A semi-dynamic traffic assignment model and its application

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In most cities traffic condition varies significantly within a day, in which static traffic assignment model may not be able to sufficiently represent time-varying congestion phenomena in transportation network analysis. On the other hand, a dynamic traffic assignment model needs much computational load and does not necessarily have a unique solution in most models. A semi-dynamic traffic assignment model is one of the alternatives for describing within-day traffic dynamics of large-scale networks. The semi-dynamic approach is basically to formulate static network equilibrium in each period, but considers flow propagation between periods. The flow propagation is that the flow on a link which cannot exit the link in a period is propagated to the next period.

***Key words**: semi-dynamic assignment model, residual flow, flow propagation, sensitivity analysis.*

1. INTRODUCTION

A semi-dynamic traffic assignment model is one of the alternatives for describing within day traffic dynamics of large-scale networks. In the fact that on a link there are number of vehicle cannot reach the destination within the time period, is propagated to the next period.

We assume for simplicity that inflow enters a link continuously at the same rate, travel cost is constant within the same period, and travel cost is the function of its inflow. On a link there are number of vehicle that cannot reach the destination within the time period. This is the residual flow of that link, and propagates to the next period. The residual flow on a link is determined by the inflow and the link travel time (which is a function of its inflow) in this study. Therefore, this does not travel on the following links in the same period but it runs on them in the next period. Also, the residual flow on a link is added to demand between the end node of that link and the original destination in the next period. In the present period, this residual flow on next links should be eliminated. In this study, residual flows for propagation are approximately calculated using sensitivity analysis in orter to reduce computational cost.

In this paper, semi-dynamic traffic assignment model will be formulated within the framework of logit-based model (Sheffi 1985). In the next section, semi-dynamic traffic assignment model will be presented, and the sensitivity analysis method is formulated. A simple example and a case study of Kanazawa city urban network are provided in the next sections for demonstrating the correctness and implement ability of our method. In the last section, we summarize the study.

2. OVERVIEW LOGIT-TYPE NETWORK EQUILIBRIUM MODEL

Static equilibrium is reached in each period and logit-type route choice is assumed. The probability of choose route j between OD pair i as below:

$$p_{\tau,ij} = \frac{\exp(-\theta c_{\tau,ij})}{\sum_{j \in J_i} \exp(-\theta c_{\tau,ij})} \qquad (1)$$

Logit-based traffic assignment is assumed:

$$f_{\tau,ij,k} = q_{\tau,ij}\, p_{\tau,ij,k} = q_{\tau,ij} \frac{\exp(-\theta c_{\tau,ij,k})}{\sum_{k \in J_{ij}} \exp(-\theta c_{\tau,ij,k})} \qquad (2)$$

where q_{ij} is demand between the i-th node and j-th node, $f_{ij,k}$ is the flow on the k-th route between the i-th node and j-th node, $p_{ij,k}$ is the probability of choosing the k-th route between the i-th node and j-th node, $c_{ij,k}$ is the travel cost on the k-th route between the i-th node and j-th node, J_{ij} is the set of routes between the i-th node and j-th node, θ is a positive parameter, τ is τ-th period.

An expression of the above equation is as follows:

$$\mathbf{f}_\tau = \mathbf{Q}_\tau \mathbf{p}_\tau \qquad (3)$$

where f_τ is the vector of all route flows, Q_τ is diagonal matrix of travel demands.

The link vector is given as below by using route flows and the link-route incidence matrix:

$$\mathbf{x}_\tau = \Delta \mathbf{f}_\tau \qquad (4)$$

where x_τ (= $x_1, x_2, \ldots, x_{|A|}$) is the vector of all link flows, Δ (= $\{\delta_{a,ijk}\}$)is the link-route incidence matrix, and A is the set of links, and $|A|$ is the number of links. Also, $\delta_{a,ijk}$ is the link-route incidence variable and $\delta_{a,ijk}$ = 1 if the k-th route between the i-th node and j-th node includes the link a-th; otherwise, $\delta_{a,ijk}$ = 0. The route travel cost function is given as:

$$\mathbf{c}(\mathbf{f}_\tau) = \Delta^T \mathbf{t}(\Delta \mathbf{f}_\tau) \qquad (5)$$

where c(f) is the vector-valued function of route travel cost, t(Δf) is the vector-valued function of link travel cost. Because travel cost is the function of its inflow, otherwise, probability of route choice is a function of its travel cost. We have:

$$\mathbf{f}_\tau = \mathbf{Q}_\tau \mathbf{p}(\mathbf{c}(\mathbf{f}_\tau)) \qquad (6)$$

3. SEMI-DYNAMIC TRAFFIC ASSIGNMENT MODEL

(1) Semi-dynamic traffic assignment model

In the practice, not all vehicle depart from Origin to Destination can reach their destinations. One part of them cannot exit the link and becomes flow propagation. The flow propagation is that the flow on a link which cannot exit the link in a period is propagated to the next period.

The length of discrete time period should be determined according to accuracy of OD data and others. Too detail description of flow propagation is not necessarily effective. Therefore, for practical applications, the period length may be from 15 min to 90 min. in many cases. Needless to say, we can determine the length much shorter if OD demand data are accurate and dynamically detail. The object of this study is the case that OD data are not detail, and the period (or the length of discrete time) is not so short (approximate from 15 min to 90min.)

Semi-dynamic traffic assignment model is shown in below figure.

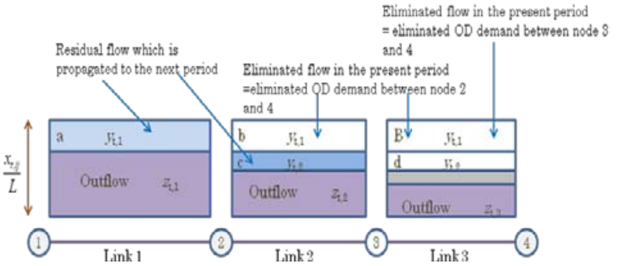

Figure 1. Semi-dynamic traffic assignment model

The inflow into link 1 cannot exit this link and become residual flow which is propagated to the next period. $x_{\tau,a}$ denotes inflow to the a-th link in the τ -th period, $c_a(x_{\tau,a})$ denotes travel cost on the a-th link in the τ -th period, $y_{\tau,a}$ denotes residual-flow which is propagated to the next period on the a-th link in the τ -th period, z_τ denotes link flow after the residual flows are eliminated. Thus, the travel time on link 2 should be: $c_2(x_{\tau,2}) = c_{\tau,2}(x_{\tau,1} - y_{\tau,1})$. The travel time on link 3 should be: $c_3(x_{\tau,3}) = c_{\tau,3}(x_{\tau,1} - y_{\tau,1} - y_{\tau,2})$. Where $x_{\tau,1}$, $x_{\tau,2}$, $x_{\tau,3}$ denote inflow to link 1, 2 and 3. $y_{\tau,1}$, $y_{\tau,2}$, $y_{\tau,3}$ denote residual flow on link 1, 2 and 3.

If the residual flow is eliminated, the travel cost does not only change, but also the inflow changes via network equilibrium. We consider that inflow enters a link continuously at the same rate, travel cost is constant within the same period and travel cost is the function of its inflow. On the other hand, residual flow which is the function of travel time on its link is added to demand between the end node of that link and the original destination in the next period.

As described above, the inflow rate is $x_{\tau,a}/L$. Because travel cost is constant within the same period and residual flow is the function of travel time on its link, one of the natural ways to estimate the residual flow is that the amount of the residual flow is the product of inflow rate and travel time. This means that the residual flow is still travelling on the link at the end of the period. The residual flow on a link in the τ-th period is:

$$y_{\tau,ij} = \frac{f_{\tau,ij} t_\tau \delta_{ij}}{L} \tag{7}$$

where $f_{\tau,ij}$ is the flow on the a link between i-th node and j-th node. t_τ is link travel cost in τ-th period, L is length of time period. The summary of residual flow in k-th route where n_{ijk} refers the number of link in k-th route between i-th node and j-th node:

$$s_{\tau,ij,n_{ijk}} = \sum_{k'}^{k-1} y_{\tau,ij,n_{ijk'}} \tag{8}$$

Consider that B is matrix if link i is a downstream link of link j, then (i,j)=1; otherwise (i,j)=0. Thus, the summary of residual flow is eliminated from route inflow:

$$\mathbf{s}_\tau = \sum_{i\in I}\sum_{j\in J_i} \mathbf{B}_{ij}\mathbf{y}_{\tau,ij} \tag{9}$$

$$\mathbf{s}_\tau = \frac{1}{L}\boldsymbol{\delta}\mathbf{T}(\Delta\mathbf{f}_\tau)\sum_{i\in I}\sum_{j\in J_i} f_{\tau,ij}\mathbf{B}_{ij}\ {}_{ij} \tag{10}$$

We consider that $r_{ij}=B_{ij}\delta_{ij}$, and $R=(r_{11}, r_{12}, \ldots, r_{|I|\,|J_i|})$, so:

$$\mathbf{s}_\tau = \frac{1}{L}\mathbf{T}(\Delta\mathbf{f}_\tau)\mathbf{R}^T\mathbf{f}_\tau \tag{11}$$

The link flow after the residual flows are eliminated z_τ is given by:

$$\mathbf{z}_\tau = \Delta\mathbf{f}_\tau - \mathbf{s}_\tau \tag{12}$$

From Eq. (6), we have:

$$\mathbf{f}_\tau = \mathbf{Q}_\tau\mathbf{p}(\Delta^T\mathbf{t}(\Delta\mathbf{f}_\tau - \mathbf{s}_\tau)) \tag{13}$$

(2) Sensitivity analysis

Sensitivity analysis is important method for confirming the robustness of the model to such uncertainties and for identifying those parameters to which the equilibrium flows are most sensitive. This output information is important to obtain as good an equilibrium solution as possible under the cost constraint of obtaining accurate data.

Base on the sensitivity analysis of logit-type network equilibrium model, we have:

$$\mathbf{f}(\mathbf{s}) = \mathbf{f}(\mathbf{0}) + \nabla_\mathbf{s}\mathbf{f}(\mathbf{0})\mathbf{s} \tag{14}$$

where f(0) denotes the flow in static assignment without residual flow.

From Eq. (11) and (14) we can obtain that:

$$\mathbf{s} = \frac{1}{L}\mathbf{T}(\Delta\mathbf{f}(0))\mathbf{R}^T\mathbf{f}(0) \tag{15}$$

Base on equations above flow f is given as follows:

$$\mathbf{f} = \Delta\mathbf{f}(\mathbf{0}) - \frac{1}{L}\nabla_\mathbf{s}\mathbf{f}(\mathbf{0})\mathbf{T}(\Delta\mathbf{f}(0))\mathbf{R}^T\mathbf{f}(\mathbf{0}) \tag{16}$$

Using Eq. (13), define the following function:

$$\mathbf{d}(\mathbf{f},\mathbf{s}) = \mathbf{f} - \mathbf{Q}\mathbf{p}(\Delta^T\mathbf{t}(\Delta\mathbf{f} - \mathbf{s})) \tag{17}$$

This is a gap between the both sides of Eq. (13) and should be 0, that is, d(f, s) = 0, under the network equilibrium.

Implicit function of Eq. (17) is given as follows:

$$\nabla_s\mathbf{f} = -\nabla_\mathbf{f}\mathbf{d}^{-1}\nabla_s\mathbf{d} \tag{18}$$

Therefore, we obtain the following equation with I is a unit matrix:

$$\nabla_s\mathbf{f} = -(\mathbf{I} - \mathbf{Q}\nabla_\mathbf{c}\mathbf{p}\nabla_\mathbf{x}\mathbf{t}\Delta)^{-1}\mathbf{Q}\nabla_\mathbf{c}\mathbf{p}\nabla_\mathbf{x}\mathbf{t} \tag{19}$$

Thus, route flows with respect to demand perturbation s, are given by:

$$\mathbf{f} = \Delta\mathbf{f_0} + \frac{1}{L}\left[\left(\mathbf{I} - \mathbf{Q}\nabla_\mathbf{c}\mathbf{p_0}\nabla_\mathbf{x}\mathbf{t_0}\Delta\right)^{-1}\mathbf{Q}\nabla_\mathbf{c}\mathbf{p_0}\nabla_\mathbf{x}\mathbf{t_0}\right]\mathbf{T}(\Delta\mathbf{f_0})\mathbf{R}^T\mathbf{f_0} \tag{20}$$

4. NUMERICAL APPLICATIONS

(1) Simple Application

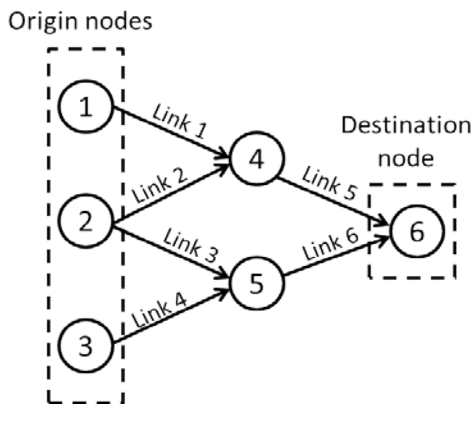

Figure 2: Example network

The network has 6 nodes and 6 links as shown in Figure 2 and with data in table 1 and 2. Two time periods of 60 min length are considered. Network includes 4 routes, route 1 passes link 1, 5, route 2 passes link 2, 5, route 3 passes link 3, 6, route 4 passes link 4, 6. The parameters are assumed that $\theta = 0.5, \alpha = 1, \beta = 2$

Table 1: Free-flow time and capacities

Link	Free-flow time	Capacity
1 ⇨4	10	150
2 ⇨4	10	175
2 ⇨5	10	125
3 ⇨5	10	150
4 ⇨6	10	200
5 ⇨6	10	200

Table 2: Travel demand in 2 time periods

O-D	Time 1	Time 2
1 ⇨6	70	60
2 ⇨6	350	300
3 ⇨6	70	60

After calculated and used semi-dynamic traffic assignment model with flow propagation based on sensitivity analysis, we obtained the results as the table below:

Table 3: The results of inflow, link travel time and residual flow

	Time 1			
Link	Link 2	Link 3	Link 5	Link 6
Inflow	194.08	155.93	212.64	178.75
Link travel tin	12.27	13.63	12.38	11.23
Residual flow	39.68	35.43	43.86	33.47
	Time 2			
Link	Link 2	Link 3	Link 5	Link 6
Inflow	161.4	138.6	232.98	207.4
Link travel tin	11.09	12.27	12.76	11.73
Residual flow	29.82	28.34	49.55	40.56

In comparison with other models, we show the differences of inflow, travel time and residual flow between some models. Model 1 is *Quasi-dynamic traffic assignment model* (Nakayama Sho-ichiro 2008). Model 2 is *Semi-dynamic traffic assignment models with queue evolution and elastic OD demand* (Akamatsu-1991). Model 3 is *Modeling of the time-of-day traffic assignment over a traffic network* (Fujita 1988-1989)

Table 4: The results of inflow in 2 time periods

	Time 1			
Link	Link 2	Link 3	Link 5	Link 6
Model 1	189.8	160.2	245	195
Model 2	188	162	245	195
Model 3	191.8	158.2	261.8	228.1
Semi-dynamic model	184.08	155.93	212.64	178.75
	Time 2			
Link	Link 2	Link 3	Link 5	Link 6
Model 1	163	137	237.8	220.2
Model 2	165.1	134.9	238.1	222
Model 3	161.9	138.1	221.9	198.1
Semi-dynamic model	161.4	138.6	232.98	207.4

Table 5: The results of travel time in 2 time periods

	Time 1			
Link	Link 2	Link 3	Link 5	Link 6
Model 1	17.15	30.95	26.88	11.36
Model 2	16.46	31.98	26.88	11.36
Model 3	17.91	29.81	32.93	21.02
Semi-dynamic model	12.27	13.63	12.38	11.23
	Time 2			
Link	Link 2	Link 3	Link 5	Link 6
Model 1	11.13	17.92	24.34	18.27
Model 2	11.19	16.77	24.46	18.87
Model 3	11.1	18.51	18.85	11.44
Semi-dynamic model	11.09	12.27	12.76	11.73

Table 6: The results of residual flow in 2 time periods

	Time 1			
Link	Link 2	Link 3	Link 5	Link 6
Model 1	14.8	35.2	45	0
Model 2	13	37	45	0
Model 3	16.8	33.2	61.8	28.2
Semi-dynamic model	39.68	35.43	43.86	33.47
	Time 2			
Link	Link 2	Link 3	Link 5	Link 6
Model 1	0	12	37.8	20.2
Model 2	0	9.9	38.1	22
Model 3	0	13.1	21.9	0
Semi-dynamic model	29.82	28.34	49.55	40.56

It is easy to recognize that the result of inflow between models does not have big differences, and travel time result also. But in the residual flow, the results in link 24 and 56 are absolutely different between models.

(2) Kanazawa city urban network application

Kanazawa urban network includes 272 nodes and 964 links. The parameters are assumed that

$\theta = 0.5, \alpha = 1, \beta = 2$

Figure 3: Kanazawa urban network

We calculated in rush-hour periods (6am, 7am, 8am and 9am). Because in time period 7am and 8am are in peak hours period. Thus, numbers of vehicle in these time periods are greater than time period 6am and 9am many times. The input database used personal trip data.

Table 7: Number of OD pair and number of paths in Kanazawa urban network

	Time 6	Time 7	Time 8	Time 9
Number of OD	383	2409	1590	348
Number of path	668	6351	3395	633

After apply Semi-dynamic traffic assignment model with flow propagation based on sensitivity analysis in Kanazawa urban network, we have output results of each time period. Below chart show the summary inflow data of all time periods. From the result we prove that semi-dynamic traffic assignment model can be applied in large-scale network.

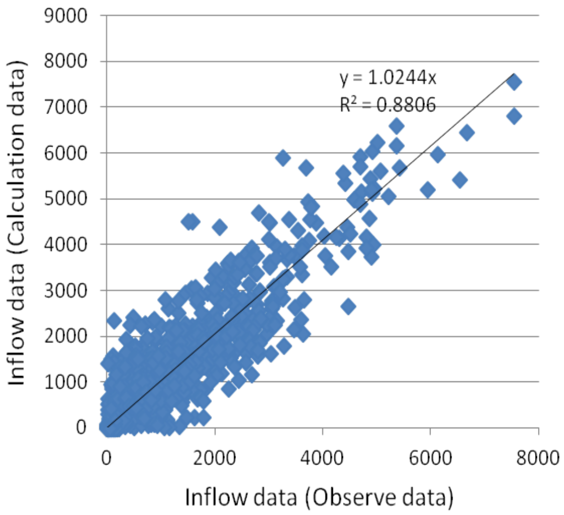

Figure 4: Inflow result of summary data in rush hours

5. SUMMARY

First of all, in semi-dynamic model, sensitivity analysis of logit type network equilibrium model is used. Secondly, static network equilibrium is reached in each period, and dynamics of network flow (flow propagation) is considered between periods. Last but not least, the semi-dynamic traffic assignment model can apply within-day traffic of large scale networks.

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