Stochastic Modeling on the Relationship between Breakdown and Discharge Flow Rates at Intercity Expressway Bottlenecks

Danpeng MA¹, Hideki NAKAMURA² and Miho ASANO³

¹Student member of JSCE, Doctoral course student, Dept. of Civil Eng., Nagoya University
(C1-2(651) Furo-cho, Chikusa-ku, Nagoya 464-8603, Japan)
E-mail: ma@genv.nagoya-u.ac.jp
²Fellow of JSCE, Professor, Dept. of Civil Eng., Nagoya University
(C1-2(651) Furo-cho, Chikusa-ku, Nagoya 464-8603, Japan)
E-mail: nakamura@genv.nagoya-u.ac.jp
³Member of JSCE, Dr. Eng., Assistant Professor, Dept. of Civil Eng., Nagoya University
(C1-2(651) Furo-cho, Chikusa-ku, Nagoya, Nagoya 464-8603, Japan)
Email: asano@genv.nagoya-u.ac.jp

Breakdown and discharge flow rates are significant indicators to represent breakdown phenomena on intercity expressways. This paper proposes a methodology for stochastic modeling on the relationship between them based on three diverge and three merge bottlenecks on intercity expressways in central Japan. Firstly, breakdown probability models are developed based on breakdown flow rates which are extracted through the lane-based breakdown identification method. Secondly, discharge flow rates are modeled in a stochastic way by considering their relationship with breakdown flow rates. Furthermore, the relationship between breakdown and discharge flow rates is generalized by referring to site-specific geometries which enables the breakdown estimation at potential bottlenecks. Finally the developed models are applied to reproduce breakdown phenomena through Monte Carlo method. The results demonstrate that the developed models can well reproduce breakdown phenomena.

Key Words: breakdown flow rate, discharge flow rate, intercity expressway, diverge section, merge section

1. INTRODUCTION

The concept of capacity plays an important role in the planning, design, and operation stages of intercity expressway facilities. Breakdown and discharge flow rates represent two distinct aspects of bottleneck capacity which are significant indicators to represent breakdown phenomena.

With respect to breakdown flow rate (BDF), considerable number of studies have adopted breakdown probability model¹, ², ³ to describe and quantify it in light of its stochastic nature. As for discharge flow rate (DCF), it is typically characterized in a deterministic manner by the existing studies. In other words, after breakdown occurs, it is assumed that the queue discharges at a deterministic flow rate. Based on field data, some studies have indicated that discharge flow rate is also stochastic in nature⁴, ⁵. However, few efforts² have been made on modeling discharge flow rate by considering its stochastic characteristic.

In addition, the relationship between breakdown and discharge flow rates has not been well understood, which significantly impacts intercity expressway performance accompanied with breakdown phenomena. Limited studies on the investigation of this relationship is to estimate capacity drop due to breakdown occurrence in a deterministic way⁶, ⁷, ⁸, ⁹. However, the stochastic nature of discharge flow rate has not been interpreted. Therefore, this paper proposes a methodology for stochastic modeling on the relationship between breakdown and discharge flow rates based on three diverge and three merge bottlenecks on intercity expressways in central Japan.

The rest of this paper is organized as follows. In section 2, the features of the study sites are presented. Lane based breakdown identification method is applied for modeling breakdown probability in section 3. Then section 4 introduces the methodology for stochastic modeling on
discharge flow rate by considering its relationship to breakdown flow rate. In section 5, Monte Carlo simulation is carried out to reproduce breakdown phenomena by applying the developed models. Finally, section 6 offers recommendations for practical applications and suggestions for future research.

2. DESCRIPTIONS OF TEST BED

In the intercity expressway network of central Japan, there exist breakdown phenomena at various bottlenecks at sag, diverge and merge sections. This study focuses on diverge sections and merge sections considering lane usage preference by drivers due to the influence of diverge and merge traffic flow there.

As illustrated in Fig.1, three diverge bottlenecks and three merge bottlenecks are chosen as test beds. Detailed descriptions of the test bed sites are listed in Tables 1 and 2. All the diverge and merge sections have left-side off-ramps. With respect to lane configuration, all of their mainlines have two lanes, namely shoulder lane and median lane (for Toyota diverge and merge sections on Tomei Expressway, their mainlines have been broadened to three lanes by fully utilizing shoulder from 10/21/2011).

Representative geometries, lane configurations and detector locations are presented in Fig.2 and 4 at Toyota diverge and merge sections. Fig.3(a) and 3(b) present more distinct and direct the overlooks of Toyota diverge section layout. At this section, frequent breakdown events have been observed due to high diverge traffic flow which aims at using Shinmeishin Expressway after its operation in March, 2008.

Along the mainlines and off-ramps of these bottlenecks, double-loop detectors are installed approximately every 2 kilometers. Thanks to Central Nippon Expressway Company Limited (NEXCO), traffic flow data records around these bottlenecks are available. This study investigates recurrent breakdown phenomena during the period from 3/1/2008 to 12/31/2009.

Traffic flow rates and average speeds on each lane were measured and aggregated at a 5-minute interval. Breakdown events due to other non-recurrent causes such as roadway maintenance works and accidents were excluded. Some preliminary studies have been conducted on constructing flow-speed diagrams at each detector, which help identify bottleneck locations.

---

**Table 1 Diverge bottleneck descriptions**

<table>
<thead>
<tr>
<th>Name</th>
<th>Expressway</th>
<th>Direction</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toyota diverge section</td>
<td>Tomei</td>
<td>Westbound</td>
<td>Toyota JCT</td>
</tr>
<tr>
<td>Toki diverge section</td>
<td>Chuo</td>
<td>Southbound</td>
<td>Toki JCT</td>
</tr>
<tr>
<td>Komaki diverge section</td>
<td>Tomei</td>
<td>Westbound</td>
<td>Komaki JCT</td>
</tr>
</tbody>
</table>

**Table 2 Merge bottleneck descriptions**

<table>
<thead>
<tr>
<th>Name</th>
<th>Expressway</th>
<th>Direction</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toyota merge section</td>
<td>Tomei</td>
<td>Eastbound</td>
<td>Toyota JCT</td>
</tr>
<tr>
<td>Toki merge section</td>
<td>Chuo</td>
<td>Northbound</td>
<td>Toki JCT</td>
</tr>
<tr>
<td>Komaki merge section</td>
<td>Tomei</td>
<td>Eastbound</td>
<td>Komaki JCT</td>
</tr>
</tbody>
</table>

---

**Fig.2 Toyota diverge section**

**Fig.3 Overlooks of Toyota diverge section layout**

**Fig.4 Toyota merge section**

---
3. BREAKDOWN PROBABILITY

(1) Lane-based breakdown identification
On multilane expressways, especially nearby diverge and merge sections, lane usage preference by drivers differs toward each lane. This kind of lane usage preference contributes to the fact that breakdown occurrences on each lane significantly vary. For example, the existence of semi-congestion states has already been reported at some diverge sections\(^{(10),\ (11)}\), where some lanes are congested whereas others are not. This semi-congestion state is likely to be inappropriately identified as breakdown occurrence on all lanes if applying the existing cross-section based method.

Therefore, a lane-based breakdown identification method is applied for modeling breakdown probabilities at diverge and merge sections. The identification results can be classified into two groups; semi-congestion and congestion states for expressway mainline.

As illustrated in Fig.5, semi-congestion state includes breakdown type \( S \) which is defined when breakdown only occurs on shoulder lane. Congestion state can be categorized into two types; shoulder and median lanes (SAM), first shoulder then median lanes (STM).

(2) Breakdown probability modeling
Breakdown probability estimation through applying Weibull distribution was originally proposed by Brilon\(^{(3)}\). This study also adopts this idea for modeling. The cumulative distribution function of Weibull is expressed by Equation (1):

\[
P(q) = 1 - e^{-\left(\frac{q}{\beta}\right)^\alpha} \quad (1)
\]

where, \( q \) is 5-min flow rate (veh/h), \( \alpha \) and \( \beta \) are shape and scale parameters, respectively.

Tables 3 and 4 list the estimated parameters of breakdown probability models at diverge and merge bottlenecks respectively.

(3) Impacts of traffic condition characteristics
As for parameters \( \alpha \) and \( \beta \) of breakdown probability models, impacts of traffic condition characteristics need to be further taken into consideration.

At diverge sections, diverge flow apparently has certain impact on mainline flow, and is likely to influence on breakdown occurrence. In addition, as aforementioned, lane specific breakdown characteristics have significant impacts on breakdown occurrence. Therefore, diverge rate \( DR \) and lane utilization rate \( LUR \) on median lane are chosen as candidates of the representative influencing factors on \( \alpha \) and \( \beta \). At Toyota diverge section, \( \alpha \) and \( \beta \) are further estimated as functions of \( DR \) and \( LUR \) in Equations (2a) and (2b) whose R square values are 0.571 and 0.914, respectively. The \( t \)-values for influencing factors are also indicated in these equations.

\[
\alpha = 3.42 \times 10^3 DR - 2.18 \quad (2a)
\]

\[
\beta = -2.15 \times 10^3 DR + 9.46 \times 10^3 \ LUR + 8.73 \times 10^3 \quad (2b)
\]

Similarly, at Toyota merge section, the estimation result is expressed in Equation (3). The impacts of merge rate \( MR \) and lane utilization rate \( LUR \) for median lane have been included.

\[
\alpha = 3.01 \times 10^3 MR - 7.39 \ LUR + 4.98 \quad (3a)
\]

\[
\beta = -3.31 \times 10^2 MR - 3.76 \times 10^2 \ LUR + 4.87 \times 10^3 \quad (3b)
\]

4. STOCHASTIC MODELING ON DISCHARGE FLOW RATE

This section introduces how to model the stochastic characteristics of discharge flow rate.
Fig. 6 illustrates DCF distributions of several representative intervals at Toyota diverge section. At each interval \( t \), Kolmogorov–Smirnov (K-S) test was conducted. DCF is found to follow Normal distribution according to K-S test values, e.g. 0.037 at the first 5 minute. Normal distribution parameters at \( t \) is denoted as \( \mu(t) \) and \( \sigma(t) \).

In Fig. 6, DCF distributions are found of lower values by comparing to the shape of BDF distribution. Furthermore, a general decreasing tendency can be identified with increase of breakdown duration. It suggests that when modeling DCF, the following impacts need to be carefully taken into consideration: 1) BDF value and 2) elapsed time \( t \) of breakdown duration.

(1) Impact of breakdown flow rate (BDF)

To investigate impact of BDF, every 10 BDF values are classified into a group considering sample size at this bottleneck as presented in Fig. 7. For each group, mean BDF value is adopted as the representative value corresponding to DCF distribution of this group.

Fig. 8 presents the relationship between discharge and breakdown flow rates for the first 5 minute. A positive relationship can be identified between \( \mu(5\text{min}) \) and BDF according to \( t \)-test at a 95% confidence level in Table 5. With respect to \( \sigma(5\text{min}) \), no significant impacts can be identified through \( t \)-test. Similar findings are also identified in each interval \( t \) which can be are expressed in Equation (4a) and (4b).

\[
\mu(t) = a_1 \times BDF + b_1 \quad (4a)
\]
\[
\sigma(t) = c_1 \quad (4b)
\]

where, \( a_1, b_1 \) and \( c_1 \) are the coefficients, \( t \) is the elapsed time of breakdown duration.

(2) Impact of elapsed time of breakdown duration (t)

The impact of \( t \) on DCF distribution is then analyzed. As aforementioned, \( \mu(t) \) and \( \sigma(t) \) can be expressed as functions of BDF values as shown in Equations (4a) and (4b). The estimated coefficients of \( \mu(t) \) functions are illustrated in Fig. 9(a).

For both \( a_1, b_1 \), the changing tendencies are steep in the beginning and then get stable with increase of \( t \). Similar tendency can be found for \( c_1 \) as shown in Fig. 9(b). The characteristics of such tendencies suggest that a logarithmic function is probably applicable as expressed in Equations (5a), (5b) and (5c).

\[
a_1 = a_t \ln(t) + a_2 \quad (5a)
\]
\[
b_1 = b_1 \ln(t) + b_2 \quad (5b)
\]
\[
c_1 = c_1 \ln(t) + c_2 \quad (5c)
\]
Table 6 Estimated parameters for DCF models

<table>
<thead>
<tr>
<th>Name</th>
<th>Geometry</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$c_1$</th>
<th>$c_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toyota diverge section</td>
<td>$L_d=550m$</td>
<td>-0.82×10^{-2}</td>
<td>5.20×10^{-1}</td>
<td>2.27×10^{2}</td>
<td>1.34×10^{3}</td>
<td>-2.31×10^{4}</td>
<td>4.34×10^{2}</td>
</tr>
<tr>
<td>Toki diverge section</td>
<td>$L_d=300m$</td>
<td>-1.61×10^{-1}</td>
<td>4.11×10^{-1}</td>
<td>3.35×10^{2}</td>
<td>1.16×10^{3}</td>
<td>-6.67×10^{4}</td>
<td>5.40×10^{2}</td>
</tr>
<tr>
<td>Komaki diverge section</td>
<td>$L_d=250m$</td>
<td>-1.38×10^{-1}</td>
<td>4.83×10^{-1}</td>
<td>2.40×10^{2}</td>
<td>9.58×10^{2}</td>
<td>-7.63×10^{3}</td>
<td>7.54×10^{2}</td>
</tr>
<tr>
<td>Toyota merge section</td>
<td>$L_m=500m$</td>
<td>-2.23×10^{-1}</td>
<td>1.40</td>
<td>5.37×10^{2}</td>
<td>3.95×10^{3}</td>
<td>-2.69×10^{4}</td>
<td>3.52×10^{2}</td>
</tr>
<tr>
<td>Toki merge section</td>
<td>$L_m=400m$</td>
<td>-1.14×10^{-1}</td>
<td>6.70×10^{-1}</td>
<td>2.26×10^{2}</td>
<td>1.34×10^{3}</td>
<td>-1.87×10^{3}</td>
<td>4.26×10^{2}</td>
</tr>
<tr>
<td>Komaki merge section</td>
<td>$L_m=325m$</td>
<td>-1.68×10^{-1}</td>
<td>8.83×10^{-1}</td>
<td>4.29×10^{2}</td>
<td>6.67×10^{3}</td>
<td>-1.27×10^{4}</td>
<td>3.92×10^{2}</td>
</tr>
</tbody>
</table>

Note: considering limited sample size, stochastic DCF model for semi-congestion state at Toyota diverge section has not been estimated. $L_d$: deceleration lane length, $L_m$: acceleration lane length.

(5) Generalizing DCF model

The estimated parameters of the developed DCF models for diverge and merge bottlenecks are listed in Table 6. The difference in the site-specific models can be attributed to various geometric characteristics at each site. At diverge sections, a general DCF model is further developed by considering the impact of deceleration lane length as expressed in Equations (8a) to (8f).

$$a_1 = 2.28 \times 10^{-4} L_d - 2.10 \times 10^{-1} \quad (8a)$$
$$a_2 = 4.71 \times 10^{-1} \quad (8b)$$
$$b_1 = 2.67 \times 10^2 \quad (8c)$$
$$b_2 = 1.11 L_d + 7.48 \times 10^3 \quad (8d)$$
$$c_1 = -1.78 \times 10^{-1} L_d - 1.20 \times 10^{-2} \quad (8e)$$
$$c_2 = -8.59 \times 10^{-1} L_d + 8.91 \times 10^2 \quad (8f)$$

It can be found that longer deceleration lane length tends to result in greater DCF values as $a_1$, $a_2$, $b_1$ and $b_2$ influence on the coefficients of $\mu(t)$. In reality, it indicates that more gaps among the mainline flow due to longer $L_d$ result in less impact from diverge flow on mainline traffic.

Similarly, at merge sections, a general DCF model is developed considering influence of acceleration lane length as illustrated in Equations (9a), (9b), (9c), (9d), (9e) and (9f).

$$a_1 = 1.90 \times 10^{-4} L_a - 7.81 \times 10^{-2} \quad (9a)$$
$$a_2 = 1.49 \times 10^{-3} L_a + 2.67 \times 10^{-1} \quad (9b)$$

where, $a_1$, $a_2$, $b_1$, $b_2$, $c_1$ and $c_2$ are coefficients for estimation.

(3) Developing DCF model

DCF model can finally be developed by incorporating Equations (5a), (5b) and (5c) into Equations (4a) and (4b) as shown in Equations (6a) and (6b). Its cumulative function is shown by Equation (7).

$$\mu(t) = (a_1 \ln(t) + a_2) \times BDF + b_1 \ln(t) + b_2$$
$$\sigma(t) = c_1 \ln(t) + c_2$$

$$F(DCF(t)) = \frac{1}{\sqrt{2\pi}\sigma(t)} \int_0^{DCF} e^{-\frac{(x-\mu(t))^2}{2\sigma(t)^2}} dx$$

(4) Validating DCF model

The validation of the developed DCF model is conducted by following the flowchart shown in Fig.10. DCF distribution can be generated in a stochastic way through Monte Carlo method based on $BDF$ distribution. At Toyota diverge section, it is found that DCF model can well reproduce DCF distribution according to Mean Absolute Percentage Error (MAPE) value, e.g. 4.13% at the first 5 minute as shown in Fig.11.
5. SIMULATION

By applying the developed models, breakdown phenomena can be simulated. Such a simulation tool would help plan countermeasures to alleviate breakdown occurrence like traffic demand management, lane usage recommendation.

Simulation is performed by incorporating models as demonstrated in Fig.12. Breakdown occurrence is reproduced by using breakdown probability model based on the simulated lane utilization rate from LUR model\(^{(2)}\). Then DCF model is applied to determine breakdown duration.

(1) Simulation procedure

Simulation is conducted in the way as follows. Firstly, traffic conditions like LUR, DR are set as they impact breakdown probability model as expressed in Equations (2a) and (2b). Fig.13 demonstrates the simulated LUR on shoulder lane during one week at Toyota diverge section. Fig.14 presents the measured DR during this week.

Secondly, diverge bottleneck capacity is computed as illustrated in Fig.15 by using Monte Carlo method. Breakdown occurrence is then judged when capacity is lower than the arriving demand.

\[
b_1 = 4.58 \times 10^{-1} L_u + 1.76 \times 10^2 \quad (9c)
\]

\[
b_2 = -3.27 L_u + 2.12 \times 10^3 \quad (9d)
\]

\[
c_1 = -2.92 \times 10^{-2} L_u + 5.30 \quad (9e)
\]

\[
c_2 = -1.29 \times 10^{-1} L_u + 4.52 \times 10^2 \quad (9f)
\]
two categories: weekdays/non-holidays and weekends/holidays. By comparing Fig.17 and 18, it is found that simulation performs better for weekends/holidays by comparing its MAPE value to that of weekdays/non-holidays.

(c) The simulated BDF
Another significant output from simulation is the simulated BDF distribution which will impact DCF distributions. Fig.19 illustrates the comparison between the simulated and the estimated BDF distributions. According to this comparison, simulation can perform well to reproduce BDF distribution at an acceptable level (MAPE=5.64%).

(d) Breakdown duration
Breakdown duration depends on 1) DCF which represents the ability of bottleneck to dissipate the queued vehicles and 2) the arriving demand from upstream. Breakdown duration is estimated by comparing the generated DCF value and the cumulative arriving demand from upstream. The interval is determined as breakdown end when the cumulative traffic flow can be dissipated.

Fig.20 presents the comparison between the measured and simulated breakdown duration. A general tendency can be simulated whereas some underestimation or overestimation exists.

(3) Simulation result at merge section
Simulation is also performed at merge section following the produces as described above.

Fig.21 presents the simulation result at Toyota merge bottleneck. At this merge bottleneck, breakdown occurrence concentrates during morning peak hours. It can be attributed to tide traffic phenomena on Tomei Expressway by comparing to the diverge section in Fig.16. In general, simulation can reproduce this time-dependent frequency at an acceptable level (MAPE=8.05%).

6. CONCLUSIONS AND FUTURE WORK
In this study, a methodology for stochastic modeling on the relationship between breakdown and discharge flow rates was developed. The findings can recommend for application in practice as follows:

When planning projects on intercity expressway, the management agency can predict breakdown occurrence and estimate breakdown duration by using the established stochastic models. Evaluation can be conducted at the planning stage for relief projects which aims at alleviating breakdown phenomena such as traffic demand management, signs to recommend lane usage. Breakdown
occurrence and duration can be predicted by assuming change of traffic condition characteristics due to the planned relief projects.

This kind of evaluation can offer support when discussing effectiveness of the planned projects. Expressway performances like travel time reliability can be then further investigated as they are quite associated with breakdown occurrence and its duration.

Furthermore, capacity estimation of potential bottlenecks at diverge and merge sections can be performed by applying the general models which refers to site-specific geometries. This enables forecasting whether breakdown will be triggered under the future traffic conditions.

There are still several issues that should be further considered in future work. With respect to DCF distributions at different elapsed time $t$, sample size decreases with the increase of $t$. Such impacts on modeling need to be taken into account in future work. Also when generalizing DCF model, some weights can be further discussed based on number of breakdown events at each bottleneck. In addition, the analysis at three-lane sections need to be carried out in future.

ACKNOWLEDGMENT: The authors would like to express their gratitude to Central Nippon Expressway Company Limited for their invaluable assistance for this research.

REFERENCES


(Received May 7, 2013)