Lane-based breakdown identification

On multilane expressways, especially nearby diverge and merge sections, lane usage preference by drivers differs toward each lane. This kind of lane usage preference contributes to the fact that breakdown occurrences on each lane significantly vary. For example, the existence of semi-congestion states has already been reported at some diverge sections[10), 11)], where some lanes are congested whereas others are not. This semi-congestion state is likely to be inappropriately identified as breakdown occurrence on all lanes if applying the existing cross-section based method.

Therefore, a lane-based breakdown identification method is applied for modeling breakdown probabilities at diverge and merge sections. The identification results can be classified into two groups; semi-congestion and congestion states for expressway mainline.

As illustrated in **Fig.5**, semi-congestion state includes breakdown type *S* which is defined when breakdown only occurs on shoulder lane. Congestion state can be categorized into two types; shoulder and median lanes (*SAM*), first shoulder then median lanes (*STM*).

(2) Breakdown probability modeling

Breakdown probability estimation through applying Weibull distribution was originally proposed by Brilon[1)]. This study also adopts this idea for modeling. The cumulative distribution function of Weibull is expressed by Equation (1):

$$P(q) = 1 - e^{-\left(\frac{q}{\beta}\right)^{\alpha}} \qquad (1)$$

where, q is 5-min flow rate (veh/h), α and β are shape and scale parameters, respectively.

Tables 3 and **4** list the estimated parameters of breakdown probability models at diverge and merge bottlenecks respectively.

(3) Impacts of traffic condition characteristics

As for parameters α and β of breakdown probability models, impacts of traffic condition characteristics need to be further taken into consideration.

Table 3 Estimated parameters at diverge bottlenecks

Name	State	Sample	α	β	R square
Toyota diverge section	Semi-congestion	15	9	2512	0.352
	Congestion	194	12	4836	0.684
Toki diverge section	Congestion	36	13	4750	0.495
Komaki diverge section	Congestion	32	13	4620	0.539

Table 4 Estimated parameters at merge bottlenecks

Name	State	Sample	α	β	R square
Toyota merge section	Congestion	256	18	4422	0.661
Toki merge section	Congestion	196	15	4276	0.411
Komaki merge section	Congestion	37	14	4198	0.584

At diverge sections, diverge flow apparently has certain impact on mainline flow, and is likely to influence on breakdown occurrence. In addition, as aforementioned, lane specific breakdown characteristics have significant impacts on breakdown occurrence. Therefore, diverge rate *DR* and lane utilization rate *LUR* on median lane are chosen as candidates of the representative influencing factors on α and β. At Toyota diverge section, α and β are further estimated as functions of *DR* and *LUR* in Equations (2a) and (2b) whose R square values are 0.571 and 0.914, respectively. The *t*-values for influencing factors are also indicated in these equations.

$$\alpha = \underset{t=4.14}{3.42\times 10}\,DR - \underset{t=-0.503}{2.18} \qquad (2a)$$

$$\beta = -\underset{t=-2.64}{2.15\times 10^{3}}\,DR + \underset{t=8.44}{9.46\times 10^{3}}\,LUR + \underset{t=0.771}{8.73\times 10^{2}} \qquad (2b)$$

Similarly, at Toyota merge section, the estimation result is expressed in Equation (3). The impacts of merge rate *MR* and lane utilization rate *LUR* for median lane have been included.

$$\alpha = \underset{t=3.55}{3.01\times 10}\,MR - \underset{t=-2.91}{7.39}\,LUR + \underset{t=1.04}{4.98} \qquad (3a)$$

$$\beta = -\underset{t=-5.86}{3.31\times 10^{2}}\,MR - \underset{t=-4.82}{3.76\times 10^{2}}\,LUR + \underset{t=20.1}{4.87\times 10^{3}} \qquad (3b)$$

4. STOCHASTIC MODELING ON DISCHARGE FLOW RATE

This section introduces how to model the stochastic characteristics of discharge flow rate.

Fig.6 illustrates *DCF* distributions of several representative intervals at Toyota diverge section. At each interval *t*, Kolmogorov–Smirnov (K-S) test was conducted. *DCF* is found to follow Normal distribution according to K-S test values, e.g 0.037 at the first 5 minute. Normal distribution parameters at *t* is denoted as $\mu(t)$ and $\sigma(t)$.

In **Fig.6**, *DCF* distributions are found of lower values by comparing to the shape of *BDF* distribution. Furthermore, a general decreasing tendency can be identified with increase of breakdown duration. It suggests that when modeling *DCF*, the following impacts need to be carefully taken into consideration; 1) *BDF* value and 2) elapsed time *t* of breakdown duration.

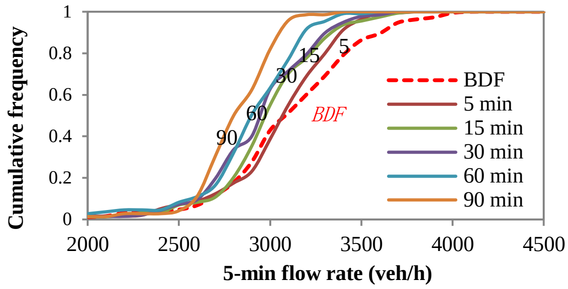


Fig.6 Empirical *DCF* distribution

(1) Impact of breakdown flow rate (*BDF*)

To investigate impact of *BDF*, every 10 *BDF* values are classified into a group considering sample size at this bottleneck as presented in **Fig.7**. For each group, mean *BDF* value is adopted as the representative value corresponding to *DCF* distribution of this group.

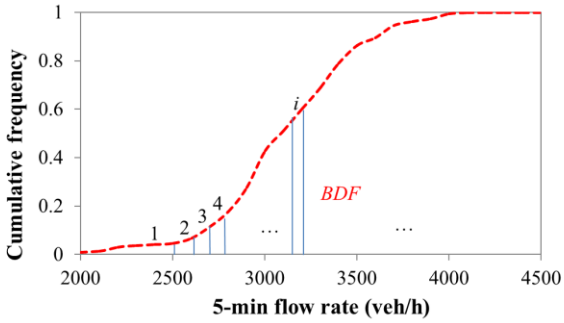
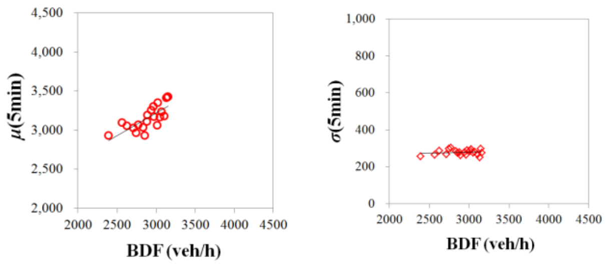


Fig.7 Classification of *BDF* distribution

Fig.8 presents the relationship between discharge and breakdown flow rates for the first 5 minute. A positive relationship can be identified between μ(5min) and *BDF* according to *t*-test at a 95% confidence level in **Table 5**. With respect to σ(5min), no significant impacts can be identified through *t*-test. Similar findings are also identified in each interval *t* which can be are expressed in Equation (4a) and (4b).



(a) Mean values **(b)** Standard deviations

Fig.8 Impact of *BDF* on *DCF* distribution

$$\mu(t) = a_t * BDF + b_t \tag{4a}$$

$$\sigma(t) = c_t \tag{4b}$$

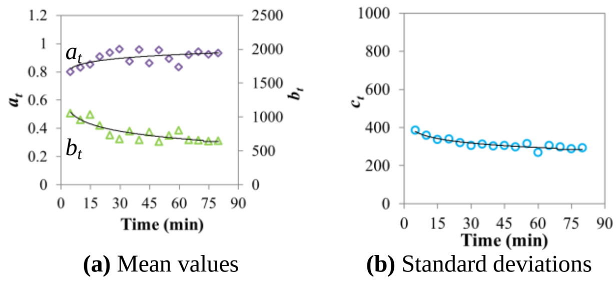
where, a_t, b_t and c_t are the coefficients, *t* is the elapsed time of breakdown duration.

Table 5 Regression result for the first 5 minute

t=5 minute	Mean values		Standard deviations	
	Coef.	*t*-value	Coef.	*t*-value
BDF	5.79×10^{-1}	5.09	8.81×10^{-3}	0.05
Constant	1.48×10^{3}	4.49	2.52×10^{2}	5.41
R square	0.577		0.015	

(2) Impact of elapsed time of breakdown duration (*t*)

The impact of *t* on *DCF* distribution is then analyzed. As aforementioned, $\mu(t)$ and $\sigma(t)$ can be expressed as functions of *BDF* values as shown in Equations (4a) and (4b). The estimated coefficients of $\mu(t)$ functions are illustrated in **Fig.9(a)**.



(a) Mean values **(b)** Standard deviations

Fig.9 Impact of *t* on *DCF* distribution

For both a_t and b_t, the changing tendencies are steep in the beginning and then get stable with increase of *t*. Similar tendency can be found for c_t as shown in **Fig.9(b)**. The characteristics of such tendencies suggest that a logarithmic function is probably applicable as expressed in Equations (5a), (5b) and (5c).

$$a_t = a_1 \ln(t) + a_2 \tag{5a}$$

$$b_t = b_1 \ln(t) + b_2 \tag{5b}$$

$$c_t = c_1 \ln(t) + c_2 \tag{5c}$$

Table 6 Estimated parameters for *DCF* models

Name	Geometry	a_1	a_2	b_1	b_2	c_1	c_2
Toyota diverge section	L_d=550m	-0.82×10^{-2}	5.20×10^{-1}	2.27×10^{2}	1.34×10^{3}	-2.31×10	4.34×10^{2}
Toki diverge section	L_d=300m	-1.61×10^{-1}	4.11×10^{-1}	3.35×10^{2}	1.16×10^{3}	-6.67×10	5.40×10^{2}
Komaki diverge section	L_d=250m	-1.38×10^{-1}	4.83×10^{-1}	2.40×10^{2}	9.58×10^{2}	-7.63×10	7.54×10^{2}
Toyota merge section	L_a=500m	-2.23×10^{-2}	1.40	5.37×10^{2}	3.95×10^{2}	-2.69×10	3.52×10^{2}
Toki merge section	L_a=400m	-1.14×10^{-1}	6.70×10^{-1}	2.26×10^{2}	1.34×10^{3}	-1.87×10	4.26×10^{2}
Komaki merge section	L_a=325m	-1.68×10^{-1}	8.83×10^{-1}	4.29×10^{2}	6.67×10^{2}	-1.27×10	3.92×10^{2}

Note: considering limited sample size, stochastic *DCF* model for semi-congestion state at Toyota diverge section has not been estimated.
L_d: deceleration lane length, L_a: acceleration lane length.

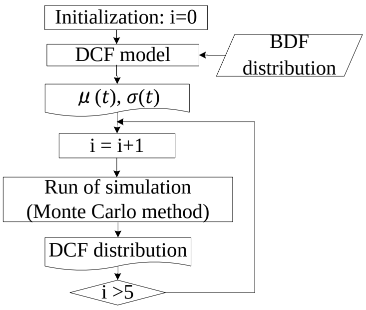


Fig.10 Flowchart of *DCF* model validation

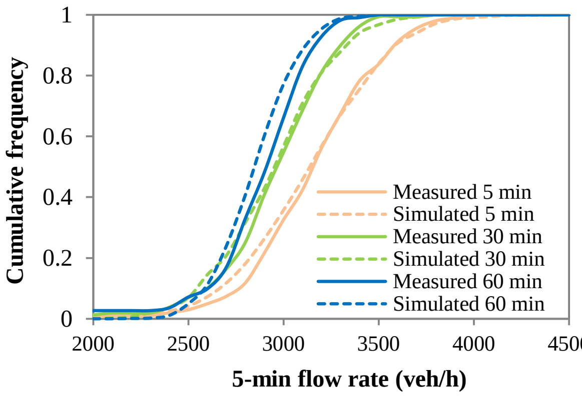


Fig.11 Validation of *DCF* model

where, a_1, a_2, b_1, b_2, c_1 and c_2 are coefficients for estimation.

(3) Developing *DCF* model

DCF model can finally be developed by incorporating Equations (5a), (5b) and (5c) into Equations (4a) and (4b) as shown in Equations (6a) and (6b). Its cumulative function is shown by Equation (7).

$$\mu(t) = (a_1 \ln(t) + a_2) * BDF + b_1 \ln(t) + b_2 \quad (6a)$$

$$\sigma(t) = c_1 \ln(t) + c_2 \quad (6b)$$

$$F(DCF(t)) = \frac{1}{\sqrt{2\pi}\sigma(t)} \int_0^{DCF} e^{-\frac{(x-\mu(t))^2}{2\sigma^2(t)}} dx \quad (7)$$

(4) Validating *DCF* model

The validation of the developed *DCF* model is conducted by following the flowchart shown in **Fig.10**. *DCF* distribution can be generated in a stochastic way through Monte Carlo method based on *BDF* distribution. At Toyota diverge section, it is found that *DCF* model can well reproduce *DCF* distribution according to *Mean Absolute Percentage Error*(*MAPE*) value, e.g. 4.13% at the first 5 minute as shown in **Fig.11**.

(5) Generalizing *DCF* model

The estimated parameters of the developed *DCF* models for diverge and merge bottlenecks are listed in **Table 6**. The difference in the site-specific models can be attributed to various geometric characteristics at each site. At diverge sections, a general *DCF* model is further developed by considering the impact of deceleration lane length as expressed in Equations (8a) to (8f).

$$a_1 = \underset{t=3.59}{2.28\times10^{-4}}\, L_d - \underset{t=-13.6}{2.10\times10^{-1}} \quad (8a)$$

$$a_2 = 4.71\times10^{-1} \quad (8b)$$

$$b_1 = 2.67\times10^{2} \quad (8c)$$

$$b_2 = \underset{t=5.13}{1.11}\, L_d + \underset{t=-5.87}{7.48\times10^{3}} \quad (8d)$$

$$c_1 = -\underset{t=-15.3}{1.78\times10^{-1}}\, L_d - \underset{t=-2.68}{1.20\times10^{-2}} \quad (8e)$$

$$c_2 = -\underset{t=2.61}{8.59\times10^{-1}}\, L_d + \underset{t=4.51}{8.91\times10^{2}} \quad (8f)$$

It can be found that longer deceleration lane length tends to result in greater *DCF* values as a_1, a_2, b_1 and b_2 influence on the coefficients of $\mu(t)$. In reality, it indicates that more gaps among the mainline flow due to longer L_d result in less impact from diverge flow on mainline traffic.

Similarly, at merge sections, a general *DCF* model is developed considering influence of acceleration lane length as illustrated in Equations (9a), (9b), (9c), (9d), (9e) and (9f).

$$a_1 = \underset{t=4.07}{1.90\times10^{-4}}\, L_a - \underset{t=-1.38}{7.81\times10^{-2}} \quad (9a)$$

$$a_2 = \underset{t=3.27}{1.49\times10^{-3}}\, L_a + \underset{t=0.785}{2.67\times10^{-1}} \quad (9b)$$

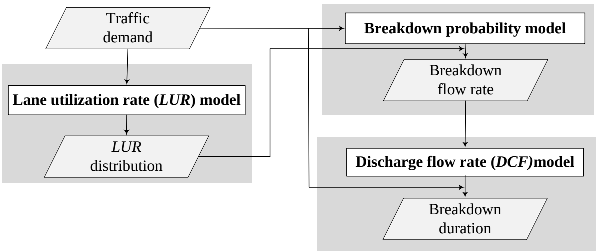


Fig.12 Framework of simulation

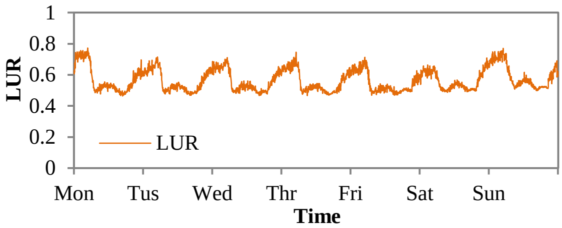


Fig.13 The simulated *LUR* on shoulder lane at Toyota diverge section

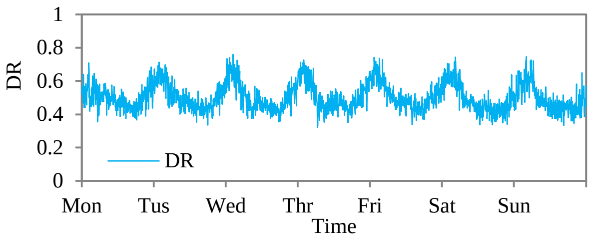


Fig.14 The measured *DR* at Toyota diverge section

$$b_1 = \underset{t=2.28}{4.58\times10^{-1}}\, L_a + \underset{t=1.91}{1.76\times10^{2}} \tag{9c}$$

$$b_2 = \underset{t=-2.69}{-3.27}\, L_a + \underset{t=2.35}{2.12\times10^{3}} \tag{9d}$$

$$c_1 = \underset{t=-2.75}{-2.92\times10^{-2}}\, L_a + \underset{t=2.19}{5.30} \tag{9e}$$

$$c_2 = \underset{t=-9.62}{-1.29\times10^{-1}}\, L_a + \underset{t=1.42}{4.52\times10^{2}} \tag{9f}$$

5. SIMULATION

By applying the developed models, breakdown phenomena can be simulated. Such a simulation tool would help plan countermeasures to alleviate breakdown occurrence like traffic demand management, lane usage recommendation.

Simulation is performed by incorporating models as demonstrated in **Fig.12**. Breakdown occurrence is reproduced by using breakdown probability model based on the simulated lane utilization rate from *LUR* model[12]. Then *DCF* model is applied to determine breakdown duration.

(1) Simulation procedure

Simulation is conducted in the way as follows. Firstly, traffic conditions like *LUR*, *DR* are set as they impact breakdown probability model as expressed in Equations (2a) and (2b). **Fig.13** demonstrates the simulated *LUR* on shoulder lane during one week at Toyota diverge section. **Fig.14** presents the measured *DR* during this week.

Secondly, diverge bottleneck capacity is computed as illustrated in **Fig.15** by using Monte Carlo method. Breakdown occurrence is then judged when capacity is lower than the arriving demand.

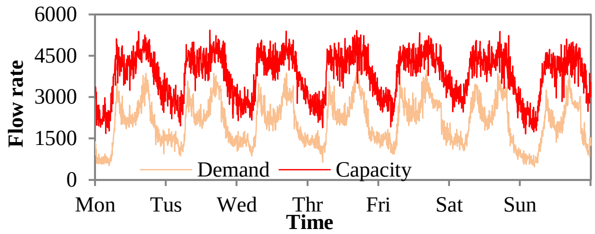


Fig.15 Capacity and demand at Toyota diverge section

Table 7 Number of breakdown occurrence

Measured	Simulated	
	Mean of 20 runs	Standard deviation
185	170	12

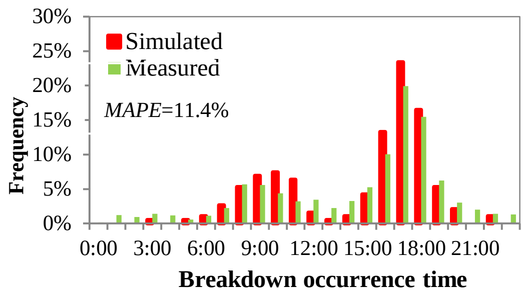


Fig.16 Time-dependent breakdown frequency

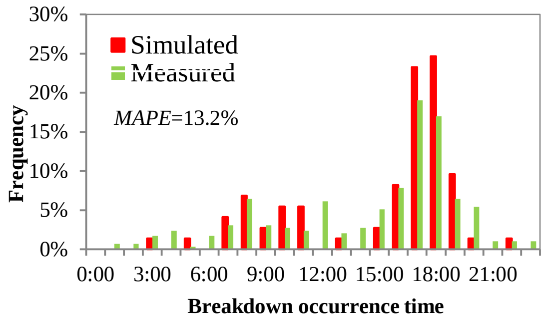


Fig.17 Breakdown frequency on weekdays/non-holidays

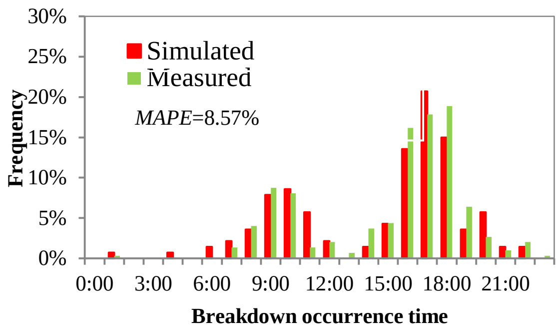


Fig.18 Breakdown frequency on weekends/holidays

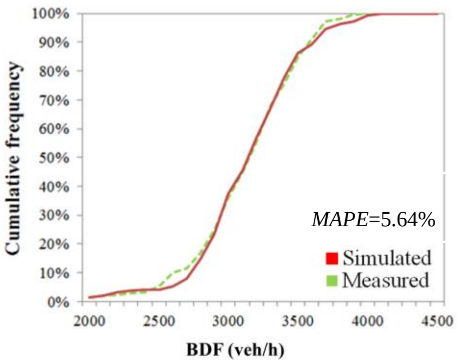


Fig.19 *BDF* distributions

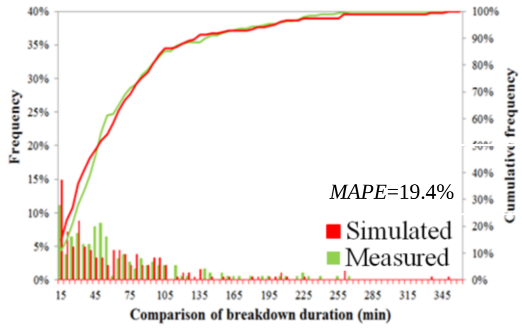


Fig.20 Breakdown duration

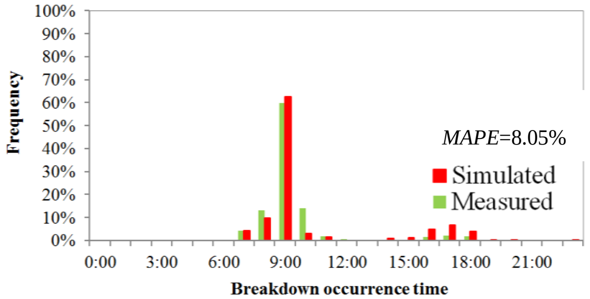


Fig.21 Time-dependent breakdown frequency at Toyota merge section

(2) Simulation result at diverge section

Several performance measures are adopted to evaluate simulation results.

(a) The number of breakdown occurrence

At Toyota diverge section, the number of breakdown occurrence is computed during the period (3/1/2008-12/31/2009) by aggregating the results of 20 simulation runs as listed in **Table 7**. Mean value is calculated as 170 which is close to the measured value 185.

(b) Time-dependent breakdown frequency

As illustrated in **Fig.16**, the measured breakdown occurrence concentrates on peak hours especially afternoon peak time around 16:00-18:00. This time-dependent frequency can be reproduced at an acceptable level (*MAPE*=11.4%). For morning peak hours, some underestimation exists.

In order to rationally evaluate simulation performance, the analysis period is classified into two categories: weekdays/non-holidays and weekends/holidays. By comparing **Fig.17** and **18**, it is found that simulation performs better for weekends/holidays by comparing its *MAPE* value to that of weekdays/non-holidays.

(c) The simulated *BDF*

Another significant output from simulation is the simulated *BDF* distribution which will impact *DCF* distributions. **Fig.19** illustrates the comparison between the simulated and the estimated *BDF* distributions. According to this comparison, simulation can perform well to reproduce *BDF* distribution at an acceptable level (*MAPE*=5.64%).

(d) Breakdown duration

Breakdown duration depends on 1) *DCF* which represents the ability of bottleneck to dissipate the queued vehicles and 2) the arriving demand from upstream. Breakdown duration is estimated by comparing the generated *DCF* value and the cumulative arriving demand from upstream. The interval is determined as breakdown end when the cumulative traffic flow can be dissipated.

Fig.20 presents the comparison between the measured and simulated breakdown duration. A general tendency can be simulated whereas some underestimation or overestimation exists.

(3) Simulation result at merge section

Simulation is also performed at merge section following the produces as described above.

Fig.21 presents the simulation result at Toyota merge bottleneck. At this merge bottleneck, breakdown occurrence concentrates during morning peak hours. It can be attributed to tide traffic phenomena on Tomei Expressway by comparing to the diverge section in **Fig.16**. In general, simulation can reproduce this time-dependent frequency at an acceptable level (*MAPE*=8.05%).

6. CONCLUSIONS AND FUTURE WORK

In this study, a methodology for stochastic modeling on the relationship between breakdown and discharge flow rates was developed. The findings can recommend for application in practice as follows:

When planning projects on intercity expressway, the management agency can predict breakdown occurrence and estimate breakdown duration by using the established stochastic models. Evaluation can be conducted at the planning stage for relief projects which aims at alleviating breakdown phenomena such as traffic demand management, signs to recommend lane usage. Breakdown occurrence and duration can be predicted by assuming change of traffic condition characteristics due to the planned relief projects.

This kind of evaluation can offer support when discussing effectiveness of the planned projects. Expressway performances like travel time reliability can be then further investigated as they are quite associated with breakdown occurrence and its duration.

Furthermore, capacity estimation of potential bottlenecks at diverge and merge sections can be performed by applying the general models which refers to site-specific geometries. This enables forecasting whether breakdown will be triggered under the future traffic conditions.

There are still several issues that should be further considered in future work. With respect to *DCF* distributions at different elapsed time *t*, sample size decreases with the increase of *t*. Such impacts on modeling need to be taken into account in future work. Also when generalizing *DCF* model, some weights can be further discussed based on number of breakdown events at each bottleneck. In addition, the analysis at three-lane sections need to be carried out in future.

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