# Modeling Gap Choice at Urban Expressway Merging Sections

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Understanding the driver behavior on urban expressway merging sections is very important for assuring a design of merging sections that satisfies users' expectations and hence leads to a reduction of traffic congestion and crashes. The objective of this paper is to gain an insight into gap acceptance by analyzing and modeling gap choice behavior using empirical data. The results reveal that relative speed and space gap, traffic conditions, acceleration lane length, and remaining distance significantly affect the gap choice behavior of merging drivers.

*Key Words: gap choice, gap acceptance, traffic conditions, acceleration lane length, binominal logit model.*

## 1. INTRODUCTION

Merging sections on expressways are designed to allow vehicles coming from ramp to safely and comfortably merge into the mainline of traffic, and they have been recognized to be one of the places where traffic congestion and crashes are likely to occur. A design of merging sections that is consistent with drivers' expectations and behavior promises to reduce traffic congestion and improve safety. In order to satisfy users' expectations, merging sections should allow adequate acceleration lane lengths to enable drivers (i) to search an acceptable gap and (ii) adjust their speed prior to merging into the mainline. Existing guidelines such as those of AASHTO[1] and of the Japan Road Association[2] define the length of an acceleration lane simply as the length required to allow merging vehicles to accelerate without any consideration of the interaction between merging and mainline vehicles. However, this definition may not be suited for actual situations in which drivers' behavior is prone to vary dependent on the mainline traffic conditions and in which merging vehicles have to react to the movements of mainline vehicles in order to position them into the mainline. Morever,
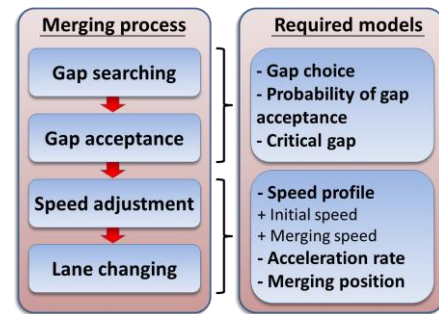


**Fig.1** Merging process and models required to represent merging maneuver

existing guidelines do not give any rational consideration to the lengths of road required for gap searching and gap acceptance. The AASHTO simply recommends a minimum length of 90 -150m while the Japanese guideline supplies adjustment factors for the length of the acceleration lane when the design speed of the expressway is under 50km/h to take account of "merging opportunities".

The process of merging at merging sections is complicated; but can be divided into four steps as shown in **Fig.1**: (i) gap searching, (ii) gap acceptance, (iii) speed adjustment, and (iv) lane changing.

As illustrated in **Fig. 1**, several models are needed to represent the merging maneuver. In order to design an acceleration lane that appropriately satisfies users' expectations, it is crucial to gain a close insight into drivers' behaviors at each of these steps. Accordingly, the aim of this paper is to analyze and model gap choice behavior so as to arrive at an understanding of how gaps are located and accepted, taking account of the effects of mainline traffic conditions, acceleration line length, and the reactions of merging vehicles to the traffic in the mainline.

## 2. LITERATURE REVIEW

Gap acceptance, an important component of the lane changing process, has received a good deal of attention. Most studies have been concerned with determining the critical gap required for estimating the capacity of unsignalized intersections. Herman and Weiss[3], and Miller[4] were pioneers in the development of gap acceptance models based on critical gap. Following that, Daganzo[5], Heckman[6], and Mahmassani and Sheffi[7] used a probit model to estimate critical gaps assuming a normal distribution. However, not all of these models are applicable for expressway merging sections where drivers have to change lane within limited length of road and where no complete stop situation occurs before a lane change (Goswami and Bham[8]).

At expressway merging sections, several studies have been done to model gap acceptance. Kita[9] made use of a binary logit model and found that the gap length, remaining distance to the end of acceleration lane, and relative speed were significant explanatory variables. Hwang and Park[10] modeled gap acceptance with variables such as lead gap, lag gap, front gap, presence of heavy vehicles and remaining distance. They demonstrated that the space gap is a more important variable than the time gap because drivers have a tendency to be more restrained by space than by time and consider distance as a more important factor for judging the safety of a particular lane change. However, all of these models are applicable only under uncongested conditions. In addition, Kita's study focused only on the cases in which the merging vehicles were moving more slowly than the ones in the mainline.

To overcome this limitations, several models have more recently been developed to represent gap acceptance for vehicles merging under congested conditions (Ahmed, *et al.*[11], Hidas[12, 13]). Under congested conditions, where there are very few acceptable gaps, they proposed "forced" and "cooperative" lane change models. These models are capable of representing instances of merging through the creation of gap either by yielding of the following vehicle in the target lane or by forcing the following vehicle to slow down. However, the influence of acceleration lane length on gap acceptance has not been considered in this research.

It should be emphasized that the previous studies did not discuss the situation in which acceleration lanes are located in the middle of expressway, which can be found in several big cities of Japan like Nagoya. In these cases, drivers have some difficulties because they have to merge into the median lane of expressway which usually has higher speed than shoulder lane. Also, the drivers are not easy to check their "blind spot" when merging in these cases. Considering this situation, Chu, *et al.*[14] have recently analyzed merging speed and the relationship between merging position and merging speed at merging sections where acceleration lanes are located in the middle of expressway. They found that a longer acceleration lane would encourage drivers to accelerate thus closing the difference of speeds between vehicles on the acceleration lane and on the mainline. Also, the further merging position is, the higher merging speed becomes.

## 3. METHODOLOGY

### 3.1 Study sites and data collection

Two merging sections named Horita and Takatsuji on Route No. 3 of Nagoya Expressway were chosen for this study. At both of the sites, acceleration lanes are located in the middle of expressway and they were extended in October 2011, as a countermeasure against congestion. Also, in the process of extending the sections, the acceleration lanes were temporarily slightly shortened for the construction work. In this study, the situations of before, during and after the extension of acceleration lanes are denoted as "before", "during" and "after", whose geometric characteristics are presented in **Fig.2** a), b), and c), respectively.

The video data was collected in various periods of the day and day of the week for both Horita and Takatsuji to observe various mainline traffic conditions, covering periods of not only "before" and "after" but also "during" the extension of the acceleration lanes. Notably, since the different lengths of acceleration lane were observed at the same merging section, it is expected that the effects of location of merging section on drivers' behavior can be avoided. Moreover, because of the open of Meinikan expressway which goes parallel to the observed sites in March 2011, significant demand has moved to this
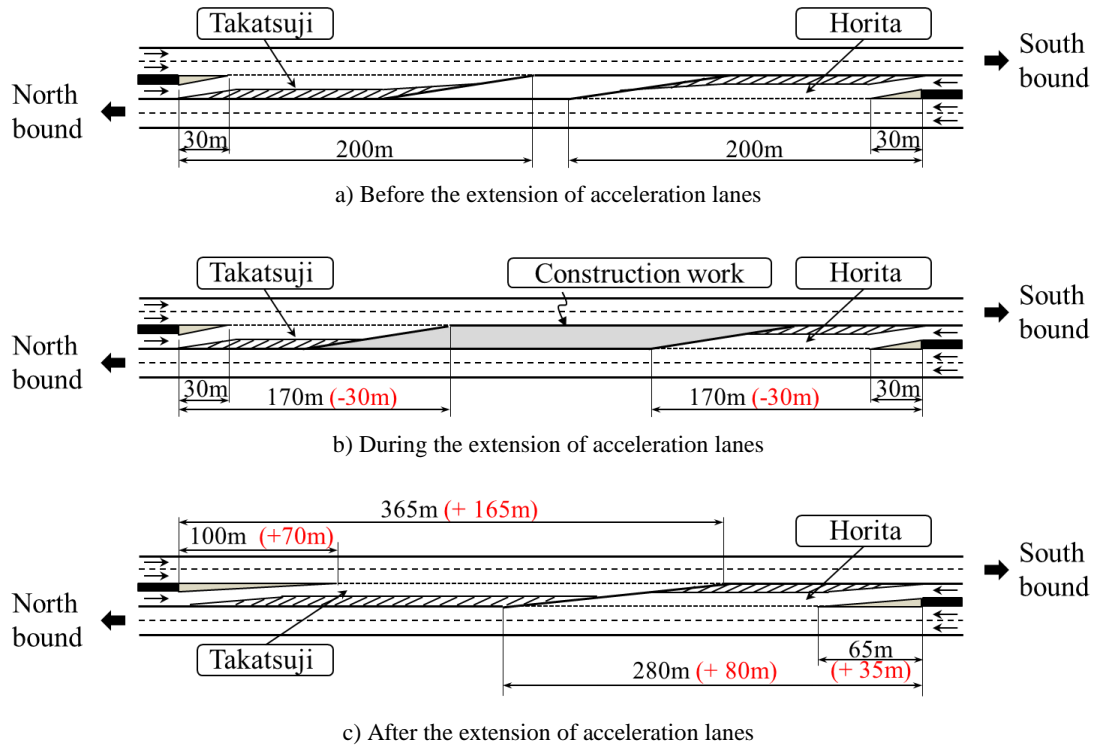
a) Before the extension of acceleration lanes



b) During the extension of acceleration lanes



c) After the extension of acceleration lanes

**Fig.2** Geometry of Horita and Takatsuji.

**Table 1** Video survey period and mainline traffic conditions

| Merging section | Situation | Survey date | Day | Survey time | Mainline flow rate [veh/h-2lane] (Min-Max) |
|---|---|---|---|---|---|
| **Horita** | Before | 09/16/2005 | Friday | 14:00 - 17:00 | 1735 – 3158 |
| | During | 07/26/2011 | Tuesday | 06:00 - 10:50 | 0588 - 3240 |
| | | | | 15:14 - 18:00 | 2484 - 3444 |
| | | 07/30/2011 | Saturday | 05:45 - 09:00 | 0432 – 2232 |
| | After | 11/10/2011 | Thursday | 14:00 - 18:00 | 2064 - 3348 |
| | | 11/13/2011 | Sunday | 07:30 - 10:00 | 1008 – 2580 |
| **Takatsuji** | Before | 01/18/2005 | Tuesday | 08:00 - 10:00 | 2650 - 3325 |
| | During | 08/02/2011 | Tuesday | 09:00 - 11:00 | 2400 - 3072 |
| | | | | 15:00 - 18:00 | 1800 - 2652 |
| | | 08/06/2011 | Saturday | 12:00 - 15:00 | 1500 – 2316 |
| | After | 1/10/2011 | Thursday | 14:00 - 18:00 | 1800 - 2820 |
| | | 01/13/2012 | Friday | 06:45 - 09:30 | 2154 - 3242 |
| | | 01/21/2012 | Saturday | 08:00 - 12:15 | 1584 – 2496 |

route and traffic conditions of during and after become less congested compared to before situation. The observation dates, the duration of survey and the mainline traffic situation are summarized in **Table 1.**

### 3.2 Classification of mainline traffic conditions

Mainline traffic conditions are firstly classified into uncongested and congested regimes by assuming a critical speed of 60[km/h] to investigate their impacts on merging maneuver. Then, the uncongested condition is divided into three levels A, B, and C with the thresholds of flow rate Q [veh/h/2-lane] as follows: A (Q < 1800), B (Q = 1800~2400) and C (Q > 2400). Moreover, the congested condition is de-

noted F as shown in **Fig.3**. The average mainline speed of each condition A, B, C and F is 85.2, 78.0, 72.6 and 38.5 [km/h], respectively.

### 3.3 Classification of gap choice

In order to analyze and model the gap choice behavior, there is a very important question; where or when merging vehicles decide to accept or to reject an available gap. Such a decision point is not fixed but distributed dependent on driver behavior, visibility, speed of mainline vehicles, etc. Since determining this decision point is very difficult, it is assumed as the position when the drivers reach the soft nose for "before" and "during" situation. Note that,

the distance from physical nose to soft nose is 30m for "before" and "during" situation at both Horita and Takatsuji. For "after" situation, since the distance from physical nose to soft nose was extended (65m at Horita and 100m at Takatsuji), considering the consistency, it is assumed that drivers would make decision when they reach the distance of 30m from physical nose as illustrated in **Fig.4**.

Based on that assumption, the gap choices are classified into three patterns considering the reaction of merging vehicles to mainline's adjacent gap at decision point as explained in **Fig.5**. If merging vehicle accepts adjacent gap, it is denoted as "direct merging" while it is called "yield merging" if merging vehicle rejects adjacent gap and merge into the following gap. In case merging vehicle increases speed and chooses the gap forward the adjacent gap, it is called "chase merging".

### 3.4 Data processing

The trajectories of merging vehicles as well as corresponding mainline leading and following vehicles are extracted from video data by using the image processing system TrafficAnalyzer developed by Suzuki and Nakamura[15]. The position and timing of each vehicle was extracted every 1.0 sec and then, by using Kalman Smoothing function, vehicle trajectories were estimated for every 0.1 sec. The reference observation point, which also is considered as the position of the vehicle at each time interval, is right-rear wheel of the vehicle. It is important to note here that only leading merging vehicles are considered in the data processing. Leading merging vehicles are defined as the vehicles that did not face any other merging vehicles while entering the physical nose. Then, for each merging vehicle, speed and position of the merging vehicle as well as corresponding mainline leading and following vehicles were extracted from trajectory data.

## 4. ANALYSIS ON MERGING MANEUVER

### 4.1 Merging position with acceleration lane length

**Fig.6** gives the comparison of merging position dependent on acceleration lane lengths at Horita merging section. The figure shows that with the same mainline traffic condition and within the range available at this site, the longer acceleration lane length is, the further merging position becomes.
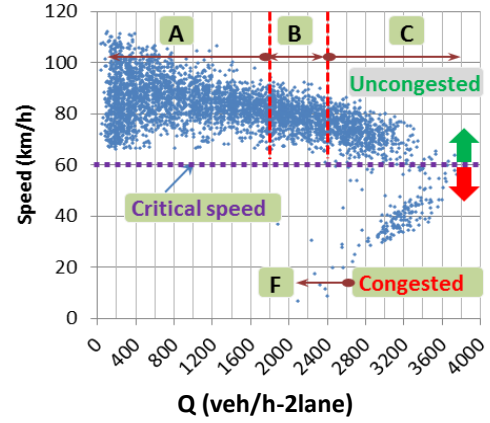


**Fig.3** Classification of mainline traffic conditions (*)
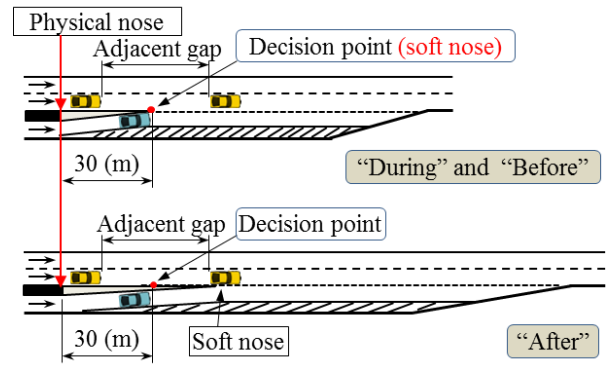(*) Using detector data at Horita from 06/06 to 06/10/2007
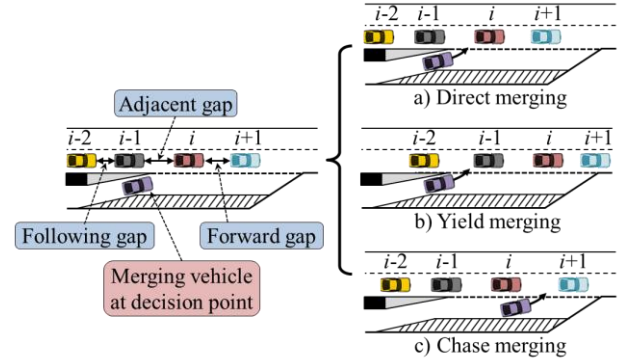


**Fig.4** Assumed decision point



**Fig.5** Classification of gap choice



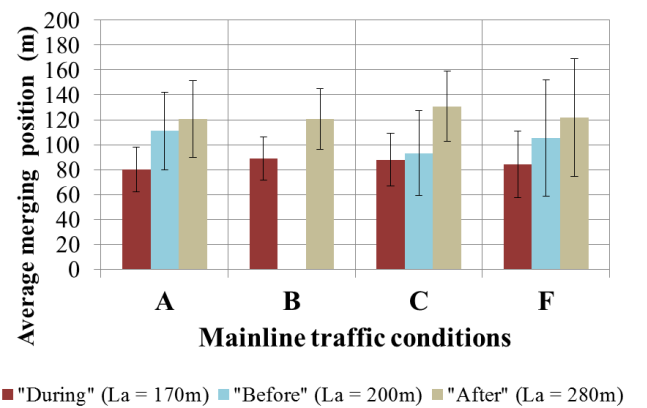■ "During" (La = 170m)  ■ "Before" (La = 200m)  ■ "After" (La = 280m)
**Fig.6** Comparison of merging position with different acceleration lane lengths at Horita

**Table 2** Summary of gap choice at study sites for different mainline traffic conditions.

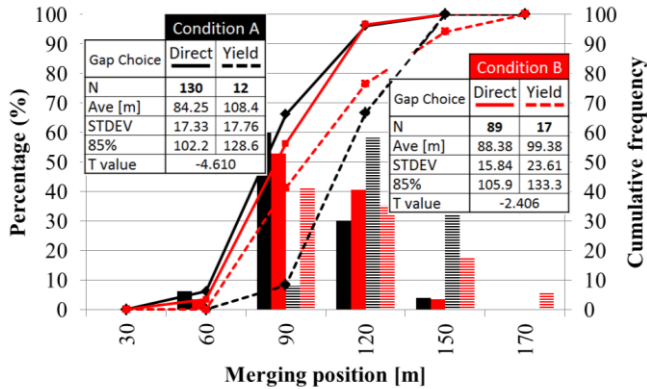| | | Before | | | | During | | | | After | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Traffic condition** | | **A** | **B** | **C** | **F** | **A** | **B** | **C** | **F** | **A** | **B** | **C** | **F** |
| **Horita** | *Total* | - | *46* | *96* | *96* | *64* | *55* | *45* | *44* | *49* | *34* | *58* | *38* |
| | Direct | - | 40 | 44 | 44 | 60 | 45 | 39 | 28 | 48 | 31 | 51 | 32 |
| | Yield | - | 1 | 0 | 0 | 4 | 10 | 1 | 0 | 1 | 3 | 6 | 1 |
| | Chase | - | 5 | 52 | 52 | 0 | 0 | 5 | 16 | 0 | 0 | 1 | 5 |
| **Takatsuji** | *Total* | - | - | *81* | *81* | *78* | *51* | *23* | *82* | *81* | *41* | *90* | - |
| | Direct | - | - | 56 | 56 | 70 | 44 | 16 | 59 | 80 | 40 | 88 | - |
| | Yield | - | - | 3 | 3 | 8 | 7 | 4 | 1 | 1 | 1 | 1 | - |
| | Chase | - | - | 22 | 22 | 0 | 0 | 3 | 22 | 0 | 0 | 1 | - |

Note: "-" not available



**Fig.7** Merging position distributions of direct and yield merging (traffic conditions A and B)
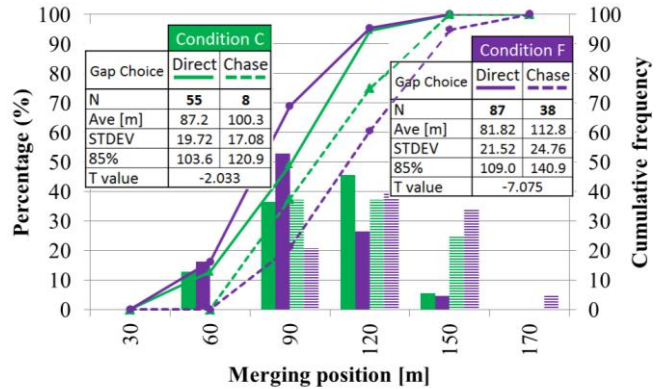


**Fig.8** Merging position distributions of direct and chase merging (traffic conditions C and F)
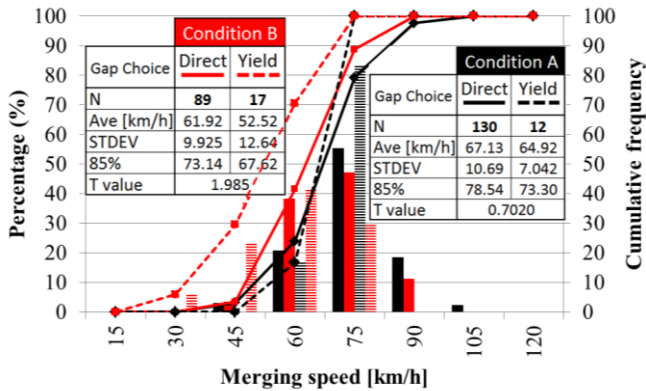


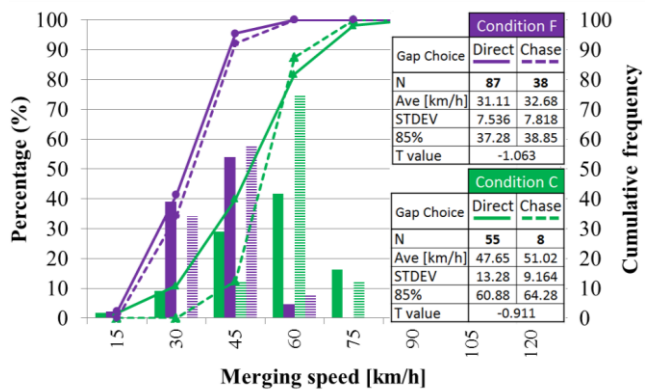**Fig.9** Merging speed distributions of direct and yield merging (traffic conditions A and B)



**Fig.10** Merging speed distributions of direct and chase merging (traffic conditions C and F)

### 4.2 Gap choice behavior with mainline traffic conditions

**Table 2** shows proportion of gap choice at study sites for "before", "during", and "after" situations, respectively. From this table, it is obvious that the high proportion of direct merging pattern at both sites were observed. It means for all conditions, drivers tend to utilize the adjacent gap available for them at decision point. For yield merging pattern, it can be frequently observed under conditions A and B while seldom be observed under conditions C and F. The reason might be that under conditions A and B, the speeds of the mainline vehicles are usually higher

than that of merging vehicles; therefore, drivers feel unsafe to merge and reject the adjacent gap. On the contrary, under condition F, the higher proportion of chase merging pattern can be observed compared to other conditions. It is because under condition F, the speeds of the mainline vehicles are low and also the gap is sometime small to be accepted. As a result, some merging vehicles would go to the last half of merging section to merge in.

### 4.3 Gap choice behavior with merging speed and merging position

Due to lack of yield and chase sample size, the

data of Horita and Takatsuji of "during" situation was combined to analyze the relationship between gap choice behavior and merging speed as well as merging position. As mentioned in section **4.2**, yield merging pattern can be frequently observed under conditions A and B while chase merging pattern can only be observed under conditions C and F. Therefore, the relationship between direct and yield is analyzed under conditions A and B as shown in **Fig.7** and **Fig.8**. Also, the relationship between direct and chase is analyzed under conditions C and F as presented in **Fig.9** and **Fig.10**.

**Fig.7** and **Fig.8** show that both yield and chase choice result in significantly further merging position compared to direct choice under the same traffic condition. Such behavior is necessary to be considered when designing acceleration lane. Regarding merging speed, from **Fig.9** and **Fig.10**, it is obvious that merging speeds of yield merging vehicles are lower than that of direct merging vehicle and merging speeds of chase merging vehicles are higher than that of direct merging vehicles. It is understandable since in case of yield, the speeds of merging vehicles are lower than that of mainline vehicles so they can not directly merge. Meanwhile, in case of chase, the speeds of merging vehicles are higher than that of mainline vehicles. So, merging vehicles should reduce their speed to directly merge while they would keep their current speed or increase their speed if they want to overtake the mainline vehicles. It may result in higher merging speed compered to direct merging.

## 5. GAP CHOICE BEHAVIOR MODELING

Empirical analysis of merging maneuver as discussed above showed that both yield and chase choice result in further merging position compared to direct one. This fact is necessary to be considered when designing the length of acceleration lane. Therefore, understanding under what kind of conditions drivers will choose yield or chase is very important. In this section, gap choice behavior is modeled by applying binomial logit model. The general form of the model is shown in equation (1):

$$P_i = \frac{e^{v_i}}{1 + e^{v_i}} \qquad (1)$$

$$v_i = \beta_o + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_n x_n \qquad (2)$$

Where $P_i$ is probability of choosing pattern $i$, $v_i$ is dependent variable, $x_j$ $(j=1\text{-}n)$ is independent variable and $\beta_k$ $(k=0\text{-}n)$ is parameter.

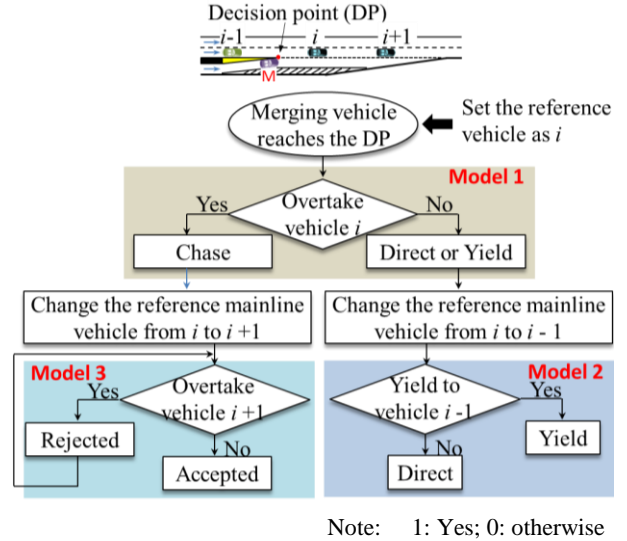**Fig.11** shows a framework of gap choice model. When merging vehicles arrive at the decision point,



<center>Note: 1: Yes; 0: otherwise</center>

**Fig.11** Framework of gap choice model



a) At decision point $t = t_o$ (apply for **model 1** and **model 2**)



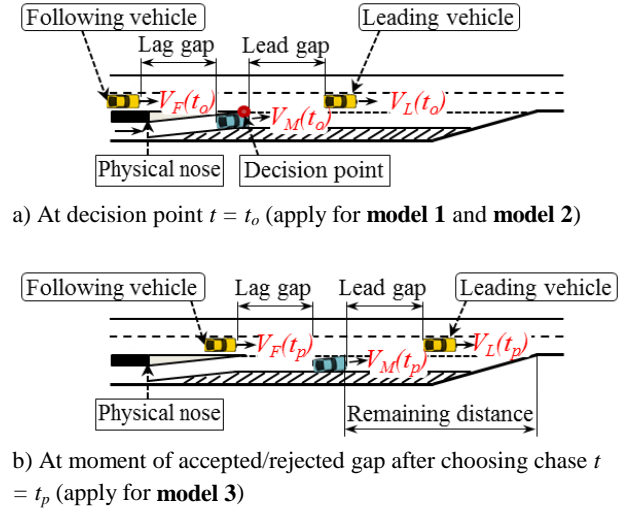b) At moment of accepted/rejected gap after choosing chase $t = t_p$ (apply for **model 3**)

**Fig.12** Definition of variables

they would first adjust their relative speed and relative distance to the mainline vehicle to decide whether to overtake the leading vehicle $i$ (chase merging) or not (direct or yield merging, Y-D). This behavior is illustrated by **model 1**. Then, **model 2** is developed for representing the direct or yield choice. Notably, it was observed from video data that when merging vehicles chose yield merging, they immediately accepted the following gap. On the contrary, for the chase merging, it was observed from video data that merging vehicle may overtake several mainline vehicles before choosing a certain gap to merge in. Therefore, **model 3** is developed for representing this behavior.

**Fig.12** explains variables which are used for modeling gap choice. These variables are lead gap [m], lag gap [m], relative speed between leading mainline vehicle and merging vehicle ($V_L(t) - V_M(t)$, [km/h]), relative speed between merging vehicle following and mainline vehicle ($V_M(t) - V_F(t)$,

<center>6</center>

[km/h]), acceleration lane length [m], and traffic conditions. Notably, the gap and relative speed are defined at different moments. Firstly, they are defined at the moment $t = t_o$ when merging vehicles reach decision point (**Fig.12**a). These gap and relative speed are used for estimating the **model 1** and **model 2**. In case of chase merging, the gap and relative speed at the moment $t = t_p$ when merging vehicles choose to accept the forward gap or continue to chase (reject). In this case, remaining distance variable is added. It is defined as the distance from the front wheel of merging vehicle to the end of acceleration lane [m] (**Fig.12**b). Notably, all the variables were tested for each model; however, significant variables only (95% confidence level) were integrated in the models. Moreover, beside the space gaps, the time gaps were also considered, but excluded because they were found to be not significant. This finding is consistent with conclusion by Hwang and Park[10].

**Table 3** shows the estimated results of **model 1**. The gap choice to choose chase or D-Y is significantly affected by relative speed and space gap between mainline vehicles and merging vehicles. Furthermore, uncongested/congested condition becomes a significant variable. When the traffic is more congested, drivers are more likely to select chase. It is understandable since with the same relative speed, when the traffic becomes more congested, the absolute value of the speed is small; merging vehicle is easy to overtake mainline vehicles.

The estimated results of **model 2** are shown in **Table 4**. It is noted that the relative speed and gap between mainline leading vehicles and merging vehicles were found to be not significant (95% confidence level). This is logical because when drivers decide to choose yield or direct merging, they would adjust their speed and position to the following mainline vehicles. Moreover, acceleration lane length and traffic condition were found to have significant effects on choosing yield/direct. The probability of choosing direct increases as acceleration lanes become longer. The reason might be that, long acceleration lane, drivers will try to accelerate to close the difference of speed with mainline vehicles, and therefore, they can directly merge. Regarding the traffic conditions, when traffic becomes more congested, since the speeds of mainline vehicles are smaller, merging drivers are easy to directly merging into the mainline.

In contrast to **model 2**, in **model 3** (**Table 5**), the relative speed and gap between mainline following vehicles and merging vehicles were found to be not significant (95% confidence level). It is because when merging drivers decide to continue to overtake

**Table 3** Estimated results of model 1.

| Dependent variables | Coef. | T-value (Sig.) |
|---|---|---|
| Lead relative speed ($V_L$ - $V_M$, [km/h]) | -0.177 | 37.1 (0.000) |
| Lag relative speed ($V_M$ - $V_F$, [km/h]) | 0.070 | 6.91 (0.009) |
| Lead gap [m] | -0.197 | 17.9 (0.000) |
| Lag gap [m] | 0.113 | 8.27 (0.004) |
| Traffic condition<br> - Uncongested (A~C): 0<br> - Congested (F): 1 | 1.73 | 12.8 (0.000) |
| Constant | -3.13 | 23.2 (0.000) |
| -2 Log likelihood | 204 | |
| Percentage correct | 96.0 | |
| Sample size | 1008 | |

**Table 4** Estimated results of model 2.

| Dependent variables | Coef. | T-value (Sig.) |
|---|---|---|
| Lag relative speed ($V_M$ - $V_F$, [km/h]) | -0.108 | 42.6 (0.000) |
| Lag gap [m] | -0.265 | 36.3 (0.000) |
| Acceleration lane length [m] | -0.023 | 18.6 (0.000) |
| Traffic condition<br> - Condition A and B: 0<br> - Condition C: 1 | -0.957 | 3.57 (0.039) |
| Constant | 4.34 | 15.9 (0.000) |
| -2 Log likelihood | 138 | |
| Percentage correct | 96.3 | |
| Sample size | 575 | |

**Table 5** Estimated results of model 3.

| Dependent variables | Coef. | T-value (Sig.) |
|---|---|---|
| Lead relative speed ($V_L$ - $V_M$, [km/h]) | -0.221 | 32.3 (0.000) |
| Lead gap [m] | -0.245 | 7.36 (0.007) |
| Acceleration lane length [m] | -0.061 | 10.4 (0.001) |
| Remaining distance [m] | 0.106 | 31.7 (0.000) |
| Constant | -2.79 | 0.968 (0.325) |
| -2 Log likelihood | 69.0 | |
| Percentage correct | 96.3 | |
| Sample size | 375 | |

**Table 6** Comparison of estimated and observed values of **model 1**.

| | | Estimated | |
|---|---|---|---|
| | **Gap choice** | Direct/Yield | Chase |
| **Observed** | Direct/Yield | **746** | 18 |
| | Chase | 22 | **222** |

**Table 7** Comparison of estimated and observed values of **model 2**.

| | | Estimated | |
|---|---|---|---|
| | **Gap choice** | Direct | Yield |
| **Observed** | Direct | **522** | 6 |
| | Yield | 15 | **32** |

**Table 8** Comparison of estimated and observed values of **model 3**.

| | | Estimated | |
|---|---|---|---|
| | **Gap choice** | Accepted | Rejected |
| **Observed** | Accepted | **122** | 8 |
| | Rejected | 6 | **238** |

or not, they would adjust their speed and position to leading mainline vehicle rather than following vehicles. **Table 5** shows that the probability to accept gap increase as acceleration lanes becomes longer. The reason might be that with longer acceleration lane, the gap size is large so merging drivers can accept that gap. Also, beside acceleration lane length, remaining distance was found to be a significant variable. When the remaining distance is shorter, drivers are likely to accept the gap rather than continue to chase.

It should be noted that, due to unbalance of gap choice sample, it is important to check whether the models can well represent the gap choice or not. **Table 6** to **Table 8** show the comparison of estimated and observed values of **model 1** to **model 3**. In these tables, the underline values show correct values resulting from the models compared to the observed one. From theses table, it can be concluded that the **model 1** and **3** can nearly able to reproduce the observed values without bias. However, **model 2** cannot well reproduce the yield choice due to unbalance sample size of yield choice compared to direct one. Therefore, collecting enough sample size of yield choice is necessary for developing more accurate model.

# 6. CONCLUSIONS

In this paper, gap choice behavior at urban expressway merging sections was analyzed and modeled considering the effects of mainline traffic conditions and acceleration lane length.

Empirical analysis showed that mainline traffic conditions significantly impact on the proportion of gap choice. Yield choice can be observed more frequently in traffic condition A and B due to high mainline speed of these conditions while chase choice can be observed mostly in traffic condition F. For direct choice, it can be observed in all conditions. It means that drivers are likely to utilize every gap closed to them at decision point. Yield and chase choice result in further merging position compared to direct one.

The gap choice behavior was modeled by applying binomial logit model. Generally, it is found that relative speed, space gap between mainline vehicles and merging vehicles, traffic conditions, acceleration lane length, and remaining distance to the end of acceleration lane are the most significant influencing factors.

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