Integrated Approach for Location-Routing Problem using Branch-and-Price Method

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It is proved that the location-routing problem could reduce the total cost over long planning period in city logistics. Since the classical facility location and vehicle routing problems are NP-hard, the computational process must be well-organized. An exact solution of LRP with time windows using branch-and-price algorithm is proposed in this study. The decomposition by relaxing integrality separates the problem into a master problem and a sub problem. The problem is iteratively solved between simplex algorithm in master problem and elementary shortest path problem with resource constraint in sub problem until the non-negative reduced cost column exist.

Keyword: *Location-routing problem, branch-and-price, linear programming, column generation*

1. INTRODUCTION

Delivery goods with an effective manner has remained a key aim of transportation and logistics management throughout decades. Solving mathematical problems early in the planning horizon provides benefits and positive impacts for both operators and socials. Traditionally, determining the depot location and vehicle routing are carried out at totally different stages; strategic and operational stages, respectively. Routing problems can be solved more frequently in short term while depot must be located earlier and during the long term planning. However, this separation is arguable since the integration of them can lower total cost and vehicle traveling distances. Dealing with large number of customers in routing and depot location simultaneously requires more computational time and resources. A proper and effective method must be developed for the optimization model. This study, therefore, proposes the integrated approach for location-routing problem by using branch-and-price method.

2. LOCATION-ROUTING PROBLEM

Determining a location of business center, classically called facility location problem, attracted much attention of mathematician and scientist for several decades. Since it involves in a particular stage from upstream to downstream, it is considered one of the most essential step in supply chain management, i.e. setting up factory, warehouse, retail store, or public services, i.e. hospital, police station, fire station[1)].

The facility location problem determines the place(s) where the total cost of satisfying customer's demand is minimum with a set of constraints. Those costs include fixed cost to establish facility and distribution cost. There are two types of distribution to be considered; direct trip and tour trip as shown in **Fig.1**. The direct trip requires vehicle visiting only one customer and return base like a fire service. The tour trip, on the other hand, requires vehicle to visit more than one customer before return to base such as a postman[1)]. If the service is direct trip, it is considered a location-allocation problem. If the service is a tour trip, it means Location-Routing Problem (LRP)[2)].

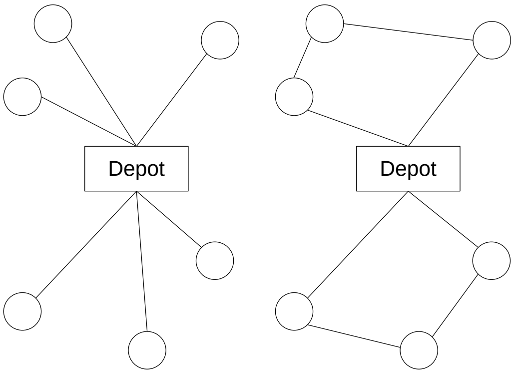

Fig.1 Direct Trip (left) and Tour Trip (right)

However, it is argued that combination of location problem and vehicle routing problem is impractical since they are in different planning framework. Setting up the depot location belongs to strategic stage while routing is in a tactical or operational stage which can be calculated on the daily basis. It is inappropriate to calculate them together. Nonetheless, it was proved that the combination of location-routing problem reduced the cost over the long term horizon[3) 4) 5)].

Since LRP is NP-hard problem, most of the studies are significantly referred to heuristics solution[2) 6)]. All methodologies decompose the problem into ordinary location, allocation, and routing problems, and find the solution of each part repeatedly, iteratively, or simultaneously[7)]. Nagy and Salhi[6)] categorize those techniques into 4 methods, namely sequential, clustering-base, iterative, and hierarchical methods. Additional analysis is required to ensure and yield a good quality solution, such as robustness and debility analysis[8).]

Exact solution studies which are successful for more specific cases and provide significant insights into problems exist fewer in LRP. A classical branch-and-bound algorithm, consisted of systematic of all candidate solutions, is firstly used by Larporte and Nobert[9)]. It is applied the subtour elimination and chain barring constraints later[10)]. More complex algorithm is developed lately by using more sophisticated integer linear programming called branch-and-price and branch-and-cut algorithms[11) 12) 13) 14)]. It is observed the development of solving LRP comes from the successful of solving VRP in different transportation problem types, such as time windows, nature of demand/supply, pickup and delivery, capacity or distance constraints[6)]. However, only two literatures review on LRP with time windows using heuristic approach. Solving LRP with time windows using exact solution has not been reviewed before. The contribution of this study therefore is to review and explore a strategic framework in solving LRP with time windows. The systematic method of branch-and-price algorithm decomposed the restricted master problem and sub problem is proposed together with additional accelerating steps.

3. FORMULATION

The LRP can be stated as follow: we define the graph *G = (V, A)* where *V* is the set of nodes and *A* is the set of arcs. Let *I* denote the set of customers and *M* denote the set of potential depot locations. For each $i \in I$, let d_i be the customers' demand and x_{ijk} be the binary variable equal to 1 if route *ij* is used by vehicle *k* which its capacity equal to *q*. For each $m \in M$, let f_m be the cost of set up the depot at site *m* and Q_m be the depot capacity. The decision variable y_m is equal to 1 if depot *m* is opened. In addition, c_{ijk} is the operating cost matrix between depots to customers and customers to customers. Let K_j be the set of vehicles located at depot *m* and *K* be the set of all vehicles. Given a set of customers and depots, the LRP is to find a set of opened depots and a set of routes of minimal total cost and meet customers' demand. The vehicle must start and return to the same depot and visit the customer exactly once. The standard formulation for the LRP is shown below;

$$\min \sum_{m \in M} f_m y_m + \sum_{i \in A} \sum_{j \in A} \sum_{k \in K} c_{ijk} x_{ijk} \tag{1}$$

subject to

$$\sum_{k \in K} \sum_{j \in V} x_{ijk} = 1 \qquad \forall i \in I \tag{2}$$

$$\sum_{j \in V} x_{ljk} = 1 \qquad \forall l \in M, \ \forall k \in K \tag{3}$$

$$\sum_{i \in V} x_{ilk} = 1 \qquad \forall l \in M, \ \forall k \in K \tag{4}$$

$$\sum_{i \in V} x_{ihk} - \sum_{j \in V} x_{hjk} = 0 \qquad \forall h \in I, \ \forall k \in K \tag{5}$$

$$\sum_{i \in I} d_i x_{ik} \le q \qquad \forall k \in K \tag{6}$$

$$\sum_{k \in K} \sum_{i \in I} d_i v_{ik} \le Q_m y_m \qquad \forall m \in M, \ K = \bigcup_{m \in M} K_m \tag{7}$$

$$s_{ik} + t_{ij} - s_{jk} \le (1 - x_{ijk}) M_{ijk} \qquad \forall i, j \in A, \ \forall k \in K \tag{8}$$

$$a_i \le s_{ik} \le b_i^{'} \qquad \forall i \in V, \forall k \in K \tag{9}$$

$$x_{ijk} \in \{0,1\} \qquad \forall i, j \in A, \; \forall k \in K \qquad (10)$$

$$y_m \in \{0,1\} \qquad \forall m \in M \qquad (11)$$

Constraint (2) indicates each customer must be visited exactly once. Constraints (3) and (4) are flow conservation constraints meaning that if vehicle leaves the depot *l*, it must come return to the same depot while constraint (5) requires the vehicle to leave the customer *h* after visited it. Constraints (6) and (7) are vehicle and depot capacity constraints, respectively. Constraint (8) is time windows constraint implying that if vehicle goes from *i* to *j*, it must serve customer *i* before *j*. Constraint (9) is the relaxed time windows [a_i, b_i']. Constraints (10) and (11) refer the binary decision variables if the arc and depot are selected, respectively.

4. MATHAMETIC ALGORITHM

(1) Column Generation

It is observed from the model that the feasible solutions grow exponentially with the number of depots and customers. By relaxing linear equation and solving with branch-and-bound algorithm directly is totally difficult or unmanageable. Hence, the idea of decomposition by using column generation is introduced to separate the problem in to a master problem and a sub Problem. The master problem is a linear programming with an integer relaxation while the sub problem (or pricing problem) is a dynamic programming called elementary shortest path problem with resource constraints. At each iteration, the master problem determines new multiplier sending to the sub problem to add new columns. After obtain the solution of linear relaxation, the optimal integer solution can be solved by using branch-and-bound[15].

The master problem can be described as follow: Let *P* is a set of feasible paths. c_p and a_{ip} are the operating cost of path *p* and number of time path *p* serves customer *i* respectively. The decision variable y_p is equal to 1 if path *p* is selected or 0 otherwise. f_m and y_m are depot fixed cost and decision variable as mentioned before in equation (1). The initial solution of master problem starts with a feasible solution that meets all constraints[16]. That is the *depot-i-depot* path. By optimizing master problem, the current optimal objective function and dual prices (π_i *and* μ_{im}) are obtained. The master problem can be formulated as;

$$\min \sum_{m \in M} f_m y_m + \sum_{p \in P} c_p y_p \qquad (12)$$

subject to

$$\sum_{p \in P} a_{ip} y_p \geq 1 \qquad \forall i \in C \qquad (13)$$

$$y_m - y_p \geq 0 \qquad \forall m \in M, \forall p \in P \qquad (14)$$

$$y_p = \{0,1\} \qquad \forall p \in P \qquad (15)$$

$$y_m = \{0,1\} \qquad \forall m \in M \qquad (16)$$

The sub problem can be solved by an Elementary Shortest Path Problem with Resource Constraints (ESPPRC). It has more advantage than ordinary Shortest Path Problem with Resource Constraints since it reduces the duality gap and can be applied with some specify problem in column generation[17]. The cost explicit in the objective function (17) $\bar{c}_{ij}$ is equal to $c_{ij} - \sum_{p \in P} a_{ip}(\pi_i - \mu_{im})$, called reduced cost where π_i and μ_{im} are dual variables for constraint (13) and corresponding to customer *i* and depot *m*, respectively. The idea of the sub problem is to find the path with minimal reduced cost. The process ends when the optimal objective is equal to non-negative value, or zero. The sub problem can be formulated as;

$$\min \sum_{i \in A} \sum_{j \in A} \bar{c}_{ij} x_{ij} \qquad (17)$$

subject to

$$\sum_{i \in V} x_{lj} = 1 \qquad \forall l \in M \qquad (18)$$

$$\sum_{i \in V} x_{il} = 1 \qquad \forall l \in M \qquad (19)$$

$$\sum_{i \in V} x_{ih} - \sum_{j \in V} x_{hj} = 0 \qquad \forall h \in I \qquad (20)$$

$$\sum_{i \in I} d_i x_i \leq q \qquad (21)$$

$$s_{ik} + t_{ij} - s_{jk} \leq (1 - x_{ijk}) M_{ijk} \qquad \forall i, j \in A, \; \forall k \in K \qquad (22)$$

$$a_i \leq s_{ik} \leq b_i' \qquad \forall i \in V, \forall k \in K \qquad (23)$$

$$x_{ij} \in \{0,1\} \qquad \forall i, l \in A \qquad (24)$$

A labeling algorithm is used in ESPPRC. The individual labels are assigned to each node indicated the cost and resource consumption of each paths. At each iteration, all new labels are extended toward possible successor node until no new labels are created. To limit the proliferation of nodes, the dominance rules are introduced to compare 4 criteria of candidate nodes, including less visited nodes, less time, less resource consumption, and less cost[18]. It helps to remove paths that do not extend further. Nonetheless, since the sub problem consume most of computational time and resources, the stabilization and acceleration plan are needed. Qureshi et al[18] recommends stopping the iteration if more than 500 feasible solutions with negative reduced cost were found or more than 100 columns were generated to the master problem. Feillet et al[17] illustrates the procedure of ESPPRC as follow;

Initialization
$\wedge_p \leftarrow \{(0, \ldots, 0)\}$
for all $v_i \in V \backslash \{p\}$
  do $\wedge_i \leftarrow \varnothing$
$E = \{p\}$

repeat
  Exploration of the successors of a node
  Choose $v_i \in E$
  for all $v_i \in Succ(v_i)$
    do $F_{ij} \leftarrow \varnothing$
    for all $\lambda_i = (vis(L_i), t(L_i), q(L_i), c(L_i)) \in \wedge_i$
      do if $V_i^j = 0$
        then $F_{ij} \leftarrow F_{ij} \cup \{$**Extend** $(\lambda_i, v_j)\}$
    $\wedge_j \leftarrow$ **EFF** $(F_{ij} \cup \wedge_j)$
    if $\wedge_j$ has changed
      then $E \leftarrow E \cup \{v_j\}$
  Reduction of E
  $E \leftarrow E \cup \{v_i\}$
until $E = \varnothing$

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while $\wedge_i$ is list of labels on node vi; Succ(v_i) is a set of successors of node v_i; E is a list of nodes waiting to be treated; F_{ij} is a set of labels extended from node v_i to v_j; and $EFF(\wedge)$ is a procedure to keep only nondominated label in the list of label $\wedge$.

(2) Branch-and-Bound

The branch-and-bound algorithm is used if the integer value does not exist in the final solution, which much likely to occur. The branching strategies are based on the original three-index flow formulation from constraints (2) to (11). The branching on flow variables (x_{ij}) is used if the number of vehicle used is integer but the flow variables are fractional[15]. Equation (25) shows a best-first strategy to make a decision when branching in the sub problem.

$$\max c_{ij} \left(\min\{x_{ijk}, 1\text{-}x_{ijk}\}\right) \tag{25}$$

5. CONCLUSION

This paper presents the framework and integrated idea to solve the combination of location and vehicle routing problems. The location-routing problem attracts the researchers and practitioners since it can tackle the mathematic problem in both strategic and operational level.

Since LRP is NP-hard, develop the exact algorithm must be aware of computational resources. This study proposes the method of decompose the integer linear programming. The branch-and-price is intensively explored in order to produce an acceptable calculation process. This research contributes the new idea to cooperate time windows into LRP which has not been developed before using exact algorithm.

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