Reliability and Vulnerability of Road Network: A Research Review from Practicability Perspective

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Abstract: Reliability of transportation network, an emerging field of research, attracts the numerous researchers around the world. Reliability study covers the various aspect of the systems performance quality. This paper presents a review of reliability and vulnerability of road network studies in the context of practicability in two parts. First part covers the subdivision of reliability studies under the broad range of relevant criteria which includes conceptual studies, mathematical theory, practical methodology of evaluation, descriptive studies, application or case studies, and the ways to improve reliability. Second part presents a detailed assessment of existing practical methodology of evaluation under the multidimensional perspective of practicability such as evaluation index, socioeconomic impact, area isolation, theoretical and practical importance, data requirement, calculation time and process, area wise impact of single link failure and probability of disaster. It is found that existing practical evaluation methodologies cover various relevant criteria of practicability, but these methodologies cannot consider the level of threat from the disaster event and to take an account of multiple link failure condition. This study concludes the necessity of the new practical methodology for the evaluation of road network at the emergency condition to overcome the deficiencies of the existing methodologies.

Key Words: *Road network, Reliability, Vulnerability, Accessibility, Redundancy, Network evaluation methodology.*

1. INTRODUCTION

Reliability and vulnerability of a road transportation network, a performance measure of a system or component, plays an important role for the transport network evaluation for the efficient allocation of resources. Transportation network suffers from the natural disaster and human activities, causes partial or complete disruption in the transportation system. However, the severity and weakness in the network are differing from location to location. Identification of weakest location and critical links in a network and prioritize them for the improvement projects is the aim of the evaluation methodology.

Indeed, developing a practical methodology to evaluate the road network, finding the weakest link and prioritize among them is quite difficult and complex. Numerous studies have been done to develop the methodology of evaluation; however practical methodology is still lacking. This paper presents the thorough review of the reliability and vulnerability studies of road network.

Several authors have written review articles on reliability measures, often focusing on certain perspectives such as vulnerability, travel time reliability, connectivity reliability, capacity reliability and encounter reliability (Berdica 2002; Nicholson, Schmocker et al. 2003). Our review differs from the existing review articles in the viewpoint of practical application in the real world. Studies are reviewed 1) under the subdivision of existing studies i.e. (a) Conceptual studies, (b) Mathematical theoretical study of reliability, (c) Practical methodology of evaluation, (d) Descriptive study, (e) Application study of evaluation methodology and (f) The ways to improve reliability. And 2) detailed assessment of the existing practical methodology of evaluation under multi-dimensional criteria including evaluation index, data requirement, calculation time , probability of adverse events, theoretical and practical importance, one link failed and area wise impact and socioeconomic impact of road network disruption . This review is based on extensive literature study, will approach the subdivision of existing studies in section 2, detailed assessment of the existing practical evaluation methodology in section 3, and conclusion of the review paper in the last section.

2. REVIEW OF ROAD NETWORK RELIABILITY STUDIES

This section reviews the existing study of the reliability of road network under different subdivided criteria. The division of the existing reliability study is based on practicality of the methodology of evaluation.

2.1 Conceptual studies: Reliability, Vulnerability and others.

Several concept of reliability studies cover definition, conceptual classification of the road network reliability including review articles from different perspective. (Iida 1999) explains three area of reliability research, first one is "Model development which is capable to investigate the reliability of road network". Second "establishment of traffic management system is able to provide high level of network performance to the users". The last one is "development of new evaluation procedure for optimizing planning, construction and management of road network that incorporates road network reliability." (Nicholson 2003) demonstrates that network users and planners have different viewpoints. From users viewpoint, some questions may arise about reaching the destination, usual route likely to close or not, possibility of encountering the unusual event, delay on usual route or confusion about the decision due to changing the route, cancel the trip or postpone the trip, or change the destination. Similarly, from the planner's viewpoint, how many users could not reach the destination? Which links will be congested or closed (i.e. are weak links)? Which are the important links in the network? Which are critical (important and weak) links? What are the expected economic costs of closure? This implies the significance of reliability studies under emergency condition.

2.1.1 Definition of terms:

Reliability:

Reliability has been defined mainly from two perspectives. First one has been taken from system reliability perspective such as "Reliability is the probability of a device performing its purpose adequately for the period of time intended under the operating conditions encountered" (Wakabayashi and Iida 1992; Berdica 2002). Such definition does not clearly define the reliability of road network. The characteristics of road network are different from the system engineering concept (Iida 1999). Second, from service quality perspective such as "the ability of the transport system to provide the expected level of service quality, upon which users have organized their activities" (OECD 2010). Therefore in the transportation sector all service standard of a network is already achieved at the construction period. However the reliability studies relate to the reduction in the service qualities due to natural and human made adverse events.

Vulnerability:

The concept of vulnerability as defined by the Berdica (2002) " a susceptibility to incidents that can result in considerable reduction in road network serviceability." (Taylor, Sekhar et al. 2006) define the vulnerability in terms of node vulnerability and link criticality. "A network node is vulnerable if loss (or substantial degradation) of a small number of links significantly diminishes the accessibility of the node, as measured by a standard index of accessibility" and "a network link is critical if loss (or substantial degradation) of the link significantly diminishes the accessibility of the network or of particular nodes, as measured by a standard index of accessibility." (Husdal 2004) suggests that vulnerability and reliability can be analyzed through the cost benefit perspective; such as vulnerability is a consequential cost of an operational degradation and reliability is a consequential benefits of an operational improvement. However, quantification of every term in monetary value is quite difficult and complicated.

Risk:

Risk is defined as the probability of hazard and the consequences of the result (Berdica 2002) i.e. expectation of hazard or threat. The evaluation of risk includes level of probability of hazard i.e. lower to highest ; consequence of hazard minor to major and identify the critical links (Nicholson, Schmocker et al. 2003). (Husdal 2004) demonstrates that the risk is a product of vulnerability of the occurrence of external events and probability of threat.

Resilience:

Resilience is the capability of a system that can return to the normal system or can recover quickly from the difficult situation. In the road transportation, (Berdica 2002) defines the resilience as a capability of network to restore the serviceability or capability of reaching the new state of equilibrium .

Robustness:

"Robustness is the extent to which, under pre specified circumstances, a network is able to maintain the function for which it was originally design" (Snelder 2010). And vulnerability is the opposite of the robustness(Berdica 2002).

Redundancy:

The backup alternative i.e. existence of alternative route/link in a road network between origin and destination can result less serious consequence in case of disruption in some part of network (Berdica 2002). Robustness of network increases with the presence of redundancy (Snelder 2010). However, redundancy depends upon the nature of threat, for instance, huge rainfall or snowfall can close all the route and no redundancy presence(Berdica 2002).

2.1.2 Classification of reliability:

A large and growing body of literature has classified the reliability in terms of travel time reliability, terminal/connectivity reliability, capacity reliability, encounter reliability and flow decrement reliability.

Travel time reliability:

A considerable amount of literature has been published on travel time reliability. These studies are focused on whether the travel time taken to reach the destination is within a specified time or not. (Iida 1999) defines "Travel time (or performance) reliability is defined as the probability that traffic can reach a given destination within a stated time". Similarly (Nicholson, Schmocker et al. 2003) give a definition as " The probability that a trip can successfully finish within a specified time interval (or less than a specified cost)". And Berdica (2002) defines "the probability that travel time between two given nodes will not exceed a given travel time." So higher the travel time variance is, the lower will be the travel time reliability (Nicholson, Schmocker et al. 2003). A stochastic model is proposed by (Yin and Ieda 2001) to investigate the day to day travel variation of travelers on travel time reliability .

Connectivity/Terminal Reliability:

Numerous studies have been attempted to explain the connectivity reliability. It has a long history; (Garrison 1960) studies the interstate highway system of USA . The established definition of the terminal reliability is " the probability that nodes are connected, i.e. it is possible to reach the destination"(Nicholson, Schmocker et al. 2003) or " a probability that there exists at least one path without disruption or heavy delay to a given destination within a given time period"(Iida 1999) . The major concern in the connectivity reliability is weather the links are opened or closed, with the state of link "a" is represented by the 0-1 variable (Wakabayashi and Iida 1992; Bell and Iida 1997; Iida 1999). The stochastic variable x_i represents the state of link i with the value 1, if the link is functional and 0 otherwise (Berdica 2002). However connectivity reliability is largely focused on the congested network (Nicholson, Schmocker et al. 2003) It is very important to study the network connectivity after disaster therefore planner can allocate the resources at weakest location.

Capacity, Encounter, and Flow Decrement Reliability:

Various kinds of reliability concept have been introduced by different studies. Capacity reliability is introduced first by (Chen, Yang et al. 1999) as a "probability that the road network can accommodate a certain level of traffic demand". Encounter reliability measures the "likelihood of users encountering a disruption on their preferred route"(Nicholson, Schmocker et al. 2003). They also introduce flow decrement reliability which measures the reliability by using the probability of reduction in flow in a degraded link. Flow will be affected in a degraded link due to affected cost of travel between one or more OD pairs (Nicholson and Du 1997).

2.2 Mathematical, theoretical study of reliability:

There is a large volume of published studies which used mathematical theory of reliability. Mathematical model have been used to evaluate the reliability of road network mathematically rather than practical application. Academically these types of studies have high importance, however, practically not so useful. (Wakabayashi and Iida 1992) introduce a methodology for the evaluation of terminal reliability of a road network by calculating upper and lower bounds of reliability. A new algorithm for the Boolean absorption has been developed for the calculation. (Chen, Yang et al. 1999) develop a Monte Carlo simulation procedure to estimate the capacity reliability. The method calculates the maximum network capacity with the assumption of every OD pair having uniform change of demand. (Asakura 1999) calculates the reliability of the road network with providing information to the user by using stochastic user equilibrium model. The expected result was supposed to increase the reliability but actually it does not always increase. (Nicholson and Du 1997) propose a mathematical model based on supply and demand and traffic equilibrium. The methodology carried out the reliability of the multi model degradable (partially operating) transport network by using integrated equilibrium model. The analysis is based on the different mode not necessarily affected by the same incident. A sensitivity analysis is used to identify the important component in degradable transportation system with the analysis of the socioeconomic impact of system degradation. (Bell 2000) proposes a game theory approach for the measurement of road network reliability. In the non-cooperative game, network user and evil entity are the two players. Network user seeking the path to minimize the expected trip cost and on the other side evil entity choosing the link performance scenario to maximize the expected trip cost, finally result has got an equilibrium condition, the user unable to reduce the expected trip cost by changing his path choice probabilities and evil entity also unable to increase the expected trip cost.

Numerous methodologies have been introduced mathematically for the evaluation of road network link for the resource allocation. A two stage stochastic program proposed by (Peeta, Sibel Salman et al. 2010) , in the first stage they identify the links to be invested and second stage, the minimum traversal cost between O-D pairs is determine. Strength of each link is measured based on the probability of the link component (Bridges and Viaducts) remain operational after disaster. (Sánchez-Silva, Daniels et al. 2005) propose the operational reliability of the transport network for the efficient allocation of resources, considering the state of network, is defined by the Markov model, through physically related failure and repair rate of each link, such that rate can be changed with investment . They also consider reaction of network user with the failure of links along route and waiting time for the user until the link is repaired.

2.3 Practical methodology of evaluation:

Several practical methodologies have been proposed for the identification of critical link in the network. (Taylor, Sekhar et al. 2006) suggest the condition of node vulnerability and link criticality by calculating the accessibility index of the link at the emergency situation. The accessibility index gives a socioeconomic impact to the society and finds out the link which has highest change in the accessibility index after disaster compared to normal condition. (Sohn 2006) also suggests accessibility index in a flood plain, the considered link is taken only from the 100 year floodplain. (Jenelius, Petersen et al. 2006) suggest very similar concept to the (Taylor, Sekhar et al. 2006) such as link importance and municipality exposure, each and every link is evaluated through the change in generalized travel cost. (Scott, Novak et al. 2006) suggest a network robustness index (NRI) of a link. The NRI calculates the change in total travel cost in a network due to failure of particular link. (MLIT 2011b) evaluates the degree of weakness of the specific network link which lies under vulnerable scenario and calculate the ratio of total travel time in a network before and after disaster. (MLIT 2011a) analyzes the degree of isolation by calculating the detour ratio and disaster tolerance function. A detailed assessment from multiple viewpoints is presented of these practical methodologies in section 3.

2.4 Descriptive study:

A large volume of studies about the road network disruption and its impact has been published describing the practical problem and requirement of policy for resources allocation. Not only research paper but also case study or some official report about the road network closure reviewed under this category. (Kawasaki 2011) explains in his presentation, importance of the redundancy and Japanese expressway served as a redundant network on the great east Japan earthquake in 2011. (Consortium 1996) explains the socioeconomic impact in the society due to disruption of transportation network, causes losses of accessibility, disrupt the economic activity across the region and nation, rescue and evacuation problem. It argues the need of pre-disaster policies and raises the issues of prioritizing the allocation of the resources to the multiple impacted areas. (Krik and Chen 2007) argue in their work route disruption analysis as an evaluation criteria and should be focused on the i) defense (national strategic importance), ii) economic importance- Identify the potential impact on the economy due to roadway closure, iii) How many critical path in a network share a given component ,iv) availability of alternative route. (Reconstruction 2011) emphasizes on the construction of disaster resilience transport network and securing the redundancy. (FHWA 2008) suggests the necessity of the durable infrastructure system, tools to identify the critical routes. (Shizuoka 2010) explains about the possibility of village isolation from disaster. (Ramirez, Peeta et al. 2005) suggest necessity of critical route which can survive under earthquake should minimize the total travel time and cover the larger population. (Iida, Kurauchi et al. 2000) describe the necessity of the alternative reliable route or redundant route which is especially designed for the emergency condition.

2.5 Application study of evaluation methodology:

Several case studies have been attempted to analyze the real practical world as a decision tools for the efficient allocation of the resources. (Hou and Hsu 2005) carry out the dominant links under earthquake disaster applied on Kaohsiung City in Taiwan. Supply and demand node were identified based on the three types of service; medical rescue, fire rescue and logistic supply. The link which has high pass frequency to connect the identified supply and demand node among the entire replaceable paths and probability more than 0.5, defined as the dominant link. Enumeration algorithm has been developed to find out the dominant link in a complex network. (Sakakibara, Kajitani et al. 2004) purpose a topological index to quantify road network depressiveness/concentration for the evaluation of isolation of a district and applied in Hanshin region of Japan. (Susilawati and Taylor 2008) apply an accessibility/remoteness index of Australia to evaluate the road network in a Green Triangle Region of Australia. Change in accessibility after assumption of candidate link failure has been calculated. The index is based on the distance to the service center from the remote village or municipality. (Chang and Nojima 2001) develop the post disaster system performance measure of the transport network in Kobe, Japan after 1995 Hyogoken –Nambu earthquake. The performance measure estimates the ratio of post-earthquake to pre-earthquake condition of the total length of network, total distance based accessibility and areal distance based accessibility ranges from 0 (non-functional system) to 1 (fully functional system). (Dalziell and Nicholson 2001) calculate the risk of closure of the Desert road section of New Zealand's major north-south road links in terms of cost. The total cost of road closure assumed to be sum of 1) the change in the vehicle operating and occupant time cost, 2) The lost user benefit from those trips are cancelled or suppressed and 3) Change in the accident cost .

2.6 The ways to improve reliability:

Recently (OECD 2010) suggests four policy strategies to improve the reliability of the network such as 1) Physical expansion of the capacity, to reduce the unexpected disruption in service expansion of infrastructure such as upgrading and adding line capacity ,construction of new road link , built new infrastructure before any incidents take place , however, priority should be to make robust network by improvement of existing infrastructure; building new should only be the last option. 2) Better management capacity- this includes incident management of the vulnerable part of the network by using different techniques and instruments. 3) Developing mechanisms for charging directly for reliability, most of the congestion management and cost recovery according to level of reliability. This kind of charging system should be associated with cost benefit analysis 4) Mitigating the cost burden associated with unreliability using information system. With the establishment of the specific information system, impact and cost of the incident can reduce; however, information itself cannot prevent the incidents.

Table: 1 Summary of review of reliability study

Reliability study	Examples	Characteristics
Conceptual study	(Iida 1999) ,(Nicholson 2003), (OECD 2010),(Taylor, Sekhar et al. 2006) ,(Berdica 2002),(Nicholson, Schmocker et al. 2003),(Garrison 1960),(Bell and Iida 1997) (Chen, Yang et al. 1999),(Nicholson and Du 1997),(Husdal 2004),(Snelder 2010).	• Defining the various terms and concept of reliability. • Reviewing the contemporary study.
Mathematical theoretical	(Wakabayashi and Iida 1992), (Chen, Yang et al. 1999), (Asakura 1999),(Nicholson and	• Mathematical solution of the problem.

study	Du 1997),(Bell 2000),(Peeta, Sibel Salman et al. 2010),(Sánchez-Silva, Daniels et al. 2005)	• Imported from the system performance evaluation of other discipline. • Difficulty to use in practical field. • Complex solution methodology, time consuming and requires highly skilled researcher. • Results are not relevant practically.
Practical methodology of evaluation	(Taylor, Sekhar et al. 2006),(Sohn 2006),(Jenelius, Petersen et al. 2006),(Taylor, Sekhar et al. 2006), (Scott, Novak et al. 2006) ,(MLIT 2011b), (Dalziell and Nicholson 2001)	• Simple evaluation methodology. • Easy to use. • Not very complex data. • Less time consuming. • Have some limitation so result is not plausible. • Could be a foundation for the new evaluation methodology.
Descriptive study	(Kawasaki 2011), (Consortium 1996),(Krik and Chen 2007), (Reconstruction 2011),(FHWA 2008),(Shizuoka 2010),(Ramirez, Peeta et al. 2005), (Iida, Kurauchi et al. 2000)	• Addresses the problem on the field • Addresses the problem faced by the practitioner • Challenge to solve the problem. • Requirement of evaluation methodology
Application study of evaluation methodology	(Hou and Hsu 2005),(Sakakibara, Kajitani et al. 2004),(Chang and Nojima 2001),(Susilawati and Taylor 2008)	• Case study of specific area. • Typical application study.
The ways to improve reliability	(OECD 2010)	• Policy recommendation for the improvement of reliability.

3. DETAILED ASSESSMENT OF EXISTING PRACTICAL METHODOLOGY OF EVALUATION

There are six existing practical methodologies selected for the analysis. These methodologies are classified into three groups based on their index calculation. Table 2 summarizes the evaluation methodology and their objectives.

Table: 2 Summary of practical evaluation methodology

Method	Author	Objectives	Methods
Accessibility approach	(Taylor, Sekhar et al. 2006), (Sohn 2006)	• Develop the methodology to identify the critical location in a road network. • Establish the priority list for improvement of network link.	• Calculate the accessibility index of a location at normal and shortest path failure assumption scenario. • Compare the index among the entire link, higher the change in accessibility index; higher is the priority.
Generalized travel cost approach	(Jenelius, Petersen et al. 2006),(Scott, Novak et al. 2006),(MLIT 2011b)	• To develop the methodology for the evaluation of link based on the generalized travel cost.	• Index is calculated with the assumption of link failure and total change in the generalized travel cost in a network. Link having higher change in generalized travel cost is prioritized first.

Disaster prevention function approach	(MLIT 2011a)	Comparing the links by disaster tolerance and redundancy.	• By calculating the detour ratio and accessing the level of links disaster protection function.

During emergency situation, or when extreme disaster happens three kinds of problem are observed. The first one is connectivity fails between two locations; there is no other option/route/link to connect between the two locations, hence, some area becomes isolated. Second, travel time is increased due to detour route. Third, traffic flow increase in the other survived route immediately after the disaster and causes the problem of capacity/congestion. A severe impact on the community can arise such as problem on rescue and evacuation, problem on post disaster logistic supply and highly impact on economy.

Road network planners need a very simple and practical decision making tools to decide where they should concentrate their resource to make robust network. However, current methodologies cannot address the needs of practitioner but leave very important message for the development of new methodology. The major question is why these methodologies cannot address the need of practitioner? What are the weaknesses of these methodologies? Which parts of these methodologies are important and can be used to develop the new methodology? We analyze here from multi-dimensional perspective.

3.1 Accessibility Approach:

Accessibility index measures the socioeconomic impact of link damage after catastrophic events. The common characteristics of methodology (Taylor, Sekhar et al. 2006) and (Sohn 2006) both calculate the accessibility index before and after assumption of link failure and identify the most critical location on a network. However, there are many differences in these two methodologies.

3.1.1 Hansen integral accessibility index and Accessibility/Remoteness Index of Australia

(Taylor, Sekhar et al. 2006) propose two index for the evaluation of road network link.

Hansen integral accessibility index:

$$A_i = \sum_{j=1} B_j f(c_{ij}) \qquad (1)$$

Where, A_i accessibility index for a location (city) i B_j is attractiveness of location (city) j, in this research B_j has been taken as population of location j. $f(c_{ij})$ is an impedance function calculated as a reciprocal of distance between i and j ($1/x_{ij}$)

Normalized value of accessibility Index

$$A_i = \frac{\sum_{j=1} B_j f(c_{ij})}{\sum_{j=1} B_j} \qquad (2)$$

Accessibility/Remoteness Index of Australia

$$\text{ARIA}_{\text{iL}} = \sum_{L} \min\left\{3, \frac{x_i}{\overline{x}_L}\right\} \qquad (3)$$

Here $\overline{x}_L$ is the mean road distance of all localities to the nearest service center. Service center are categorized as (A, B, C, D, E) based on the population on the city highest to lowest. The maximum value of ARIA will be 15.

Accessibility is calculated at normal condition and after assumption of failure of each link one at a time. The vulnerability is calculated by change in accessibility after failure of a link. The node with higher change in accessibility is defined as vulnerable node. Critical link is that link which has higher change in accessibility in locality at the time of failure assumption. This methodology clearly defines the node vulnerability and link criticality. Evaluating road network in terms of accessibility index is very important, clear measurement and also practical; this methodology claims socioeconomic impact of network degradation and measures the regional network vulnerability. Data collection and calculation procedure are simple. However, the study fails to consider the probability of failure of link i.e. different location have different probability of failure; it assumes all shortest paths (candidate links) fails in turn and accesses the consequence of failure, in reality all shortest path do not have same probability of failure or same risk from the disaster and another problem with this approach is, fails to take the multiple link failure condition into account.

3.1.2 Accessibility index under 100 year floodplain.

The second methodology(Sohn 2006) of accessibility group also calculates accessibility index at normal condition and disaster condition and significance of link is based on the higher change in the accessibility index after a hypothetical disaster. One difference is it assumes candidate link failure under 100 year floodplain, so in the floodplain the probability of link disruption is higher. However it does not consider depth of flood and multiple link failure at the same time with same event.

Accessibility is calculated as follows

$$A_i = 4 \times \left[\alpha \frac{P_i}{\sum_{k=1}^{24} P_k} \sum_{j=1}^{23} \left(\frac{P_j}{\sum_{k=1}^{24} P_k} \frac{d_{ij}^{-\beta}}{\sum_{k=1}^{24} d_{ik^*}^{-\beta}} \right) + (1-\alpha) \frac{P_i}{\sum_{k=1}^{24} P_k} \sum_{j=1}^{23} \left(\frac{P_j}{\sum_{k=1}^{24} P_k} \frac{t_{ij}}{\sum_{k=1}^{24} t_{ik^*}} \right) \right] (i \neq j) \quad (4)$$

Where

A_i= accessibility score of county i

α= weighting factor ($0<\alpha<1$)

$P_{i\,(j)}$ =Population of county $i(j)$

d_{ij}= shortest road distance between counties i and j under a scenario

d_{ij*}= initial shortest road distance between counties i and j

β= distance decay parameter

$$t_{ij} = \frac{\sum_{m=1}^{n} AADT_m d_m}{d_{ij}}$$ Average traffic between i and j on the shortest path

AADTm= annual average daily traffic on link segment m

d_m= distance of link segment m

First part of formula is accessibility based on distance criteria only if the value of α is 1 and including second part is distance traffic volume criteria.

Accessibility deterioration:

$$A^j = \sum_{i=1}^{24} A_i - \sum_{i=1}^{24} A_i^j = \sum_{i=1}^{24} \left(Ai - A_i^j \right) \qquad (5)$$

Where

A_j= accessibility deterioration when link j is disrupted

A_i = accessibility score of County i before disruption of link

A_i^j =accessibility score of County i after disruption of link j

OR

If probability of flood damage of a link j p_j is available then accessibility deterioration is

$$A^j = p^j \sum_{i=1}^{24} \left(Ai - A_i^j \right) \qquad (6)$$

This methodology is more practical than previous one because it considers 100 year flood plain and candidate link is selected as link lies on the 100 year floodplain or considers the probability of damage. Although, the methodology considers the floodplain and specific area but it can be used as a general methodology and can consider the other disaster with disaster hazard map or probability of damage or event occurring. The main weakness of the study are 1) fails to address the level of risk in a road link i.e. how much deep or fringe of the flood affect the road network .2) Failure to consider the change in traffic flow after disruption of a link 3) It fails to address the multiple link disruption all at a time because it assumes only one link failure at a time.

3.2 Generalized Travel Cost Approach:

The methodologies consider the important factor as a transport cost (travel time) for the calculation which is somehow similar to accessibility, however, theoretically, conceptually and calculation process are different.

3.2.1 Important Link and Exposed Municipality.

A very similar to node vulnerability and link criticality (Taylor, Sekhar et al. 2006)'s methodology, (Jenelius, Petersen et al. 2006) introduce the concept of important link and exposed municipality. The main decision factor is based on change in total travel cost between link failure condition and normal condition. A detailed calculation is as follows.

$$\text{Importance}_{\text{net}}(\text{k}) = \frac{\sum_i \sum_{j \neq i} w_{ij} (c_{ij}^k - c_{ij}^0)}{\sum_i \sum_{j \neq i} w_{ij}} \quad k \in E^{nc} \quad (7)$$

Where, k= Link assumed to be failure

w_{ij}= Weight of OD pair reflects its significance in relation to the other pairs, for the calculation wij is taken as traffic demand between node i-j.

c_{ij}^k = generalized travel cost between node i-j when link k is failed

c_{ij}^0 = generalized travel cost between node i-j at normal condition

E^{nc} = non cut link

When k is cut link

$\text{Importance}_{\text{net}}(\text{k}) = \infty$

Also this study introduced the concept of unsatisfied demand when link k is closed, the travel cost between nodes become infinite. The finite and infinite unsatisfied demand is defined as

$$u_{ij}^{k} = \begin{cases} x_{ij} \text{ if } c_{ij}^{k} = \infty \\ 0 \text{ if } c_{ij}^{k} < \infty \end{cases}$$

Where x_{ij} is travel demand from demand node i to j

Importance of link k at unsatisfied demand

$$\text{Importance}_{net}^{uns}(\text{k}) = \frac{\sum_{i}\sum_{j\neq i} u_{ij}^{k}}{\sum_{i}\sum_{j\neq i} x_{ij}} k \in E \qquad (8)$$

For

$k \in E^{nc}$, $\text{Importance}_{net}^{uns}(\text{k}) = 0$

From the above equations the importance of the link is proportional to the change in generalized travel cost. Study addresses the theoretical aspect of link criticality and municipal exposure and calculation data and time are suitable for the practical application; however, this does not give the real world result. Although very important parameter generalized travel cost has been considered, the methodology has some practical limitation. 1) The calculation process is based on the removing of one link without considering any adverse event also realize by the authors " there might be need for more realistic modeling of the failure caused by the adverse event, then just removing one link at a time"(Jenelius, Petersen et al. 2006). 2) Fails to address the multiple links failure all at a time.

3.2.2 Network Robustness Index (NRI)

(Scott, Novak et al. 2006) propose a very similar methodology called network robustness index (NRI) to identify the critical link in a road network. This methodology calculates the total change in travel cost after removing a link and higher the value higher the criticality. Index is calculated as follows

Network robustness index (NRI) of a link a is

$$q_a = c_a - c \qquad (9)$$

Where,

q_a = Network robustness index (NRI) of a link a in minute

c_a = Total travel time cost after removing of link a ,

c= Total travel time cost of network at normal condition

$$c_a = \sum_{a} t_a x_a \delta_a \qquad (10)$$

$$c_a = \sum_{a} t_a x_a \qquad (11)$$

t_a = Travel time of link a

x_a = volume of traffic in a link a

$$\delta_a = \begin{cases} 1 \text{ if link } a \text{ is not the link removed} \\ 0 \text{ otherwise} \end{cases}$$

This model runs in the TransCAD program first to calculate the travel time and traffic flow at normal condition by using user equilibrium assign model and remove each link one at a time sequentially and then again calculate the travel time and traffic flow. The model argue that the network users who do not use the removed link may be rerouted based on the user equilibrium principle.

This methodology gives the value of index which can be compared easily with the other link and the ranking of link can be done, data are simple and calculation software is needed. However, the main weakness of the study is failure to address the practical field situation i.e. 1) every link do not have same probability of damage while it assume link failure in turn.2) there is the possibility of multiple link failure while the model calculates the value of link reliability one at a time.

3.2.3 Degree of Weakness

Recently (MLIT 2011b) publish a manual for the evaluation of road network at emergency condition calculates the index called degree of weakness of a link. The model calculates the total travel time in a network before and after disaster and index is calculated as the ratio of total travel time at disaster condition to total travel time at normal condition. This methodology does not remove all the links in a network like previous methodology. The model of this methodology is as follows:

The degree of weakness

$$\alpha_0^k = \frac{T_{02}^k}{T_{01}^k} \qquad (12)$$

Where, α_0^k = the degree of weakness

$$T_{01}^k = \sum_{i}\sum_{j} t_{ij(n)} \delta_{ij(n)} \qquad (13)$$

$$T_{02}^{k} = \sum_{i} \sum_{j} t_{ij(n)} \delta_{ij(n)} \qquad (14)$$

T_{01}^{k} =Total travel time at normal condition

T_{02}^{k} =Total travel time after disaster event

$t_{ij(n)}$ = Total travel time from municipalities (i) to the nearest capital of the prefectures or expressway IC j (1) and the travel time from municipalities (i) to the neighboring municipalities' j (2).

$\delta_{ij(n)}$ = 1, the route from municipalities i to the nearest capital of prefecture or expressway IC j (1) and to the neighboring municipalities' j (2) can pass evaluated link (k).

$\delta_{ij(n)}$ = 0 do not pass the evaluated link (k)

The main important approach of this methodology is its probabilistic approach, the selection of link (k) is crucial part and identified based on the probability of damage, however , it does not consider any numerical value of probability but it identifies the point where traffic cannot pass during disaster .The points to be assumed as a impassable are : 1) Points where the probability of following earthquake causes damage a)The road section lies on the Tsunami inundation zone, b) Section of road where possibility of landslide, rock sliding, avalanches. c) Bridge constructed before 1980. 2) Points where smooth traffic flow is difficult (road width less than 5.5m). This methodology considers the real world situation and significantly reduces the number of links to be evaluated. However, it has considered the link travel time only, there are other important parameter like traffic flow, length of the link, population in area are left to be considered. And there is lacking of level of threat from the disaster i.e. the risk on the particular link.

3.3 Disaster Prevention Function Approach:

The recently published manual (MLIT 2011a) gives an evaluation methodology for the evaluation of redundancy by using detour ratio. It is very simple and easy to evaluate the redundancy of a road link. The detour ratio is calculated as follows:

$$Detour\ Ratio = Min(At_i, Al_i) \qquad (15)$$

$$At_i = \frac{T_2^i}{T_1^i} \qquad (16)$$

$$Al_i = \frac{L_2^i}{L_1^i} \qquad (17)$$

Where

At_i = Detour ratio of time.

Al_i = Detour ratio of length.

T_2^i =The necessary time of possible alternative route (the shortest time route).

T_1^i =The necessary travel time of major route (The shortest time route).

L_2^i =The distance of possible alternative route (The shortest distance route).

L_1^i = The distance of major route (The shortest distance route).

Similarly, in a degree of weakness, the disaster tolerance (risk of incidents) is evaluated based on the riskiness criteria, identifies the point where traffic cannot pass during disaster. The points to be assumed as an impassable are: 1) Points where the following earthquake damage a) the road section lies on the Tsunami inundation zone, b) Section of road where possibility of landslide, rock sliding, avalanches. c) Bridge constructed before 1980. 2) Points where smooth traffic flow is difficult (road width less than 5.5m). The disaster prevention function is leveled as in table below. The four level of disaster prevention function has been proposed. Level A is highly protected where the detour ratio is less than 1.5, has low danger of disaster risk (i.e. the links is not located the impassable points) and faster route. Level B is categorized as the point which has the wide range of possibility of rescue and emergency supply. Level C is categorized as the links which lie on the impassable zone but detour ratio is less than 1.5. The lowest category of link is D, which has highest risk of disaster including detour ratio greater than 1.5. Table 3 shows the evaluation criteria.

Disaster prevention function approach categories the level of route disaster prevention functions as A, B, C, and D; but does not give the comparative result of the routes because there is a possibility of more than one route lying on the same level and single link may lie on the various route. So, more than one route could be affected by the same vulnerable link and cannot give the comparative result of the links.

Table: 3

Level of link disaster prevention function	Disaster tolerance (The riskiness of disaster)	Redundancy (Vulnerability)
	Low danger of disaster of major route = ○ and faster route = ◎	The detour ratio of alternative route that has low danger of disaster is less than 1.5 =○
A	◎	○
B(BB)	○(◎) {(BB) Where the place is located as transportation point of wide rescue and emergency supply }	- (No need to evaluate redundancy)
C	×	○
D	×	×

Table 4 Data requirement for the calculation process

Approach	Studies	Data Requirement						
		Travel time/ Cost	Link length	Traffic flow/ Demand	Population in location	Hazard map	Level of riskiness	Probability of event
Accessibility	(Taylor, Sekhar et al. 2006)	+	+	-	+	-	-	-
	(Sohn 2006)	-	+	+	+	+	-	+
Generalized Travel cost	(Jenelius, Petersen et al. 2006)	+	-	+	-	-	-	-
	(Scott, Novak et al. 2006)	+	-	+	-	-	-	-
	(MLIT 2011b)	+	-	-	-	-	+	-
Disaster Prevention function	(MLIT 2011a)	+	+	-	-	-	+	-

Score: + =Data required
Score: - = Data not required

Table 4, up summarizes the data requirement for each practical evaluation methodology. Table 5, below summarizes the consideration of multiple dimension of practicability approach of existing evaluation methodology. Table 6, below summarizes the analysis of the existing methodologies based on the multidimensional viewpoint of practicability with positive and lacking perspective. Some of the methodologies have not considered the probability of disaster event. They only assume the failure of a link at a time and calculate the change in indices, so that they do not represent real practical world. Some of the methodologies considered the probability of disaster event without considering the any numerical value of probability. They only select the links to be evaluated from the disaster possible area. Therefore comparing among the links is difficult due to lack of any measurable value.

Table: 5 Summary of existing evaluation methodology:

Approach	Paper	Evaluation index	Probabilistic approach [a]	Area Isolation [b]	Socioeconomic impact [c]	Theoretical view point	Data requirement [d]	Calculation time [e]	Practical viewpoint [f]	One link failed area wise impact [g]
Accessibility	(Taylor, Sekhar et al. 2006)	Change in accessibility index	-	+	+	Vulnerable node, critical link	-	-	±	+
	(Sohn 2006)	Change in accessibility index	+	+	+	Critical link	±	±	±	+
Generalized travel cost	(Jenelius, Petersen et al. 2006)	Change in generalized travel cost	-	+	+	Important link and exposed municipality	-	-	±	+
	(Scott, Novak et al. 2006)	Network robustness Index	-	-	+	Critical link	-	±	±	+
	(MLIT 2011b)	Degree of weakness	+	-	+	Critical link	-	-	±	+
Disaster Prevention Function	(MLIT 2011a)	Detour ratio and level of links disaster protection function	+	+	+	Redundancy Disaster protection function	-	-	±	-

[a] Score: + =considered ; - = Do not considered
[b] Score += Considered ; - = Do not considered
[c] Score: + = considered; - = Do not considered
[d] Score: + = Higher ; - = lower and ± = medium
[e] Score: + = long time ; - = short time ± = Medium time
[f] Score: ± = Partly acceptable
[g] Score: + = Addressed ; - = Do not addressed

Table: 6 Analysis of existing practical study from multidimensional viewpoint of positive part and lacking part

SNo.	Viewpoint	(Taylor, Sekhar et al. 2006)		(Sohn 2006)		(Jenelius, Petersen et al. 2006)	
		Positive	Lacking	Positive	Lacking	Positive	Lacking
1	Socioeconomic Impact of network disruption	Evaluating the impact on accessibility of the service and facilities.	Has not considered how much population is affected.	Physical damage is converted to the damage on society.	Has not consider travel time.	The index; important link and exposed municipality are based on the change in generalized travel cost.	
2	Area Isolation	Complete loss in accessibility index caused area isolation.	The index is calculated through the concept one link failure at a time. Cannot explain more than 2 link failure condition.	Accessibility index of county (area) is calculated and area isolation represents the complete loss in the accessibility of an area.	Has not addressed multiple link failure condition	The value of importance, ∞ represents the area isolation	
3	Theoretical	Node vulnerability and link criticality is due to highest impact in the socioeconomic condition when road is closed.	Vulnerability is calculated without considering the adverse event.	Methodology has been proposed for the conversion of physical damage to the socioeconomic impact.		Important link and exposed municipality (Area) is calculated on the basis of the change in generalized travel cost.	The adverse event is not considered.
4	Practical	Measurable evaluation index.	Result is not plausible because it assumes every shortest path fails at a time	Index is measurable and only links are selected from the 100 year floodplain.	Level of threat based upon the depth and velocity of flood.	Numerical value of importance is used to rank the links.	Result is not plausible because any adverse event is not considered
5	Data requirement	Very simple data is needed.	Traffic flow is not considered.	Complex data need GIS database.		Simple data are required	
6	Calculation time and process	Easy calculation process and less time consuming.	-	Complex calculation process need GIS experts		Medium level	

7	Area wise impact of single link failure	Accessibility index of each location is calculated if one link is failed		Index is calculated for each area(node) even in one link failure condition		Change in generalized travel cost is calculated for all OD pair after removing one link at a time.	
8	Probability of disaster	-	Has not considered any probability of the adverse event.	Links are selected from the 100 years floodplain and the probability of flood damage is multiplied by change in accessibility index.			Does not consider any probability of adverse event.

Table: 6 contd... Analysis of existing practical study from multidimensional viewpoint of positive part and lacking part

SNo.	Viewpoint	(Scott, Novak et al. 2006)		(MLIT 2011b)		(MLIT 2011a)	
		Positive	Lacking	Positive	Lacking	Positive	Lacking
1	Socioeconomic Impact of network disruption	Total change in travel time due to failure of link which represents the socioeconomic impact.		Travel time is taken as socioeconomic indicator		Detour ratio is the important indicator for the evaluating redundancy	Population and other travel time parameter is not considered
2	Area Isolation		This index only calculates the total change in travel time under the one link failure. Does not calculate the locational index.	The index calculates the ratio of total travel time in a network after and before the disaster events.	Individual isolation of location is not analyzed.	Degree of isolation is analyzed by the level of link disaster protection function.	Numerical value of level of link disaster protection function.
3	Theoretical	Network robustness Index (NRI) To evaluate the critical importance of network link.	This methodology has not consider any adverse events in reality, index represent the link importance.	Degree of weakness is depends on the travel time.		Redundancy is analyzed through the detour ratio which is very clear indicator.	

4	Practical	Link importance calculated based on rerouting all the traffic after removing one link	The result only represents the link importance because it does not consider the adverse events.	Selection of link to be evaluated is practical.	Numerical value of disaster risk on the link.	Method is very practical in terms of redundancy and disaster protection function	It will be better if there is numerical value of level of threat from the disaster
5	Data requirement	Simple data		Simple data		Simple data	
6	Calculation time and process		Comparatively high calculation time, repetitive process and need programming.	Very easy calculation and less time consuming.		Easy and convenient calculation time.	
7	Area wise impact of single link failure	Total impact on network (change in travel time) is calculated when particular link is failed		Total impact of travel time in a whole network is calculated.	Population or traffic volume is not considered.		Has not addressed one link failure area wise impact.
8	Probability of disaster		There is no any evidence of adverse event.	Although the numerical value of probability is not considered, links to be evaluated is selected under the probability of disaster	Numerical value of probability of adverse events	No numerical value of probability but the selection of link to be evaluated is based on the possibility of disaster.	Numerical value of level of threat is better parameter for the analysis

4. CONCLUSION AND DISCUSSION

Road network is very important lifeline during emergency situation .For the rescue and evacuation, post disaster support and reconstruction, road network is not comparable to the other mode. Road network performance evaluation is the crucial step for the efficient allocation of resource to make reliable network. We have summarized the road network reliability study from the viewpoint of practical application.

Although numerous studies have been done on the reliability of road network to find the critical links and prioritize them for the improvement, there is a lack of practical evaluation methodology. However, contemporary research left the very important concepts and ideas for the future research. Conceptual study gives various relevant concepts of reliability of road network and classification of reliability. Mathematical theoretical methodology evaluates the reliability by mathematical model. Existing practical methodologies evaluate the performance under uncertain and emergency situation. Descriptive type study explains about the problems on the practical field and problems faced by the practitioner. Application study of evaluation methodology is the case study which has been applied in the specific area. Ways to improve reliability demonstrated the policy issues for the improvement of network reliability. Practical methodology should have easy application to the practitioner, simple data, calculation procedure and appropriate calculation time. However, mathematical evaluation methodologies have complex calculation procedure and result is not so relevant. Considering the requirement of final result as a priority list for the improvement project of road network; there are six key existing practical methodologies selected as a foundation of the practical study. These methodologies give very important direction such as change in accessibility, change in generalized travel cost and redundancy. But result from these methodologies is not plausible.

We have analyzed the existing practical methodologies under various multi-dimensional criteria, such as evaluation index, data requirement, and calculation time, probability of adverse events, theoretical importance, socioeconomic impact, one link failure and area wise impact and isolation of location with the positive and lacking perspective. We have found that the existing methodology measure the link criticality indices considering the socioeconomic impact and other factor of the link disruption but they haven't considered the level of disaster risk on the road network link and the impact of multiple link failure condition. It can be argued that there is a necessity of easy, practitioner friendly and simple methodology for the evaluation of road network under emergency scenario which should address the multi-dimensional criteria. Thus a new methodology should be formulated based on the positive part of the existing methodology by overcoming the lacking part.

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