Cost-Benefit Analysis for Network Reliability Improvement

Shuming FANG[1], Hiroshi WAKABAYASHI[2]

[1]Student Member of JSCE, Faculty of Urban Science, Meijo University
(Nijigaoka 4-3-3, Kani, Gifu 509-0261, Japan)
Fangshuming999@126.com
[2]Member of JSCE, Professor, Faculty of Urban Science, Meijo University
(Nijigaoka 4-3-3, Kani, Gifu 509-0261, Japan)
wakabaya@urban.meijo-u.ac.jp

A highly reliable traffic network is very important for both abnormal and normal period. Network reliability can be improved effectively by improving the most important key link in the network. When such important link is once found, network reliability can be improved and maintained efficiently. Thus, some indexes have been proposed for finding the most important key link in the network. However, these indexes have their defects. Therefore, a good solution cannot be obtained by these indexes for evaluating network reliability improvement. In addition, the point of view of cost / benefit is also important.

This paper presents the following contents: Firstly, previously proposed indexes are summarized. Secondly, since the calculation work for network reliability increases exponentially with the number of links of the network, an enormous amount of CPU time and memory size should be needed (NP-hard problem). Therefore an efficient Calculation Algorithm for Boolean Absorption (CABA) with a partial differential is proposed. It enables to calculate reliability and importance automatically even for a very large scale network. Using CABA, the processes of network improvement with some indexes are compared for a small network. Then the features and defects of these indexes are pointed out. Thirdly, a cost-benefit analysis method is proposed in order to improve the previously proposed indexes. Series, parallel and simple bridge networks are discussed. Depending reliability-cost function, the behavior of network improvement process will be found different. A general conclusion for effective and efficient network improvement is presented.

Key Words: *Terminal Reliability; Probability Importance; Criticality Importance; Cost-Benefit Analysis; Boolean Absorption.*

1. INTRODUCTION

It is important to keep a highly reliable traffic network for abnormal period especially disaster period. However, the traffic system may be damaged seriously, and what damaged road should be selected is very difficult for reconstructing to maintain or improve the traffic network reliability. In addition, terminal reliability of the traffic network is often calculated by the combination of path sets and cut sets. For example, since path sets are a set of links, the reliability of one path set is combination of link reliability. Therefore, when a key link for improving the terminal reliability is once found, the network reliability can be improved and maintained efficiently by improving the link reliability of the key link. Thus, some indexes such as probability importance and criticality importance have been proposed. However, these indexes have own demerits for finding the most important key link in the network. In addition, the cost for repairing the traffic system is also important for improving the network reliability. Therefore, the point of view of cost/benefit for improving the traffic network reliability should be discussed.

2. CURRENT RELIABILITY INDICES

The concept of importance has been proposed for long period in the system engineering field, but has appeared in only some papers in the transportation field[1)]. Importance is defined as the degree of magnitude that improvement in reliability of a link contributes for system reliability. In this paper, importance is based on the connectivity reliability.

(1) Reliability Importance

The terminal reliability of the highway network is defined as the probability that two given nodes over the network are connected with a certain service level of traffic for a given time period[7),10)]. Similarly, link reliability in the network is defined as the probability that the traffic is in a certain service level for a given time period. Terminal reliability, R, is given by the minimal path sets expression as following[7];

$$R(r) = E[1 - \prod_{S=1}^{N}(1 - \prod_{a \in P_S} X_a)] \quad \text{(Eq.1)}$$

Where Ps is the S-th minimal path set, and N is the total number of minimal path set, and X_a is a binary indicator variable for link a (and equals 1 if link a survives or provides the certain support, and equals 0 otherwise), and link reliability $r_a = E[X_a]$.

The terminal reliability of a traffic network depends on the network structure and the link reliabilities. There are, therefore, two basic approaches to improving network reliability, namely, to improve the network structure or to improve the reliability of the links. The focus here is identifying which links should be improved, to maximize the improvement in network reliability. In order to find out the key link to improve the terminal reliability most efficiently, the Birnbaum's structural importance[2] has been proposed. The Birnbaum's structural importance is shown as Eq.2;

$$RI_a = \partial R(r) / \partial r_a \ (\text{and } 0 \leq RI_a \leq 1) \quad \text{(Eq.2)}$$

Although Birnbaum's structural importance has potentiality in improving network reliability, it has a demerit to be stated in this section.

For the case of two links in series network, the terminal reliability R_{AB} follows from Eq.1 is:

$$R_{AB} = r_1 r_2 \quad \text{(Eq.3)}$$

For the case of two links in parallel network, the terminal reliability R_{AB} follows from Eq.1 is:

$$R_{AB} = 1 - (1 - r_1)(1 - r_2) \quad \text{(Eq.4)}$$

The probability importance for these two links in parallel network, RI_1 and RI_2, are obtained from Eq.2 and Eq.4 as $RI_1 = 1 - r_2$ and $RI_2 = 1 - r_1$. If $r_1 > r_2$, $RI_1 > RI_2$ is hold.

The result indicates that in case of parallel typed network, improving the more reliable link will be more effective for improving terminal reliability. According to common sense, it is difficult to improve more reliable link whereas it is rather easy to improve less reliable link. This result is actually irrational for improving, and maintaining network.

(2) Criticality Importance

Because of the demerit of Birnbaum's structural importance, Criticality Importance was proposed which is the ratio of the proportional improvement in the network reliability to the proportional improvement in the link reliability[5].

$$CI_a = \frac{\partial R / R}{\partial r_a / r_a} = RI_a \frac{r_a}{R} \quad \text{(Eq.5)}$$

Based on the defect of Eq.5, Wakabayashi also proposed the criticality importance as the proportion of the marginal change in terminal reliability against the marginal change in the reliability engineering, and Criticality Importance CIW_a is introduced as Eq.6.

$$CIW_a = \lim_{\Delta q_a \to 0}\left\{-\frac{\Delta R(r)/R(r)}{\Delta q_a / q_a}\right\} = RI_a \frac{1 - r_a}{R} \quad \text{(Eq.6)}$$

Where $q_a = 1 - r_a$ is used for the unreliability of link a.

For the case of two links 1 and 2 in series network, it follows fromEq.3 and Eq.5 that:

$$CI_1 = \frac{r_1 r_2}{R} = CI_2 \quad \text{(Eq.7)}$$

For the case of two links 1 and 2 in series network, it follows from Eq.3 and Eq.6 that:

$$CIW_1 = \frac{1 - r_1}{r_1} \quad \text{and} \quad CIW_2 = \frac{1 - r_2}{r_2} \quad \text{(Eq.8)}$$

If $r_1 > r_2$, $CIW_1 < CIW_2$ is hold from Eq.8.

For the case of two links 1 and 2 in parallel, it follows from Eq.4 and Eq.6 that:

$$CIW_1 = \frac{(1 - r_1)(1 - r_2)}{r_1 + r_2 - r_1 r_2} = CIW_2 \quad \text{(Eq.9)}$$

From Eq.7 and Eq.9, the Criticality importance index is same for both links in some type network, so it does not help distinguish between them in terms of improving network reliability.

The formula of the criticality importance which Henley and Kumamoto proposed or Wakabayashi proposed can't make an expected result

The reliability importance and the criticality importance mentioned above cannot explain explicitly for selecting the most important key link of traffic network because of their own demerits. Therefore, a good solution cannot be obtained by these indexes for evaluating network reliability improvement. In addition, although the point of view of cost-benefit is also important[8], those indexes cannot explain explicitly the increase in cost of improving link reliability when link reliability increases. Thus the traffic network reliability increase in accordance with the variety of the cost should be discussed.

3. COST-RELIABILITY FUNCTION

According to the index CIW in Chapter 2, the less reliable link in series network should be improved in accordance with Eq.8. However, the result from Eq.9 does not provide distinguishable information which link should be improved firstly in parallel network. Thus the improvement of traffic network reliability in accordance with the variety of the cost will be proposed in this Chapter.

Table 1 Variety of cost increase with reliability increase

Reliability increase	Cost increase of Case 1	Cost increase of Case 2
0.0→0.1	500	500
0.1→0.2	1000	1500
0.2→0.3	1500	3000
0.3→0.4	2000	5000
0.4→0.5	2500	7500
0.5→0.6	3000	10500
0.6→0.7	3500	14000
0.7→0.8	4000	18000
0.8→0.9	4500	22500
0.9→1.0	5000	27500

The needed cost strategies to improve the link reliability are assumed to be three cases:

Case 1: The cost increase is a constant amount when the link reliability increases under the same increase degree of link reliability and is shown in case l of Table 1 (constant amount equals 500 (unit is 10,000 Yen)).

Case 2: The cost increase is a progressive increase when the more reliable link is improved under the same increase degree of link reliability and is shown in case 2 of Table 1 (progressive increase equals 500).

Case 3: The cost to increase the link reliability is fixed under the same increase degree of link reliability (fixed cost is 1000).

The effect of improvement of network reliability caused by cost increase may be not obvious in the short time, thus a simple cost-benefit function as the improvement of the network reliability against the cost increase for a long time is defined as Eq.10.

The effect of cost increase to improve the link reliability may be not obvious in the short time, so the cost-effect function for a long time is defined as Eq.10, where Y shows the number of years to invest, F shows the benefit of all kinds of traffic consumption, R_{ab0} means the original network reliability, and $Cost_{ab}$ shows the cost increase to improve the network reliability from R_{ab0} to R_{ab}.

$$Eff(Y,F) = \frac{\{R_{ab} - R_{ab0}\}}{Cost_{ab}} \times Y \times F \qquad \text{(Eq.10)}$$

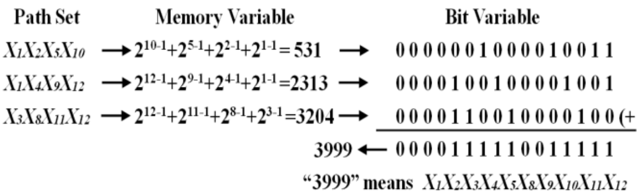

Fig.1 Process of Boolean absorption for Terminal Reliability

4. CALCULATION ALGORITHM OF BOOLEAN ABSORPTION FOR TERMINAL RELIABILITY AND *RI*

Boolean absorption is used to calculate the exact value of terminal reliability[6)]. However, the manual work, including an expansion and Boolean absorption, is very complicate and impractical as the size of the network expands. In addition, it tends to lead to miscalculation. Therefore, an algorithm for processing this calculation by a computer named as CABA has been developed. The main point of this algorithm is to expand Eq.1 directly. In addition, it is designed to generate and to unify each term by turns for efficiency. Furthermore, only one bit of the memory of a computer is used to memorize each random variable of every link of the network. In addition, the reliability importance (RI) can be obtained by Eq.2. Thus, the calculation for RI of link a is also obtained by this algorithm. The algorithm is as following:

Step 1: Let p be the number of minimal path sets to be used in this calculation. Memorize these minimal path sets. Here, every minimal path set that is composed of links expressed as binary number is memorized as a decimal number. For example, the minimal path set, $\alpha = X_1 X_2 X_5 X_{10}$, that is, {1, 2, 5, 10}, is expressed as the binary number 0000010000010011 (read this figure from the right). At this step, the number is translated into a decimal number then memorized; the binary number 0000010000010011 is memorized as the decimal number 531 (=20+21+24+29). This procedure permits reduction of the memory region size used in the computer.

Step 2: Let $m = 1$. m is the number of minimal path sets in every iteration.

Step 3: Any product composed of m minimal path sets (obtained in the expansion of Eq.1 into $2^p - 1$ terms) is expressed as: $(-1)^m \cdot \alpha_{S_1} \cdot \alpha_{S_2} \cdots \cdot \alpha_{S_n}$.

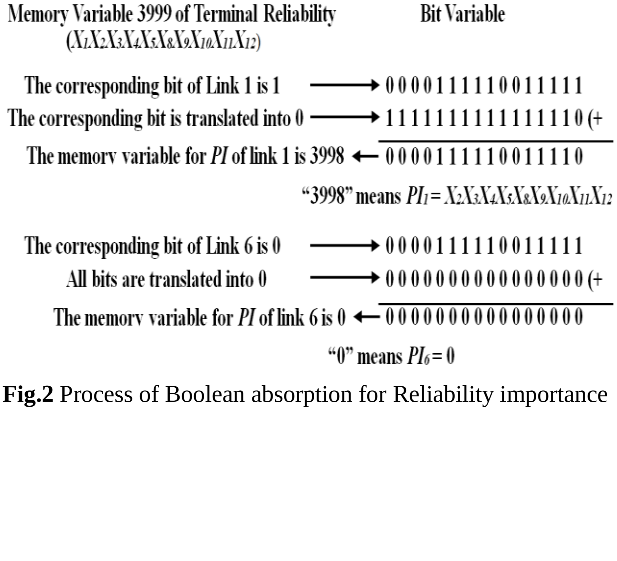

Fig.2 Process of Boolean absorption for Reliability importance

Arrange this product by Boolean absorption in terms of links. For example, the product of the minimal path sets {1, 2, 5, 10}, {1, 4, 9, 12}, {3, 8, 11, 12} is translated into the memory variable 3999 which indicates $X_1X_2X_3X_4X_5X_8X_9X_{10}X_{11}X_{12}$ .This procedure is demonstrated in Fig.1.

Based on the memory variable of product for terminal reliability, the memory variable for RI of all links can be calculated and stored in other store regions. If the corresponding bit of X_a does not exist in the memory variable of product for terminal reliability, the memory variable for RI of link a translates into 0, otherwise, the corresponding bit of X_a in memory variable of product for terminal reliability is translated into 0 and the new memory variable is stored in other store regions as the memory variable for RI of link a . For example, the product of RI_1 is $X_2X_3X_4X_5X_8X_9X_{10}X_{11}X_{12}$ based on the memory variable 3999 for terminal reliability, thus the memory variable for RI of link 1 is 3998. However, the memory variable for RI of link 6 is 0. This procedure id demonstrated in Fig.2.

Step 4: Combine like terms. The products generated in step 3 are checked whether or not the same product has been generated in the preceding process. For the above examples, the number 3999 and 3998 are checked whether or not the same number exists in the own store regions. When the same product exists, the coefficient of the product is updated; when not, it is newly stored.

Step 5: Iterate step 3 and step 4 for all combinations of $(-1)^m \cdot \alpha_{S_1} \cdot \alpha_{S_2} \cdot \cdots \cdot \alpha_{S_n}$. The number of iterations is $\binom{p}{m}$.

Step 6: Iterate step 3 to step 5 for $m = 2,3,\cdots,p$.

Step 7: Each number in the store regions corresponds to each term in the polynomial expression of X_a , for which Boolean absorption has already been carried out. If the number 3999 remains in the store region for terminal reliability, the corresponding term, $X_1X_2X_3X_4X_5X_8X_9X_{10}X_{11}X_{12}$ exists in the polynomial expression for terminal reliability. Similarly, if the number 3998 remains in the store region for reliability importance, the corresponding term, $X_2X_3X_4X_5X_8X_9X_{10}X_{11}X_{12}$ exists in the polynomial expression for reliability importance. Therefore, the value of terminal reliability and reliability importance are obtained by substituting the value for the link reliability in the corresponding terms.

5. NUMERICAL EXAMPLES

The Cost-benefit function for improvement of traffic network reliability was proposed in Chapter 3, and three cost strategies were proposed in order to find the most important key link to improve the network reliability mostly. According to the index proposed by Wakabayashi, the least reliable link in series type network should be selected as the most important key link to improve the network reliability mostly, thus, in this section, only a simple parallel network and a simple bridge network are selected to analyze the cost benefit function for improvement of network reliability.

In this Chapter, two strategies for selecting the most important key link to improve the network reliability are discussed:

The most reliable link will be selected as the most important key link according to RI and CI ;

The least reliable link will be selected as the most important key link according to common sense.

(1) The Cost-benefit Analysis for Simple Parallel Network

The effect of the cost-benefit analysis will be

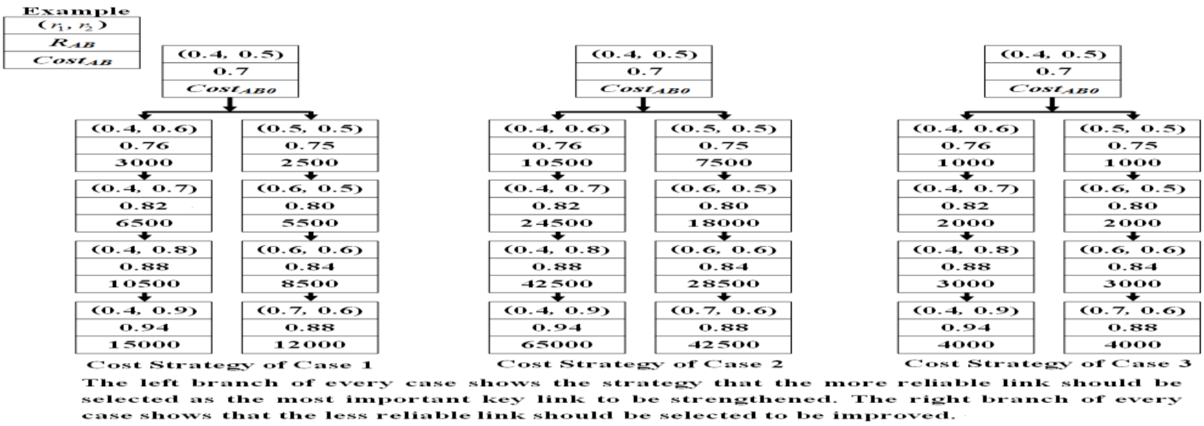

Fig.3 the effect of three cost strategies for improving the link reliability of simple parallel network

discussed in the case of two links 1 and 2 in parallel network, and the original reliability of two links are shown as r_1 =0.4 and r_2 =0.5. Y equals 50 years and F equals a hundred million every year in order to short cut calculation.

Fig.3 shows three cost strategies for improving the link reliability of the simple parallel network, and the left branch of every case in Fig.3 shows that the more reliable link should be improved, and the right branch shows that the less reliable link should be improved.

(a) Cost strategy of Case 1

The cost increase is a constant amount when the link reliability is improved with the same degree in case 1. The result $Eff(50,1)_L$ from the left branch of case 1 is 8 by using Eq.21, and the result $Eff(50,1)_R$ from the right branch of case 1 is 7.5. It means that $Eff(50,1)_L > Eff(50,1)_R$. These results suggest that the more reliable link should be selected as the most important key link based on cost strategy of Case 1.

(b) Cost strategy of Case 2

The cost increase is progressive when the link reliability is improved with the same degree in case 2. The result $Eff(50,1)_L$ is 1.85 from the left branch of case 2 by using Eq.21, and the result $Eff(50,1)_R$ from the right branch of case 2 is 2.12. It means that $Eff(50,1)_L < Eff(50,1)_R$. These results suggest that the less reliable link should be selected as the most important key link based on cost strategy of Case 2.

(c) Cost strategy of Case 3

The cost is fixed when the link reliability is improved with the same degree in case 3. The result $Eff(50,1)_L$ from the left branch of case 3 is 30 by using Eq.21, and the result $Eff(50,1)_R$ from the right branch of case 3 is 22.5. It means that $Eff(50,1)_L > Eff(50,1)_R$. These results suggest that the more reliable link should be selected as the most important key link based on cost strategy of Case 3.

Based on the cost strategies of case 1 and case 3, the more reliable link should be selected as the most important key link; on the contrary, the less reliable link should be selected as the most important key link to be improved based on the cost strategy of case 2. Therefore, the different link should be selected as the most important key link according to the different cost strategies by using the cost-benefit function Eq.10 in parallel type network.

(2) Cost-benefit Analysis for Simple Bridge Network

It is easy to calculate the exact value of terminal reliability and reliability importance of the above-mentioned parallel network. However, it is very complicate and impractical to calculate the exact value of terminal reliability as the size of the network expands.

In this section, there is a simple bridge network that has four notes and five links shown in Fig.4. The minimal path sets of this bridge are P_1 = {1, 2}, P_2 = {3, 4}, P_3 = {1, 5, 4}, P_4 = {3, 5, 2}. The independent minimal path set is a series network system[9], thus, the reliability of minimal path set is shown as following;

$$R(P_1) = r_1 r_2,\ R(P_2) = r_3 r_4,\ R(P_3) = r_1 r_5 r_4,\ R(P_4) = r_3 r_5 r_2 \qquad \text{(Eq.11)}$$

Terminal reliability of this bridge network is given by using Calculation Algorithm of Boolean Absorption (CABA) as Eq.12;

$$R(r) = r_1r_2 + r_3r_4 + r_1r_4r_5 + r_2r_3r_5 - r_1r_2r_3r_4 - r_1r_2r_4r_5 - r_1r_3r_4r_5 - r_1r_2r_3r_5 - r_2r_3r_4r_5 + 2r_1r_2r_3r_4r_5 \qquad \text{(Eq.12)}$$

Reliability importance of this bridge network is given by using CABA as followings;

$$PI_1 = r_2 + r_4r_5 - r_2r_3r_4 - r_2r_4r_5 - r_3r_4r_5 - r_2r_3r_5 + 2r_2r_3r_4r_5, \qquad \text{(Eq.13)}$$

$$PI_2 = r_1 + r_3r_5 - r_1r_3r_4 - r_1r_4r_5 - r_1r_3r_5 - r_3r_4r_5 + 2r_1r_3r_4r_5, \qquad \text{(Eq.14)}$$

$$PI_3 = r_4 + r_2r_5 - r_1r_2r_4 - r_1r_4r_5 - r_1r_2r_5 - r_2r_4r_5 + 2r_1r_2r_4r_5, \qquad \text{(Eq.15)}$$

$$PI_4 = r_3 + r_1r_5 - r_1r_2r_3 - r_1r_2r_5 - r_1r_3r_5 - r_2r_3r_5 + 2r_1r_2r_3r_5, \qquad \text{(Eq.16)}$$

$$PI_5 = r_1r_4 + r_2r_3 - r_1r_2r_3 - r_1r_2r_4 - r_1r_3r_4 - r_2r_3r_4 + 2r_1r_2r_3r_4. \qquad \text{(Eq.17)}$$

The original data of the simple bridge network is shown as r_1 =0.3, r_2 =0.4, r_3 =0.5, r_4 =0.4, r_5 =0.4, Y = 50 and F =100,000,000 yen /year.

(a) Cost strategy of Case 1

The process of cost-benefit analysis is similar as the previous description of Section 5.(1), and the result $Eff(50,1)_L$ about the simple bridge network from the left branch of case 1 is 5.21 by using Eq.10, and the result $Eff(50,1)_R$ from the right branch of case 1 is 7.55. It means that $Eff(50,1)_L < Eff(50,1)_R$. These results suggest that the least reliable link should be selected as the most important key link based on cost strategy of Case 1.

(b) Cost strategy of Case 2

The result $Eff(50,1)_L$ is 1.20 from the left branch of

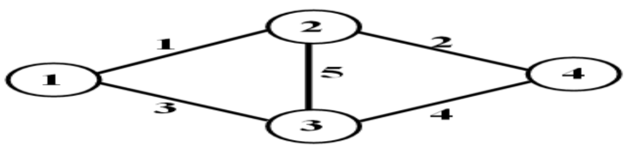

Fig.4 A simple bridge network

case 2 by using Eq.10, and the result $Eff(50,1)_R$ from the right branch of case 2 is 2.61. It means that $Eff(50,1)_L < Eff(50,1)_R$. These results suggest that the least reliable link should be selected as the most important key link based on cost strategy of Case 2.

(c) Cost strategy of Case 3

The result $Eff(50,1)_L$ from the left branch of case 3 is 19.52 by using Eq.10, and the result $Eff(50,1)_R$ from the right branch of case 3 is 17.94. It means that $Eff(50,1)_L > Eff(50,1)_R$. These results suggest that the most reliable link should be selected as the most important key link based on cost strategy of Case 3.

Based on cost strategies of case 3, the most reliable link should be selected as the most important key link; on the contrary, the least reliable link should be selected as the most important key link to be improved based on cost strategy of case 1 and case 2.

In practice, the cost strategies of case 1 and case 2 are rational, in accordance with the well-known "laws of diminishing returns"[8]. Furthermore, the conclusion, that the least reliable link in the simple bridge network should be selected as the most important key link from case 1 and case 2 based on the cost-benefit analysis, is also rational, in accordance with the well-known "it is difficult to improve highly reliable link whereas it is rather easy to improve lower reliable link"[12]. Therefore, the least reliable link in general traffic network should be selected as the most important key link to be improved for increasing the network reliability most effectively.

6. CONCLUSION

In this paper, firstly, in order to discuss the improvement of the traffic network reliability, the current indexes of reliability including PI , CI and CIW , and were introduced and the demerits of these indexes were also pointed out.

And then, the Cost-reliability function was proposed for improvement of the traffic network reliability based on the simple cost-benefit analysis and the Calculation Algorithm of Boolean Absorption for terminal Reliability and probability importance has been developed.

Lastly, two numerical examples for the parallel network and a bridge network were simulated based on the Calculation Algorithm of Boolean Absorption and the cost-reliability function. From these simulations, a general conclusion can be obtained as following:

In the very simple network, the different link should be selected as the most important key link according to the different cost strategies for improvement of the traffic network reliability.

In general network, the least reliable link should be selected as the most important key link for network reliability improvement.

However, there are only three types of traffic network have been used to certificate this conclusion. More types of traffic network should be used for finding the most important key link in some typical networks in the future studies.

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