A Probabilistic Maximal Coverage Ambulance Location Model with Uncertainty of Travel Speed

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This paper presents a maximal coverage ambulance location problem with stochastic travel speed. A model named Probabilistic Maximal Covering Location Problem (PMCLP) is proposed to identify optimal location patterns with the limited number of stations within specific response time and the expected travel speed distribution. Computational results were given using real network data derived from Osaka city and randomly generated demands. Two dynamic programming algorithms were developed to provide the exact solution; one for the MCLP model and other one is for the PMCLP model. Comparing the computational results between the PMCLP model and the MCLP model, the PMCLP model increases the demand coverage proportion that will be served within short arrival time while maintaining the specified demand coverage within standard response time.

Key Words: *emergency medical service, ambulance, location, uncertainty, travel speed, normal distribution, maximal covering*

1. INTRODUCTION

The emergency medical services (EMS) systems aim to reduce unnecessary death and disability. The key linkage between emergency case and treatment played by emergency ambulance service in its role of transporting emergency medical technician (EMT) to the scene within effective time for treatment. The ambulance location problem is macro level of planning for determining the optimal location of ambulance stations for strategic and operational consideration to provide high standard of ambulance logistics under limited of resources and dynamic of the traffic. The standards of the American College of Surgeons[1] indicate that every patient should be seen by a physician within 15 minutes of arrival, and that resuscitation cases should be seen immediately. The Ambulance location problem was focused since 1970s. The review of ambulance location models was found in [2-6]. Various maximum covering models have been proposed for example, the first model named the maximal covering location problem (MCLP) proposed in 1974 by Church and ReVelle[7] aims to maximal population covering with limited *p* ambulances as the binary linear programming problem, the tandem equipment allocation model (TEAM) proposed in 1979 by Schilling *et al.*[8] optimizes for 2 ambulance types situation, the maximal expected covering location model (MECLP) proposed in 1983 by Daskin[9] applied the busy probability of each ambulance into MCLP, the backup coverage problem (BOCAP) proposed in 1986 by Hogan and ReVelle[10] try to achieve more one ambulance covered at demand node, the maximum availability location problem (MALP) proposed in 1989 by ReVelle and Hogan[11] aims to maximal demand node covered by at least *b* ambulances, the double standard model (DSM) proposed in 1979 by Gendreau *et al.*[12] try to maximize demand node covered by at least 2 ambulance stations with 2 standards response time r_1 and r_2, the dynamic double standard model (DDSM) proposed in 2001 by Gendreau *et al.*[13] considers the redeployment problem for a fleet of ambulance by adding probability that ambulance locate at the site into DSM, the maximal expected coverage relocation problem (MECRP) proposed in 2006 by Gendreau *et al.*[14] add q_k the probability of having *k* ambulances into MCLP and the multi-period double standard model (mDSM) proposed in 2010 by Schmid and Doerner[15] adjusted DSM to cover largest possible demand every time period. There are many models adapted from MCLP by added more conditions of problems such as [8-11]. The MCLP[7] model was solved by greedy adding algorithm.

The covering function of ambulance location problem is cooperating between maximum travel time (response time) and travel speed value. Those models in literature review use the static value of maximum service distance normally was evaluate from the mean travel speed. In the real world, the travel speed is the uncertainty variable; where it is considered as an average travel speed of the road network at specific time and may vary for each road link in the road network. It is favorable to estimate the average travel speed for each road link in the road network at specific time but it will require high performance machines. The behavior of travel speed can be represented with the normal distribution[16]. Normal distribution is often used to describe real-valued random variables that cluster around a single mean value. Notation of normal distribution is $N(\mu,\sigma^2)$ where parameter μ is the *mean* and σ^2 is the *variance* (a "measure" of the width of the distribution), σ is the standard deviation. The travel speed value can obtained from normal distribution with specifics the percentile in the inverse of the normal cumulative distribution. The relation between travel speed and percentile rank in term of μ and σ is shown in **Fig.1**. To consider the congestion of road network (traffic congestion) situation it can represent the traffic congestion problem as more or less congestion by varying the percentile rank of inverse cumulative travel speed distribution related to **Fig.1** started at 0.5 percentile (the mean travel speed) by decreasing (more congestion) or increasing (less congestion) the percentile value.

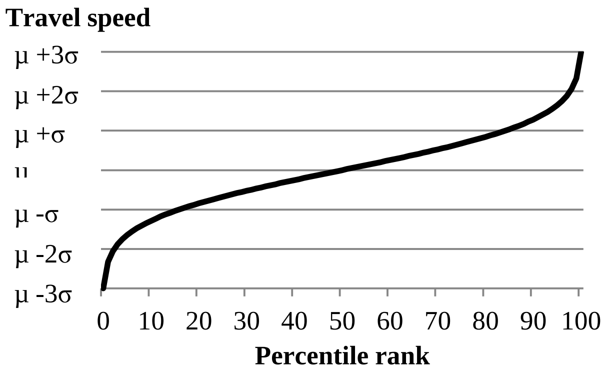

Fig.1 The percentile rank related to μ and σ of travel speed

We developed a maximal coverage model for uncertainty travel speed by using travel speed distribution. The question is how to provide the best coverage with high traffic congestion. The proposed model was adjusted from MCLP model and a dynamic programming was developed.

2. MODEL AND SOLUTION

The problem is defined on a graph G=$(V \cap W, E)$ where V represent the demand node and W represent the potential stations, and $E = \{(i,j): i \in V$ and $j \in W\}$ is an edge set.

We have designed 2 exact dynamic programming algorithms for solving the MCLP model and the new model.

(1) The model

The model proposed solves the maximal coverage problem where the objective is to maximal the weight covered at high traffic congestion to guarantee a feasible solution for stochastic travel speed. The formulation of probabilistic maximal covering location problem (PMCLP) model is:

Maximize $$Z(\mathbb{L},p) = \sum_{i\in V} \omega_i x_i^{\beta} \quad (1)$$

Subject to $$\sum_{i\in V} \omega_i x_i^{0.5} \geq \alpha^{0.5} \sum_{i\in V} \omega_i \quad (2)$$

$$\sum_{j\in W_i^{\beta}} y_j \geq x_i^{\beta} \quad (3)$$

$$\sum_{j\in W} y_j = p \quad (4)$$

$$p < m \quad (5)$$

$$x_i^{0.5} \geq x_i^{\beta} \quad \forall i \in V \quad (6)$$

$$x_i^{0.5}, x_i^{\beta} \in \{0,1\} \quad \forall i \in V \quad (7)$$

$$y_j \in \{0,1\} \quad \forall j \in W \quad (8)$$

$$0.0 < \beta < 0.5 \quad (9)$$

Where $\mathbb{L}$ = the set of station allocation pattern;

$V = \{1,2,3,\ldots,n\}$ is the set of demand nodes;

$W = \{1,2,3,\ldots,m\}$ is the set of potential stations;

m = maximum number of potential stations;

p = the number of stations to be located;

ω_i = the weight of demand node i can be ;

$y_j = \begin{cases} 1 \text{ if ambulance is located at station } j \\ 0 \text{ otherwise} \end{cases}$

$d_{i,j}$ = the shortest distance from node i to station j;

r = the standard response time;

$N(\mu,\sigma^2)$ = the travel speed distribution function with mean μ and standard deviation σ;

$\delta_{i,j}^{0.5} = \begin{cases} 1 \text{ if } d_{i,j} \leq (r \times \mu) \\ 0 \text{ otherwise} \end{cases}$

$\delta_{i,j}^{\beta} = \begin{cases} 1 \text{ if } d_{i,j} \leq (r \times \mathrm{ICDF}(\beta,\mu,\sigma)) \\ 0 \text{ otherwise} \end{cases}$

$x_i^{0.5} = \begin{cases} 1 \text{ if } \sum_{j\in W} \delta_{i,j}^{0.5} y_j > 0 \\ 0 \text{ otherwise} \end{cases}$

$x_i^{\beta} = \begin{cases} 1 \text{ if } \sum_{j\in W} \delta_{i,j}^{\beta} y_j > 0 \\ 0 \text{ otherwise} \end{cases}$

$W_i^{\beta} = \left\{ j \in W : \delta_{i,j}^{\beta} = 1 \right\}$

$\alpha^{0.5}$ = the proportion of the total weight that must be covered at the mean travel speed;

The decision variable of the problem is the pattern of stations allocated $\mathbb{L}$ by the given number of p stations. The objective function (1) represents the total demand covered at β percentile of the travel speed distribution. Constraint (2) states $\alpha^{0.5}$ is the proportion of the total demand that must be covered by an ambulance located within the mean travel speed. The left-hand side of constraint (3) counts the number of stations covering demand node i with β percentile. The number of stations allocated is restricted to equal p by constraint (4) and p must less than the number of potential station m by constraint (5). By constraint (6), a demand node i cannot be covered at β percentile if it is not covered at 0.5 percentile. The percentile β just less than 0.5 and more than 0 by constraint (9).

The travel speed distribution was given as $N(\mu, \sigma^2)$ and the uncertainty travel speed value was derived by the inverse cumulative distribution function (ICDF) with 3 parameters are percentile rank, the mean travel speed μ and the standard deviation σ.The coefficient $\delta_{i,j}^{0.5}$ and $\delta_{i,j}^{\beta}$ are binary value represent node i is covered or not covered by station j with 0.5 and β percentile of travel speed. With the station j has allocated, the binary variable $x_i^{0.5}$ and x_i^{β} will be 1 if coefficient $\delta_{i,j}^{0.5}$ and $\delta_{i,j}^{\beta}$ are 1. The W_i^{β} is the set of stations eligible to provide "cover" to demand node i with β percentile. The $\alpha^{0.5}$ is the proportion of the total demand that must be covered by an ambulance located with 0.5 percentile (the mean travel speed).

(2) Exact solution for MCLP

The MCLP model aims to maximal population covering; we replaced the values of a_i as the demand of demand nodes. We designed the dynamic programming algorithm to solve the exact solution for the MCLP model is:

```
MCLPsearch(siteList, start, spare) {
 tmp1 = demand covered by siteList
 If (spare > 0) {
   If (tmp1 >= maxDemandCover) {
     for (i = start; i < siteList.size; i++) {
       newList = siteList
       newList[i] = 0
       MCLPsearch(newList, i +1, spare -1)
 }}}
 else {
   If (tmp1 >= maxDemandCover) {
     If (tmp1 > maxDemandCover) {
       Clear the solution list
       maxDemandCover = tmp1
     }
     Add siteList into the solution list.
   }
 }
```

(3) Exact solution for PMCLP

The PMCLP model aims to maximal weight coverage at the high confidence level (β) of travel speed of the travel speed distribution and $\alpha^{0.5}$ of total demand have been covered by the mean travel speed, we assigned the demand of demand nodes to ω_i. The dynamic programming algorithm for exact solution of PMCLP model was designed as:

```
PMCLPsearch(siteList, start, spare, alpha) {
 tmp1 = demand covered at 0.5 confidence by siteList
 tmp2 = demand covered at β confidence by siteList
 if (tmp1 > alpha) {
   If (spare > 0) && (tmp2 >= maxDemandCover) {
     for (i = start; i < siteList.size; i++) {
       newList = siteList
       newList[i] = 0
       MECCsearch(newList, i +1, spare-1,alpha)
     }
   }
   else {
     If (tmp2 >= maxDemandCover) {
       If (tmp2 > maxDemandCover) {
         Clear the solution list
         maxDemandCover = tmp2
       }
       Add siteList into the solution list.
     }
   }
 }
```

In both algorithms, the *siteList* is an array of potential stations contains value of y_j; all are 1 at the beginning, the *spare* is the number of potential stations must be unselected (0) and the *maxDemandCover* starts with 0 is a global variable contains maximum demand covered by the objective function of the model. The *alpha* in PMCLPsearch algorithm represents the proportion of the total demand must be covered at the mean travel speed is given by $\alpha^{0.5}$. Both algorithms are recursive function and the results will be the list of stations allocation pattern.

3. EXPERIMENTATION

(1) Travel speed data

The vehicle information communication systems (VICS)[17] are available for providing traffic information on travel time, level of congestion, crashes and car parks in Japan. We retrieved travel speed of Osaka area from VICS's data between October 4th 2010 and November 5th 2010. The statistical summary of the travel speed distribution on the weekday is shown in **Fig.2**. The **Fig.3** shows the inverse cumulative distribution graph of travel speed distribution on the weekday.

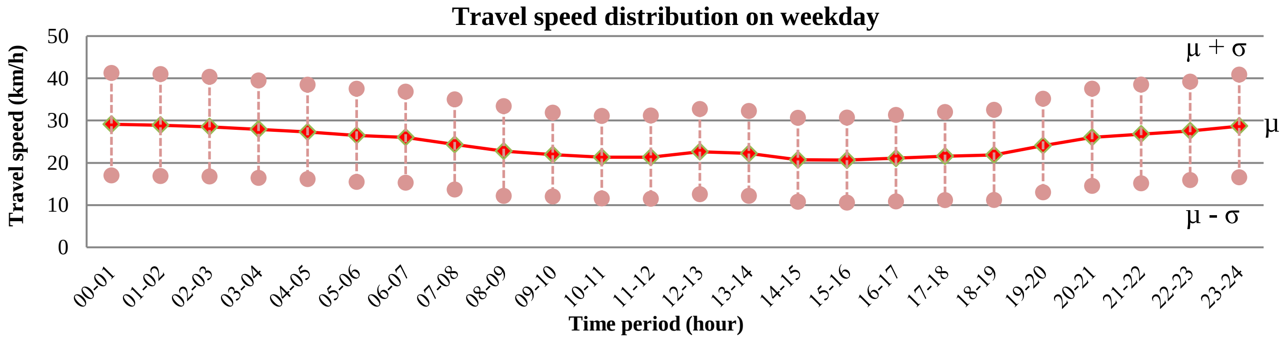

Fig.2 Travel speed distribution on weekday between Oct 4 and Nov 5, 2010 in Osaka area.

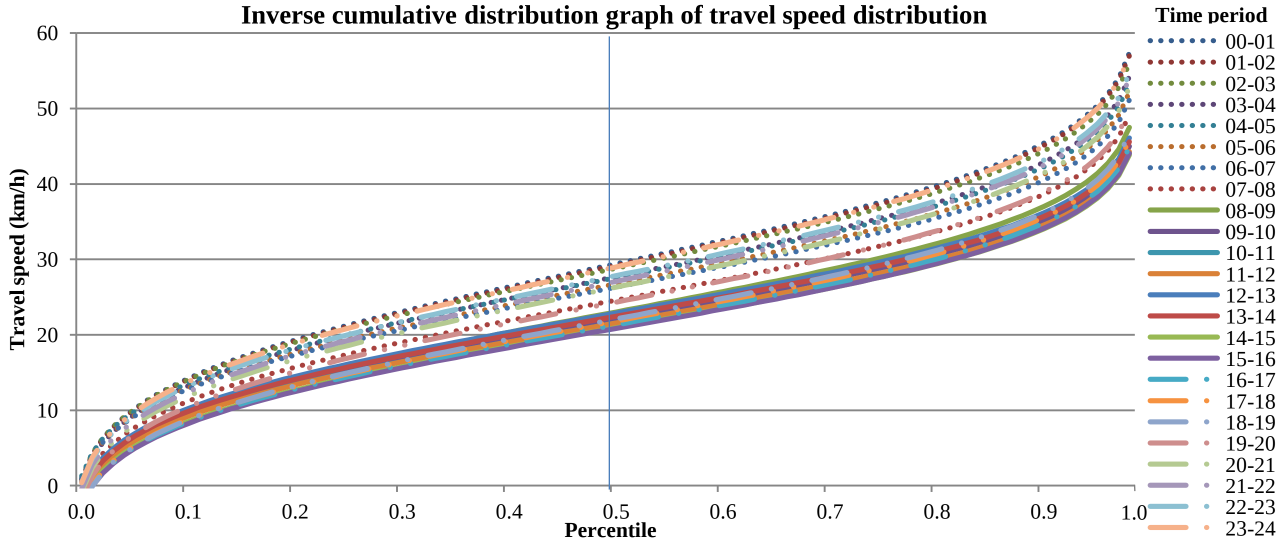

Fig.3 Inverse cumulative distribution graph of travel speed distribution each time period between Oct 4 and Nov 5, 2010 in Osaka area.

(2) Experimentation network data

The experimentation network was derived from Osaka area has 1,614 demand nodes was separated by grids size 300 x 300 meters and 26 fire stations (available online at http://www10.brinkster.com/wisitlim/NetworkData.zip). The shortest distance between all demand nodes and each potential station was given in meters by Google®™ Distance Matrix Service via Google Maps JavaScript API v3[18]. The statistical summary of the experimentation network regarding the demand and distance are shown in **Table 1**. The weight each demand node is random value of demand occurring probability as shown in **Fig.4**. The Osaka area map with grid of demand nodes and location of fire stations is shown in **Fig.5**.

Table 1 Statistical summary of the test network.

Statistics parameters	Demand proportion	Distance (meters)
Minimum	0.0000129159	46.00
Maximum	0.0012066151	31,012.00
Mean	0.0006195787	10,648.71
Median	0.0006338471	10,171.00
Standard deviation	0.0003474264	5,270.32
Summation	1.0000000000	-

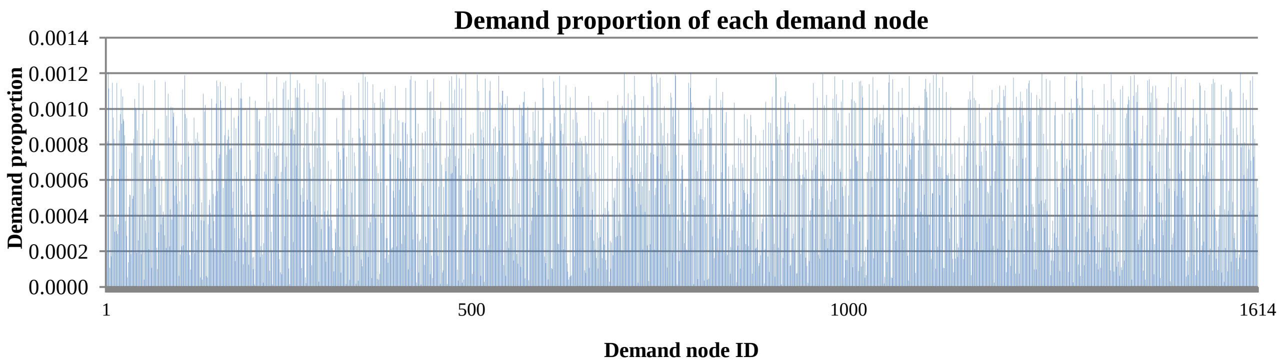

Fig.4 The event occur probability of each demand node, the total is 1

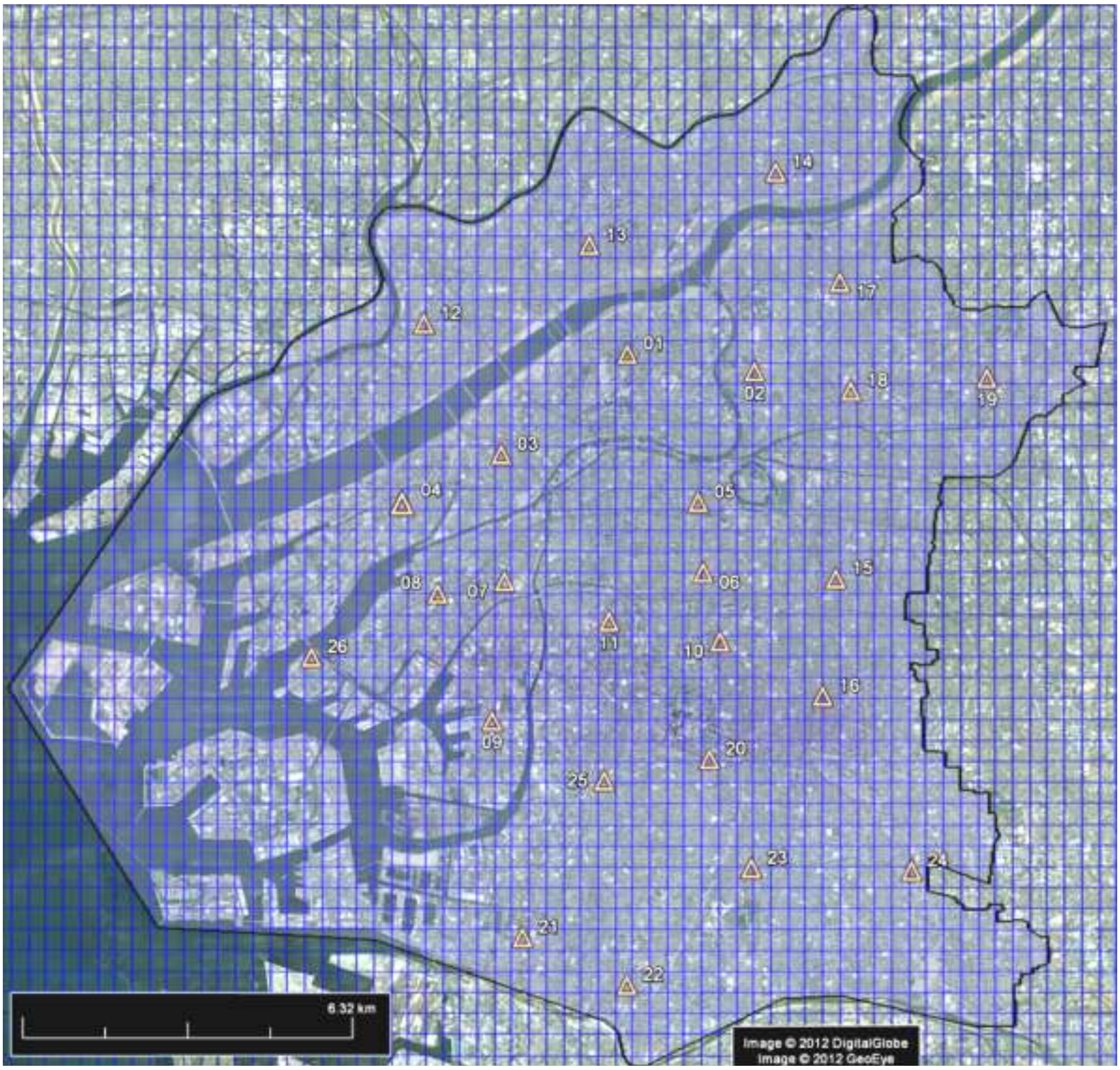

Fig.5 The Osaka area with 26 location of fire stations and 1,614 demand area grids by Google® Earth™

4. RESULTS

The experimentation was done on AMD® Athlon™ 64X2Dual Core 4800+ 2.40 GHz and 2 GB of RAM operated by Microsoft® Windows XP™ Professional 32bits with Service P1207gdagack 3. The dynamic programming algorithms were implemented with Java™ Development Kit version 1.7.0 update 5.

Both models were applied to the Osaka area with 26 fire stations and 1,614 demand nodes. The travel speed distribution derived from VICS data at period 07-08 hour is $N(24.3187{,}10.6798^2)$. The parameters are given response time r = 15 minutes, proportion of the demand covered at the mean travel speed $\alpha^{0.5}$ = 0.9 and percentile β= 0.1, 0.05.

The results are reported in **Table 2** including the proportion of demand will be served within 4, 8 and 15 minutes with the mean travel speed by the stations allocated pattern produced from each model under the given parameters p and β value. The table headings are as follows :

p	Number of stations to be allocated
MCLP_GA	MCLP solved with greedy adding
MCLP_DP	MCLP solved with dynamic programming
PMCLP10	PMCLP with β = 0.10
PMCLP05	PMCLP with β = 0.05
CT	Computing time in milliseconds
NS	Number of feasible solutions
DC4	Demand covered within 4 minutes
DC8	Demand covered within 8 minutes
DC15	Demand covered within 15 minutes

The illustrations of ambulance stations allocated pattern by each model with given p stations are shown in **Fig. 6 – 13**. The triangle with red color represents the allocated station and the other with white color represents the un-allocated station. The clients' satisfaction with 16 stations between MCLP model and PMCLP model had compared in **Fig.14** and with 22 stations is shown in **Fig.15**.

With 16 stations shown in **Fig.14**, the proportion of demand will be served within 4 minutes is increases with MCLP model solved by dynamic programming and more increases with PMCLP model with β = 0.1 and more increases with β = 0.05 but the total demand covered within standard response time by PMCLP is decreases around 3% comparing with MCLP model. With 22 stations shown in **Fig.15**, the demand coverage within standard response time of both models are same and the PMCLP model provide better demand coverage within 4 and 8 minutes and be the best coverage with β = 0.05.

Table 2 Result of the experimentations

p	Model	CT	NS	Stations Allocation Pattern (the first of list)	DC4	DC8	DC15
16	MCLP_GA	47	1	1,2,3,4,5,7,8,10,12,13,14,17,19,21,24,25	0.2544	0.6906	0.9486
	MCLP_DP	12,281	14,345	4,8,9,12,13,14,16,18,19,20,21,22,23,24,25,26	0.2699	0.7322	0.9486
	PMCLP10	108,953	1	3,5,8,9,12,13,14,15,16,17,19,21,22,23,24,25	0.2746	0.7714	0.9354
	PMCLP05	539203	1	2,5,8,9,10,12,14,15,16,18,19,20,21,22,23,24	0.2836	0.7075	0.9193
22	MCLP_GA	46	1	1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,21,24,25	0.3395	0.7764	0.9486
	MCLP_DP	625	2,715	4,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26	0.3367	0.7811	0.9486
	PMCLP10	140	1	1,2,3,4,5,8,9,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25	0.3639	0.8349	0.9486
	PMCLP05	547	1	2,3,4,5,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25	0.3696	0.8312	0.9486

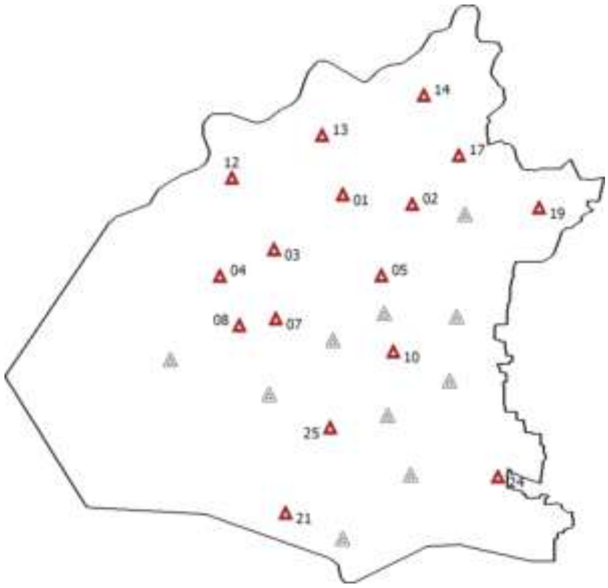

Fig.6 The stations pattern allocated by MCLP_GA with p = 16

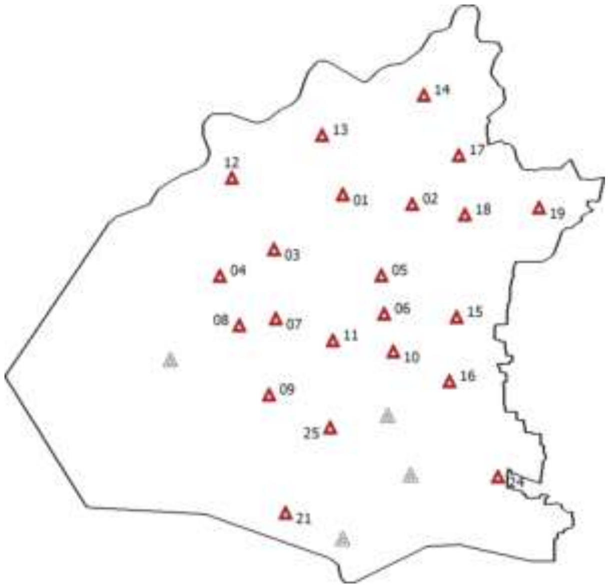

Fig.7 The stations pattern allocated by MCLP_GA with p = 22

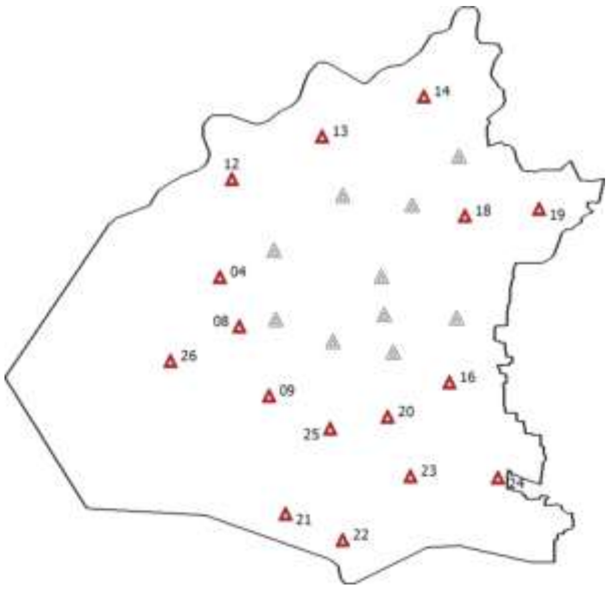

Fig.8 The stations pattern allocated by MCLP_DP with p = 16

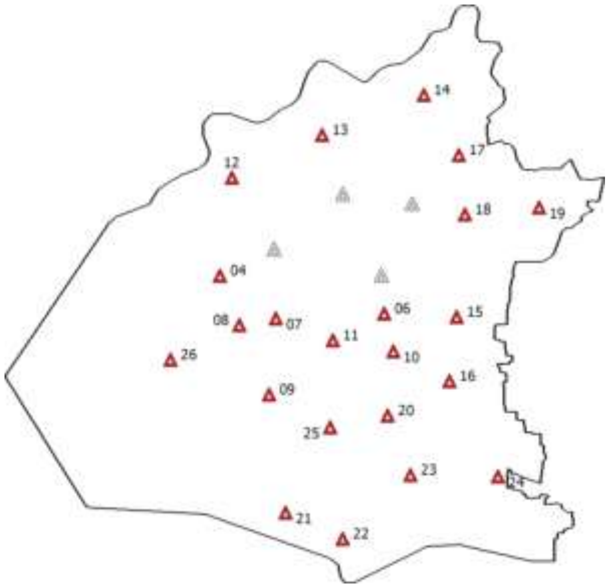

Fig.9 The stations pattern allocated by MCLP_DP with p = 22

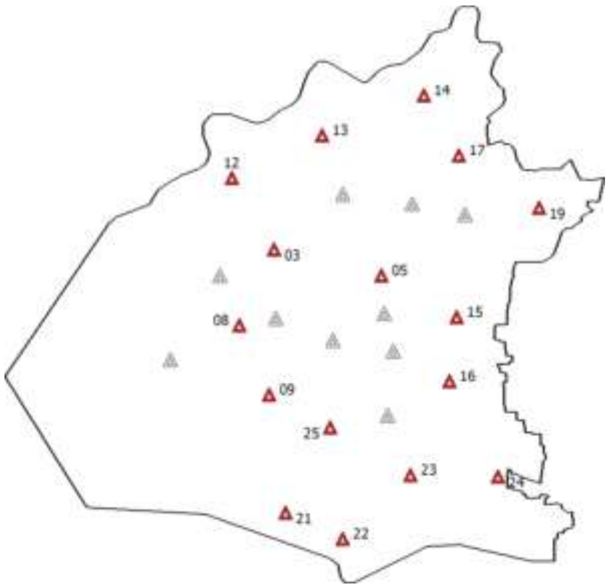

Fig.10 The stations pattern allocated by PMCLP10 with p = 16

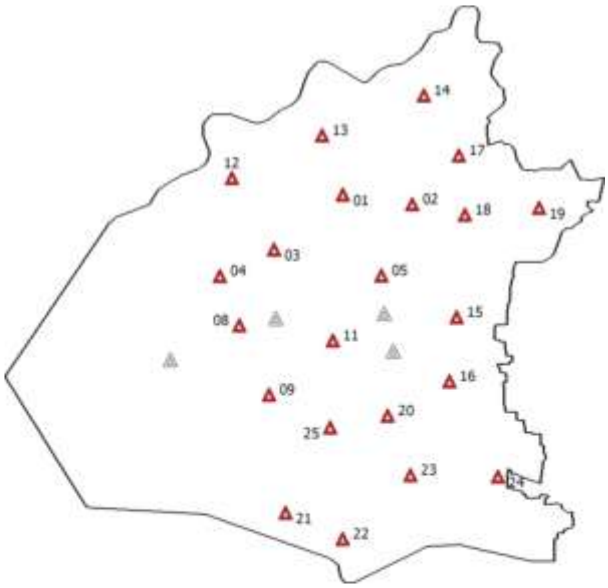

Fig.11 The stations pattern allocated by PMCLP10 with p = 22

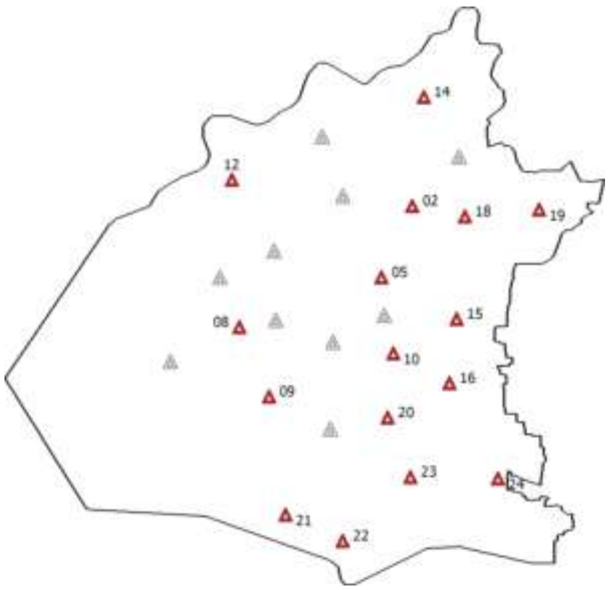

Fig.12 The stations pattern allocated by PMCLP05 with p = 16

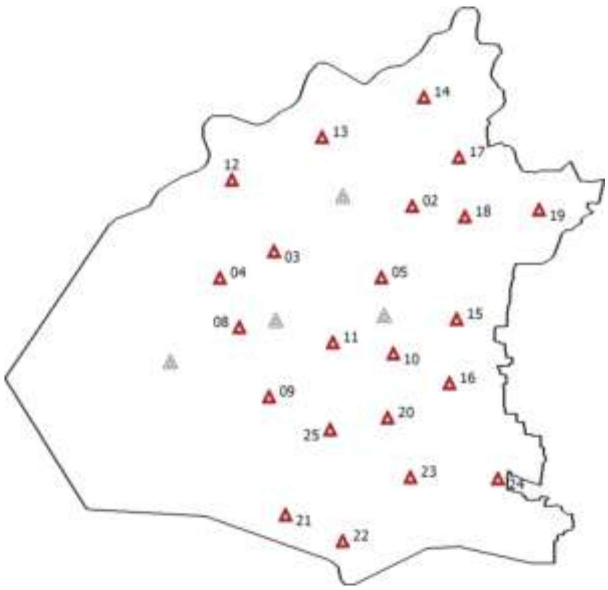

Fig.13 The stations pattern allocated by PMCLP05 with p = 22

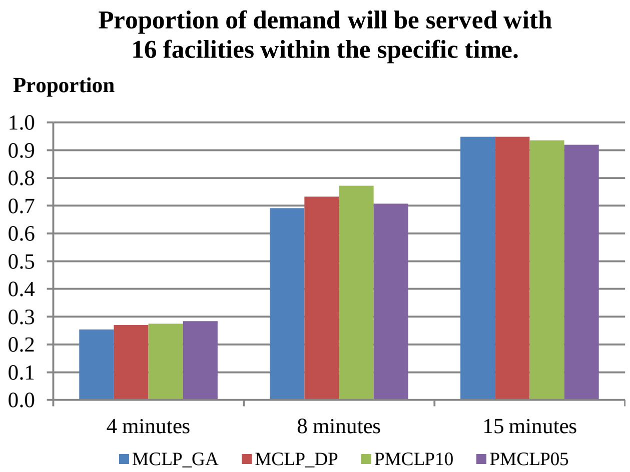

Fig.14 Comparison of demand proportion will be served with 16 stations within the specific time.

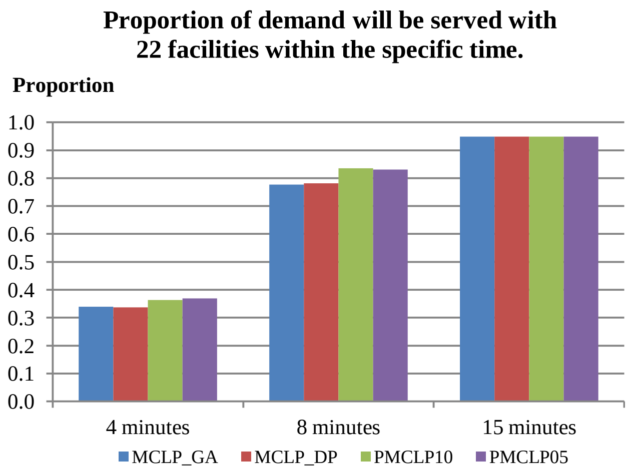

Fig.15 Comparison of demand proportion will be served with 22 stations within the specific time.

5. CONCLUSION

A new model for ambulance location problem with stochastic travel speed and 2 dynamic programming algorithms have been developed. We assumed the travel speed behavior is normal distribution and obtain the travel speed value with inverse cumulative distribution function by specified the percentile value. The proposed algorithms provide the exact solution with acceptable computing time.

Our probabilistic ambulance location model can maximize the clients satisfied by increasing the demand proportion that be served within small arrival time and maintain the specified coverage within the standard response time.

APPENDIX A MCLP MODEL

Maximize
$$z(p) = \sum_{i \in I} a_i x_i \tag{10}$$

Subject to
$$\sum_{j \in J_i} y_j \geq x_i \quad \text{for all } i \in I \tag{11}$$
$$\sum_{j \in J} y_j = p \tag{12}$$
$$y_j = (0, 1) \quad \text{for all } j \in J \tag{13}$$
$$x_i = (0, 1) \quad \text{for all } i \in I \tag{14}$$

where

I = denotes the set of demand nodes;
J = denotes the set of station stations;
r = the distance beyond which a demand point is considered “uncovered”;
d_{ij}= the shortest distance from node i to node j;
$y_j = \begin{cases} 1 \text{ if a station is allocated to station } j \\ 0 \text{ otherwise;} \end{cases}$
$x_i = \begin{cases} 1 \text{ if demand node } i \text{ is covered} \\ 0 \text{ otherwise;} \end{cases}$
$J_i = \{ j \in J \mid d_{ij} \leq r \}$;
a_i = population to be served at demand node i;
p = the number of station to be located.

The original MCLP model has solved by heuristic solution called the greedy adding (GA) algorithm to achieve a maximal cover for p stations under a given service distance. The GA algorithm starts with an empty solution set and then adds the best station into the set one at a time. The GA algorithm picks the first station that covers the most of the total population. For the second station, GA picks the station that covers the most of the population not covered by the first station. Then, for the third station, GA picks the station that covers the most of the population not covered by the first and second stations. This process is continued until either p stations have been selected.

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