A semi-dynamic traffic assignment model and its application

Pham Thi Thu HA[1], Sho-ichiro NAKAYAMA[2] and Jun-ichi TAKAYAMA[3]

[1] Graduate School of Natural Science & Technology, Kanazawa University
Kakuma-machi, Kanazawa 920-1192, Japan
Email: phamthithuha1985@gmail.com
[2]Member of JSCE, Associate Profeesor, School of Environmental Design, Kanazawa University
Kakuma-machi, Kanazawa 920-1192, Japan
Email: snakayama@t.kanazawa-u.ac.jp
[3]Fellow member of JSCE, Professor, School of Environmental Design, Kanazawa University
Kakuma-machi, Kanazawa 920-1192, Japan
Email: takayama@t.kanazawa-u.ac.jp

In most cities traffic condition varies significantly within a day, in which static traffic assignment model may not be able to sufficiently represent time-varying congestion phenomena in transportation network analysis. On the other hand, a dynamic traffic assignment model needs much computational load and does not necessarily have a unique solution in most models. A semi-dynamic traffic assignment model is one of the alternatives for describing within-day traffic dynamics of large-scale networks. The semi-dynamic approach is basically to formulate static network equilibrium in each period, but considers flow propagation between periods. The flow propagation is that the flow on a link which cannot exit the link in a period is propagated to the next period.

Key words*: semi-dynamic assignment model, residual flow, flow propagation, sensitivity analysis.*

1. INTRODUCTION

A semi-dynamic traffic assignment model is one of the alternatives for describing within day traffic dynamics of large-scale networks. In the fact that on a link there are number of vehicle cannot reach the destination within the time period, is propagated to the next period. The semi-dynamic approach is basically to formulate static network equilibrium in each period, but considers flow propagation between periods.

We assume for simplicity that inflow enters a link continuously at the same rate, travel cost is constant within the same period, and travel cost is the function of its inflow. On a link there are number of vehicle that cannot reach the destination within the time period. This is the residual flow of that link, and propagates to the next period. The residual flow on a link is determined by the inflow and the link travel time (which is a function of its inflow) in this study. Therefore, this does not travel on the following links in the same period but it runs on them in the next period. Also, the residual flow on a link is added to demand between the end node of that link and the original destination in the next period. In the present period, this residual flow on next links should be eliminated.

2. OVERVIEW SENSITIVITY ANALYSIS OF LOFIT-TYPE NETWORK EQUILIBRIUM MODEL

Static equilibrium is reached in each period and logit-type route choice is assumed. The probability of choose route j between OD pair i as below:

$$p_{\tau,ij} = \frac{\exp\left(-\theta\, c_{\tau,ij}\right)}{\sum_{j' \in J_i} \exp\left(-\theta\, c_{\tau,ij'}\right)} \tag{1}$$

Logit-based traffic assignment is assumed:

$$f_{\tau,ij,k} = q_{\tau,i\,j}\, p_{\tau,ij,k} = q_{\tau,i} \frac{\exp\left(-\theta\, c_{\tau,ij,k}\right)}{\sum_{j' \in J_i} \exp\left(-\theta\, c_{\tau,ij',k}\right)} \tag{2}$$

Where q_{ij} is demand between the i-th node and j-th node, $f_{ij,k}$ is the flow on the k-th route between the i-th node and j-th node, $p_{ij,k}$ is the probability of choosing the k-th route between the i-th node and j-th node, $c_{ij,k}$ is the travel cost on the k-th route between the i-th node and j-th node, k_{ij} is the set of routes between the i-th node and j-th node, θ is a positive parameter, τ is in τ –th period.

An expression of the above equation is as follows:

$$\mathbf{f}_\tau = \mathbf{Q}_\tau \, \mathbf{p}_\tau \tag{3}$$

Where f_τ is the vector of all route flows, Q_τ is diagonal matrix of travel demands

$$Q = \mathrm{diag}(q) \tag{4}$$

Above equation is expressed that:

$$Q_{\tau,i} = \begin{pmatrix} q_{\tau,i} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & q_{\tau,i} \end{pmatrix} \tag{5}$$

The link vector is given as below by using route flows and the link-route incidence matrix:

$$\mathbf{x}_\tau = \Delta \mathbf{f}_\tau \tag{6}$$

where x (= $x_1, x_2, \ldots, x_{|A|}$) is the vector of all link flows, Δ (= $\{\delta_{a,ijk}\}$)is the link-route incidence matrix, and A is the set of links, and |A| is the number of links. Also, $\delta_{a,ijk}$ is the link-route incidence variable and $\delta_{a,ijk} = 1$ if the k-th route between the i-th node and j-th node includes the the a-th; otherwise, $\delta_{a,ijk} = 0$. The route travel cost function is given as:

$$\mathbf{c}(\mathbf{f}_\tau) = \Delta^T \, \mathbf{t}(\Delta \mathbf{f}_\tau) \tag{7}$$

where c(f) is the vector-valued function of route travel cost, t(Δf) is the vector-valued function of link travel cost.

Because travel cost is the function of its inflow, otherwise, probability of route choice is a function of its travel cost. We have:

$$\mathbf{f}_\tau = \mathbf{Q}_\tau \mathbf{p}(\mathbf{c}(\mathbf{f}_\tau)) \tag{8}$$

2. SEMI-DYNAMIC TRAFFIC ASSIGNMENT MODEL

In the practice, not all vehicle depart from Origin to Destination can reach the Destination, one part of them cannot exit the link and becomes flow propagation. The flow propagation is that the flow on a link which cannot exit the link in a period is propagated to the next period.

The length of discrete time period should be determined according to accuracy of OD data and others. Too detail description of flow propagation is not necessarily effective. Therefore, for practical applications, the period length may be from 15 min to 90 min. in many cases. Needless to say, we can determine the length much shorter if OD demand data are accurate and dynamically detail. The object of this study is the case that OD data are not detail, and the period (or the length of discrete time) is not so short (approximate from 15 min to 90min.)

Semi-dynamic traffic assignment model is shown in follow figure.

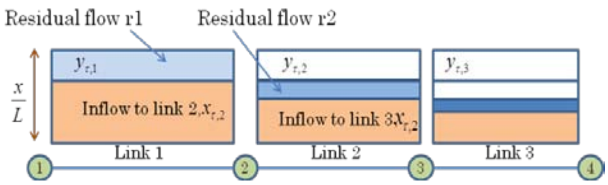

Figure 1. Semi-dynamic traffic assignment model

The inflow into Link 1 cannot exit this link and become residual flow which is propagated to the next period. $x_{\tau,a}$ denotes inflow to the a-th link in the τ -th period. $c_a(x_{\tau,a})$ denotes travel cost on the a-th link in the τ -th period. $y_{\tau,a}$ denotes residual-flow on the a-th link in the τ -th period. z_τ denotes link flow after the residual flows are eliminated.

Thus, the travel time on Link 2 should be:

$$c_2(x_{\tau,2}) = c_{\tau,2}(x_{\tau,1} - y_{\tau,1}) \tag{9}$$

And the travel time on Link 3 should be

$$c_3(x_{\tau,3}) = c_3(x_{\tau,1} - y_{\tau,1} - y_{\tau,2}) \tag{10}$$

If the residual flow is eliminated, the travel cost does not only change, but also the inflow changes via network equilibrium.

We consider that inflow enters a link continuously at the same rate, travel cost is constant within the same period and travel cost is the function of its inflow. On the other hand, residual flow which is the function of travel time on its link is added to demand between the end node of that link and the original destination in the next period and may change its original route in the next period.

As described above, the inflow rate is $x_{\tau,a}/L$. Because travel cost is constant within the same period and residual flow is the function of travel time on its link, one of the natural ways to estimate the residual flow is that the amount of the residual flow is the product of inflow rate and travel time. This means that the residual flow is still travelling on the link at the end of the period. The residual flow on the a-th link in the τ -th period is $x_{\tau,a}\, c_a(x_{\tau,a})/L$.

$$y_{\tau,ij,a} = \frac{f_{\tau,ij}\, c_{\tau,a}\, \delta_{a,ij}}{L} \tag{11}$$

Where $f_{\tau,ij}$ is the flow on the a-th link between i-th node and j-th node. $c_{\tau,a}$ is travel cost. L is length of time period.

The summary of residual flow in k-th link

$$s_{\tau,ij,n_{ijk}} = \sum_{k'=1}^{k-1} y_{\tau,ij,n_{ijk'}} \tag{12}$$

Where n_{ijk} refers the number of link in k-th route between the i-th node and j-th node

Consider that B is matrix if link i is a downstream link of link j, then (i,j)=1; otherwise (i,j)=0. Thus, the summary of residual flow is eliminated from route inflow:

$$\mathbf{s}_\tau = \sum_{i\in I}\sum_{j\in J_i} \mathbf{B}_{ij}\, \mathbf{y}_{\tau,ij} \tag{13}$$

$$\mathbf{s}_\tau = \frac{1}{L}\mathbf{T}(\Delta\mathbf{f}_\tau)\sum_{i\in I}\sum_{j\in J_i} f_{\tau,ij}\mathbf{B}_{ij}\, \boldsymbol{\delta}_{ij} \tag{14}$$

We consider that $r_{ij}=B_{ij}\delta_{ij}$, and $R=(r_{11,} r_{12}, \ldots, r_{|I|\,|Ji|})$, so:

$$\mathbf{s}_\tau = \frac{1}{L}\mathbf{T}(\Delta\mathbf{f}_\tau)\mathbf{R}^T\mathbf{f}_\tau \tag{15}$$

The link flow after the residual flows are eliminated z_τ is given by:

$$\mathbf{z}_\tau = \mathbf{\Delta f}_\tau - \mathbf{s}_\tau \tag{16}$$

In addition, from equation 8, we have

$$\mathbf{f}_\tau = \mathbf{Q}_\tau\, \mathbf{p}\left(\mathbf{\Delta}^T\mathbf{t}(\Delta\mathbf{f}_\tau - \mathbf{s}_\tau)\right) \tag{17}$$

Base on the sensitivity analysis of logit-type network equilibrium model, we have:

$$\mathbf{f}(\mathbf{s}) = \mathbf{f_0} + \nabla_\mathbf{s}\mathbf{f_0}\ \mathbf{s} \tag{18}$$

Where: f_0 denotes the flow in static assignment without residual flow.

From equation 15 and 18 we can obtain that:

$$\mathbf{s} = \frac{1}{L}\mathbf{T}(\Delta\mathbf{f_0})\mathbf{R}^T\mathbf{f_0} \tag{19}$$

Base on equations above f is given as follows:

$$\mathbf{f} = \Delta\mathbf{f_0} + \frac{1}{L}\nabla_\mathbf{s}\mathbf{f_0}\,\mathbf{T}(\Delta\mathbf{f_0})\mathbf{R}^T\mathbf{f_0} \tag{20}$$

Using Eq. (17), define the following function:

$$d(f,s) = f - Q_\tau p(\Delta^T t(\Delta(-s) \tag{21}$$

This is a gap between the both sides of Eq. (17) and should be 0, that is, d(f, s) = 0, under the network equilibrium.

Implicit function of Eq.(21) are given as follows:

$$\nabla_\mathbf{s}\mathbf{f} = -\nabla_\mathbf{f}\mathbf{d}^{-1}\nabla_\mathbf{s}\mathbf{d} \tag{22}$$

Therefore, we obtain the following equation:

$$\nabla_s\mathbf{f} = \left(\mathbf{I} - \mathbf{Q}\nabla_c\mathbf{p}\nabla_x\mathbf{t}\,\Delta\right)^{-1}\mathbf{Q}\nabla_c\mathbf{p}\nabla_x\mathbf{t} \tag{23}$$

Thus, route flows with respect to demand perturbation s, are given by:

$$\mathbf{f} = \Delta\mathbf{f_0} + \frac{1}{L}\left[\left(\mathbf{I} - \mathbf{Q}\nabla_c\mathbf{p_0}\nabla_x\mathbf{t_0}\,\Delta\right)^{-1}\mathbf{Q}\nabla_c\mathbf{p_0}\nabla_x\mathbf{t_0}\right]\mathbf{T}(\Delta\mathbf{f_0})\mathbf{R}^T\mathbf{f_0} \tag{24}$$

3. SIMPLE APPLICATION

The network has 6 nodes and 6 links as shown in Figure 2 and with data in table 1, 2 and 3. Two time periods of 60 min length are considered. Network includes 4 routes. Route 1 passes Link 1, 5, route 2 passes Link 2, 5, route 3 passes Link 3, 6, route 4 passes Link 3, 6.

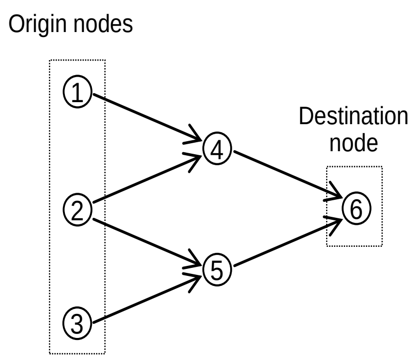

Figure 2: Example network

Table 1: OD pair in 2 time period

OD	Time 1	Time 2
1	1--6	1--6
2	2--6	2--6
3	3--6	3--6
4		4--6
5		5--6

Table 2: Free-flow time and capacities

Link	Free-flow time	Capacity
1 4	10	150
2 4	10	175
2 5	10	125
3 5	10	150
4 6	10	200
5 6	10	200

Table 3: Travel demand in 2 time periods

O-D	Time 1	Time 2
1 ⇨ 6	70	60
2 ⇨ 6	350	300
3 ⇨ 6	70	60

4. RESULT

After calculated by program, we obtained the results as the table below:

	Time 1			
	Link 24	Link 25	Link 46	Link 56
Inflow	194.08	155.93	212.64	178.75
Link Travel Time	12.27	13.63	12.38	11.23
Residual flow	39.68	35.43	43.86	33.47
	Time 2			
	Link 24	Link 25	Link 46	Link 56
Inflow	161.4	138.6	232.98	207.40
Link Travel Time	11.09	12.27	12.76	11.73
Residual flow	29.82	28.34	49.55	40.56

In comparison with other models, we show the differences of inflow, travel time and residual flow between some models.

Table 5: The results of inflow in 2 time periods

	Time 1			
	Link 24	Link 25	Link 46	Link 56
Model 1[1]	189.8	160.2	245	195
Model 2[2]	188	162	245	195
Model 3[3]	191.8	158.2	261.8	228.2
Semi-dynamic model	194.08	155.93	212.64	178.75
	Time 2			
	Link 24	Link 25	Link 46	Link 56
Model 1[1]	163	137	237.8	220.2
Model 2[2]	165.1	134.9	238.1	222
Model 3[3]	161.9	138.1	221.9	198.1
Semi-dynamic model	161.4	138.6	232.98	207.40

Table 6: The results of travel time in 2 time periods

	Time 1			
	Link 24	Link 25	Link 46	Link 56
Model 1	17.15	30.95	26.88	11.36
Model 2	16.46	31.98	26.88	11.36
Model 3	17.91	29.81	32.93	21.02
Semi-dynamic model	12.27	13.63	12.38	11.23
	Time 2			
	Link 24	Link 25	Link 46	Link 56
Model 1	11.13	17.92	24.34	18.27
Model 2	11.19	16.77	24.46	18.87
Model 3	11.1	18.51	18.85	11.44
Semi-dynamic model	11.09	12.27	12.76	11.73

Table 7: The results of residual flow in 2 time periods

	Time 1			
	Link 24	Link 25	Link 46	Link 56
Model 1	14.8	35.2	45	0
Model 2	13	37	45	0
Model 3	16.8	33.2	61.8	28.2
Semi-dynamic model	39.68	35.43	43.86	33.47
	Time 2			
	Link 24	Link 25	Link 46	Link 56
Model 1	0	12	37.8	20.2
Model 2	0	9.9	38.1	22
Model 3	0	13.1	21.9	0
Semi-dynamic model	29.82	28.34	49.55	40.56

It is easy to recognize that the result of inflow between models does not have big differences, and travel time result also. But in the residual flow, the results in link 24 and 56 are absolutely different.

5. SUMMARY

First of all, in semi-dynamic model, sensitivity analysis of logit type network equilibrium model is used. Secondly, static network equilibrium is reached in each period, and dynamics of network flow (flow propagation) is considered between periods. Last but not least, the semi-dynamic traffic assignment model can apply within-day traffic of large scale networks.

REFERENCES

1) Sho-Ichiro Nakayama.: A semi-dynamic traffic assignment model with flow propagation based on sensitivity analysis. *DTA2010*:Semi-dynamic traffic assignment with sensitivity analysis
2) 中山晶一朗:感度分析を用いた交通混雑内生 型 時間帯別配分,土木学会論文集D Vol.64 No3, 340-353, 2008.7
3) Akamatsu, T., Makino, Y., and Takahashi, E. (1991). Semi-dynamic traffic assignment models with queue evolution and elastic OD demand, *Infrastructure Planning Review 15*: 535–545
4) Fujita, M., Matsui, M., and Mizokami, S. (1988). Modelling of the time-of-day traffic assignment over a traffic network, *Journal of Infrastructure Planning and Management* 389: 111–119
5) Fujita, M., Yamamoto, K., and Matsui, M. (1989). Modelling of the time-of-day assignment over a congested network, *Proceedings of Japan Sciety of Civil Engineers* 407: 129–138