The Cost-Benefit Analysis for Improving the Traffic Network Reliability after Disaster

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A highly reliable traffic network is very important after a disaster, and the network reliability can be improved and maintained by many methods. However, it is difficult to improve more reliable links than less reliable links which has not been considered in most of current methods. Therefore, this paper proposes a method for improving link reliability which is combined the cost-benefit function with the criticality importance. Firstly, a model to identify the relationship between the cost and reliability from the point of view of the efficiency of local government is proposed based on the practice that it is difficult to improve more reliable links than less reliable links. In order to analyze the government efficiency on the network reliability, the variety of reliability increase of parallel network is discussed mainly by using the cost-reliability function based on the different government support. The effect of improvement of link reliability is discussed lastly according to the different government efficiency. The result that the strategy to improve the link reliability should be changed according to the different investment strategies and the different government efficiency has been got based on our models and discussion.

Key Words: *Network reliability, Cost-Reliability function, Local government efficiency, Cost-Benefit analysis, disaster*

1. INTRODUCTION

It is important to keep the highway network highly reliable after a disaster. If there is a serious earthquake in some regions, and many links are collapsed, the question which link should be selected to reconstruct firstly is very important. There are many factors on this question such as the damage level of some link and the number of injury people near some link and so on. At the same time, refugees' confidence and volunteers' devotion are also important factors to reconstruct the network reliability although the investment is very important to improve the network reliability most of the time, and those nonmaterial factors can replace investment sometimes. Chinese local government will provide the best resource to some disaster region where is considered as the most essential region in some ways such as strategic position. So the government can play an important role in maintaining or improving the reliability of the highway network by providing the nonmaterial support such as the labor force and the confiscation of land. But the work efficiency of government was ignored in current indices such as probability importance and criticality importance for a long time. In fact, the government works more efficient, the more nonmaterial support can be got under the same investment, so the efficiency of government should be considered and discussed carefully.

2. CURRENT RELIABILITY INDICES

The concepts of probability importance and criticality importance have been proposed long in the system engineering field, and now have been increasingly used in the transportation field.

(1) Probability Importance

Link reliability in the network is defined as the probability that the traffic is in a certain support for a given time period. And terminal reliability, R is given by the minimal path sets expression as follows:

$$R(r) = E[1 - \prod_{S=1}^{N} (1 - \prod_{a \in P_S} X_a)] \qquad \text{(Eq.1)}$$

Where Ps is the s-th minimal path set, and N is the total number of minimal path set, and X_a is a binary indicator variable for link a (and equals 1 if link a survives or provides the certain traffic support, and equals 0 otherwise), and link reliability r_a is $r_a = E[X_a]$.

It is essential to find out the key link to improve the terminal reliability most efficiently. For this purpose, The Birnbaum's structural importance has been proposed so far. The Birnbaum's structural importance, PI_a in the papers of Wakabayashi and Iida (1992), is

$$PI_a = \partial R(r) / \partial r_a \text{ (and } 0 \le PI_a \le 1) \qquad \text{(Eq.2)}$$

Although Birnbaum's structural importance has potentiality in improving network reliability, it has a defect to be stated in the next section.

For the case of two links in series network, the terminal reliability *RAB* follows from Eq.1 is:

$$R_{AB} = r_1 r_2 \qquad \text{(Eq.3)}$$

For the case of two links in parallel network, the terminal reliability R_{AB} follows from Eq.1 is:

$$R_{AB} = 1-(1-r_1)(1-r_2) \qquad \text{(Eq.4)}$$

The probability importance for these two links in parallel network, PI_1 and PI_2, are obtained from Eq.2 and Eq.4 as $PI_1 = 1-r_2$ and $PI_2 = 1-r_1$. If $r_1 > r_2$, $PI_1 > PI_2$ is hold.

This result indicates that in case of parallel typed network, improving the link of the highest reliable is most effective for improving terminal reliability. According to common sense, it is difficult to improve highly reliable link whereas it is rather easy to improve lower reliable link. This result is actually irrational for improving, managing and reconstructing network.

(2) Criticality Importance

Henley and Kumamoto proposed the Criticality importance which is the ratio of the proportional improvement in the network reliability to the proportional improvement in the link reliability:

$$CI_a = \frac{\partial R / R}{\partial r_a / r_a} = PI_a \frac{r_a}{R} \qquad \text{(Eq.5)}$$

Based on the defect of Eq.5, Wakabayashi also proposed the criticality importance as the proportion of the marginal change in terminal reliability against the marginal change in the reliability engineering, and the Criticality importance CI_a is introduced as Eq.6.

$$CI_{wa} = \lim_{\Delta q_a \to 0} \left\{ -\frac{\Delta R(r)/R(r)}{\Delta q_a / q_a} \right\} = PI_a \frac{1-r_a}{R} \qquad \text{(Eq.6)}$$

Where $q_a = 1-r_a$ is used for the unreliability of link a.

For the case of two links 1 and 2 in series network, it follows fromEq.3 and Eq.5 that:

$$CI_1 = \frac{r_1 r_2}{R} = CI_2 \qquad \text{(Eq.7)}$$

For the case of two links 1 and 2 in series network, it follows from Eq.3 and Eq.6 that:

$$CI_{w1} = \frac{1-r_1}{r_1} \text{ and } CI_{w2} = \frac{1-r_2}{r_2} \qquad \text{(Eq.8)}$$

If $r_1 > r_2$, $CI_{w1} < CI_{w2}$ is hold from Eq.8.

For the case of two links 1 and 2 in parallel, it follows from Eq.4 and Eq.6 that:

$$CI_{w1} = \frac{(1-r_1)(1-r_2)}{r_1 + r_2 - r_1 r_2} = CI_{w2} \qquad \text{(Eq.9)}$$

From the Eq.7 and Eq.9, the criticality importance index is same for both links in some type network, so it does not help distinguish between them in terms of improving network reliability.

The formula of the criticality importance which Henley and Kumamoto proposed or Wakabayashi proposed can't make an expected result

The probability importance and the criticality importance of the above mentioned indices don't allow explicitly for the increasing cost of improving link reliability as link reliability increase, and improving network reliability as network reliability increases. So we would like to discuss the link reliability increase in accordance with the variety of the cost.

3. SIMPLE COST-BENEFIT ANALYSIS ON NETWORK RELIABILITY

According to the indices of PI and CI, the higher reliable link should be strengthened from Eq.2 and Eq.5, on the contrary, the lower reliable link should be strengthened according to the index of CI_w proposed by Wakabayashi from Eq.8 in series network. The result from Eq.8 accords with the common sense that it is difficult to improve highly reliable link whereas it is rather easy to improve lower reliable link, so we discuss the link reliability increase only in parallel network based on the variety of the cost.

In general, there are many strategies on the cost of link reliability increase, and in this paper the needed cost to improve the link reliability is assumed to be three kinds of the following:

Case 1: The cost increase is a constant amount according with the reliability increase under the same increase degree of link reliability such as Table 1 (the constant amount equals 500(unit is 10,000 Yen)).

Table 1 Cost increase is a constant amount

Reliability increase	Cost increase
0.0→0.1	500
0.1→0.2	1000
0.2→0.3	1500
0.3→0.4	2000
0.4→0.5	2500
0.5→0.6	3000
0.6→0.7	3500
0.7→0.8	4000
0.8→0.9	4500

Case 2: The cost increase is a progressive increase when the more reliable link is improved under the same increase degree of link reliability such as Table 2(progressive increase equals 500(unit is 10,000 Yen)).

Table 2 Cost increase is a progressive increase

Reliability increase	Cost increase
0.0→0.1	500
0.1→0.2	1500
0.2→0.3	3000
0.3→0.4	5000
0.4→0.5	7500
0.5→0.6	10500
0.6→0.7	14000
0.7→0.8	18000
0.8→0.9	22500

Case 3: The cost to increase the link reliability is fixed under the same increase degree of link reliability such as cost is 1000(unit is 10,000 Yen) when the range of variation of reliability increase is 0.1.

The effect of cost increase to improve the link reliability may be not obvious in the short time, so the cost-effect function for a long time is defined as Eq.10, where Y shows the number of years to invest, F shows the investment amount of every year, R_{ab0} means the original network reliability, and $Cost_{ab}$ shows the cost increase to improve the network reliability from R_{ab0} to R_{ab}.

$$Eff(Y,F)=\frac{\{(1-R_{ab0})-(1-R_{ab})\}}{Cost_{ab}}\times Y\times F \quad \text{(Eq.10)}$$

The effect of the cost-benefit analysis will be discussed in the case of two links 1 and 2 in parallel network, and the original reliability of two links are shown as r_1=0.4 and r_2=0.5. Y equals 50 years and F equals a hundred million every year in order to short cut calculation.

(1) The Cost-Benefit Analysis on Constant Cost Increase

The reliability of two links in parallel network will be improved differently according to the different improvement strategies proposed by Henley and Kumamoto or Wakabayashi under the same invest strategy. Fig.1 shows the cost variety under the strategy of constant cost increase. The left branch in Fig.1 shows the strategy of reliability improvement according to the indices of PI and CI , and the right branch shows the variety according to the index of CI_w .

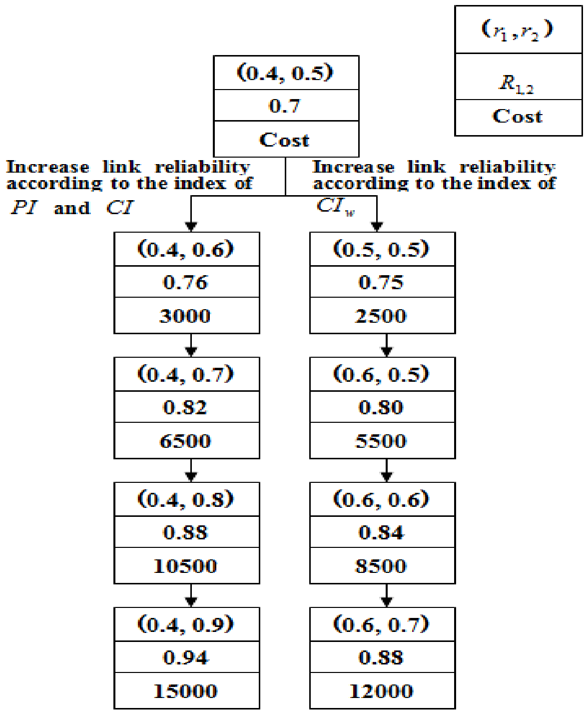

Fig.1 The cost variety based on constant cost increase

The result $Eff(50,1)_{PI,CI}$ based on the indices of PI and CI is 8 by using Eq.10 and the data from Fig.1 to calculate, and the result $Eff(50,1)_{CIW}$ based on the index of CI_w is 7.5. It means that $Eff(50,1)_{PI,CI} > Eff(50,1)_{CIW}$. This result shows that the higher reliable link should be strengthened based on the strategy of constant cost increase.

(2) The Cost-Benefit Analysis on Progressive Cost Increase

The reliability of two links is also improved differently according to the different improvement strategies under the strategy of progressive cost increase. Fig.2 shows the cost variety. The left branch in Fig.2 shows the strategy of reliability improvement according to the indices of PI and CI, and the right branch shows the variety according to the index of CI_w.

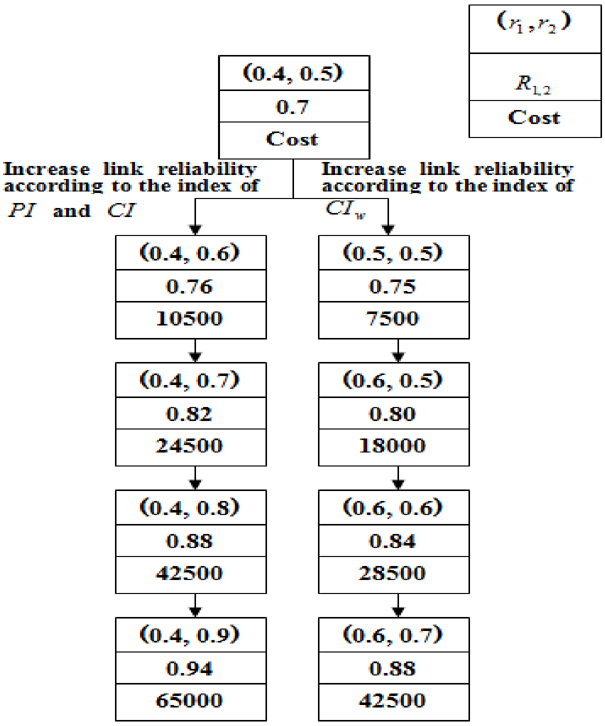

Fig.2 The cost variety based on the progressive cost increase

The result $Eff(50,1)_{PI,CI}$ based on the indices of PI and CI is 1.85 by using Eq.10 and the data from Fig.2 to calculate, and the result $Eff(50,1)_{CIW}$ based on the index of CI_w is 2.12. It means that $Eff(50,1)_{PI,CI} < Eff(50,1)_{CIW}$. This result shows that the lower reliable link should be strengthened based on the strategy of the progressive cost increase.

(3) The Cost-Benefit Analysis on Fixed Cost under Same Increase Degree of Reliability

Fig.3 shows the cost variety under the strategy of fixed cost when the range of variation of reliability increases is 0.1. The left branch in Fig.3 shows the strategy of reliability improvement according to the indices of PI and CI, and the right branch shows the variety according to the index of CI_w.

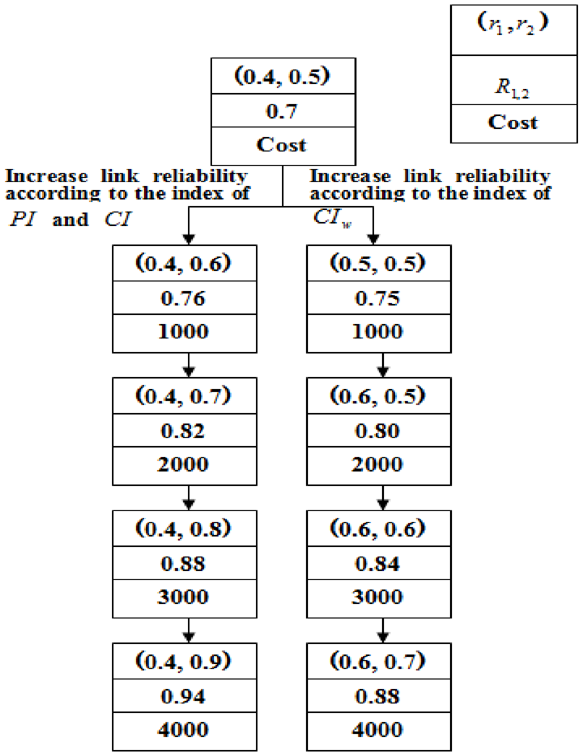

Fig.3 The cost variety based on fixed cost

The result $Eff(50,1)_{PI,CI}$ based on the indices of PI and CI is 30 by using Eq.10 and the data from Fig.1 to calculate, and the result $Eff(50,1)_{CIW}$ based on the index of CI_w is 22.5. It means that $Eff(50,1)_{PI,CI} > Eff(50,1)_{CIW}$. This result shows that the higher reliable link should be strengthened based on the strategy of the progressive cost increase.

From case 1 and case 3, the indices of PI and CI should be selected to improve the link reliability, but the index of CI_w should be selected from case 2. The different result has been described according to the different indices by using the cost-effect function from the above-mentioned discussion, and whether the evaluation index is more efficient depends on the strategy of investment. So the cost strategy to improve link reliability should be concerned and researched.

4. MODEL OF EFFICIENCY OF LOCAL GOVERNMENT ON NETWORK RELIABILITY INCREASE

As mentioned in Chapter 3, the effect of link reliability increase will be different according to the different investment strategy. In general, there may be limited funds to improve the link reliability, and the best strategy to improve the link reliability should be found under the same limited cost. The local government should play a maximum efficiency to use the same limited funds to maximize the improvement of network reliability because the local government is responsible for road reconstruction and improvement of traffic network reliability after disaster occurred, so the efficiency of local government to reconstruct and rescue after a disaster occurred is an important factor on improvement of link reliability.

After a disaster occurred, the government's working efficiency on traffic is how to get more support to repair traffic network in the shortest possible time. The support to reconstruct traffic network includes material support and nonmaterial support, and the material support is considered as investment usually, the nonmaterial support includes many aspects such as technology support and volunteers and so on.

Some assumptions are given in order to discuss the efficiency of local government on network reliability based on the same funds and these assumptions as follows.

1) The nonmaterial support which can be provided to the local government is limited.

2) If the local government works more efficient to increase the reliability of links, more nonmaterial support can be got to repair the collapsed links such as labour force and confiscation of land and volunteers and so on.

3) The cost to make the reliability reach 1.0 does not tend to infinity based on the efficiency of local government until the government can't get any support after disaster occurred.

4) The maximum cost of the reliability has a limitation.

5) When the reliability equals zero, the original cost dose not equal zero because of those basic work for increasing the link reliability.

Base on those assumptions, we try to find out the relationship between the increase in cost and the increase in link reliability.

If the cost of providing a link with reliability r_a is C_a, one can allow for the cost of improving link reliability increasing as the link reliability increases by assuming the marginal cost dC_a of a marginal improvement dr_a in the link reliability is given by Eq.11.

$$\frac{dC_a}{dr_a} = \alpha e^{\frac{r_a}{\beta_a}} \qquad \text{(Eq.11)}$$

α is a positive constant, and $\frac{dC_a}{dr_a} = \alpha$ when r_a equals zero, and β_a which stands for the efficiency of the government support for link a is a non-negative constant. β_a equals zero when the local government can’t get any support, and $\frac{dC_a}{dr_a}$ will tend to infinity. The $\delta\beta$ which means the summation of the efficiency of local government to increase the reliability of all links of traffic network is named practical efficiency. $\delta R/\delta\beta$ can reflect the efficiency of local government to improve the network reliability.

The cost-reliability function can be given by the Eq.12.

$$C_a = \alpha\beta_a e^{\frac{r_a}{\beta_a}} + C_0 \qquad \text{(Eq.12)}$$

Because of the assumption 5), C_0 is the value of $C_a - \alpha\beta_a$ when r_a equals zero. If the reliability of link a is to be increased by δr_a from r_a to $r_a + \Delta r_a$, then the cost to achieve this will be

$$\delta C_a = \alpha\beta_a e^{\frac{r_a+\Delta r_a}{\beta_a}} - \alpha\beta_a e^{\frac{r_a}{\beta_a}} = \alpha\beta_a e^{\frac{r_a}{\beta_a}} (e^{\frac{\Delta r_a}{\beta_a}} - 1) \qquad \text{(Eq.13)}$$

If there are N links in the network, then the total increasing cost of the network is

$$C_{increase} = \sum_{a=1}^{N} \delta C_a \qquad \text{(Eq.14)}$$

5. MODEL SIMULATION ON PARALLEL NETWORK RELIABILITY

In order to simplify the discussion, $C_{increase}$ is assumed to be 0.2 and α is assumed to be 1.0. Table 3 shows the results for the two links in parallel for various levels of β_1 and β_2 and various reliability of r_1 and r_2 under the same funds. It can be seen from Table 3:

Table 3 The efficiency of government on parallel network

β_1	β_2	r_1	r_2	R	δR	$\frac{\delta R}{\delta \beta}$
Case P1: $r_{10}=0.5, r_{20}=0.5, R=0.75$						
0.2	0.2	0.508	0.508	0.758	0.008	0.0200
0.4	0.4	0.528	0.528	0.777	0.027	0.0338
0.5	0.5	0.5	0.569	0.784	0.034	0.0680
0.6	0.6	0.581	0.5	0.791	0.041	0.0683
0.7	0.7	0.5	0.592	0.796	0.046	0.0657
Case P2: $r_{10}=0.5, r_{20}=0.5, R=0.75$						
0.6	0.2	0.581	0.5	0.791	0.041	0.0683
0.7	0.4	0.592	0.5	0.796	0.046	0.0657
0.8	0.6	0.600	0.5	0.800	0.050	0.0625
0.9	0.8	0.608	0.5	0.804	0.054	0.0600
Case P3: $r_{10}=0.3, r_{20}=0.6, R=0.72$						
0.2	0.2	0.340	0.6	0.736	0.016	0.0800
0.4	0.4	0.384	0.6	0.754	0.034	0.0850
0.5	0.5	0.306	0.654	0.760	0.040	0.0400
0.6	0.6	0.3	0.669	0.769	0.049	0.0817
Case P4: $r_{10}=0.4, r_{20}=0.5, R=0.70$						
0.2	0.2	0.425	0.5	0.713	0.013	0.0650
0.2	0.25	0.4	0.526	0.715	0.015	0.0600
0.2	0.3	0.4	0.536	0.721	0.021	0.0700
0.4	0.45	0.4	0.561	0.739	0.039	0.0867
Case P5: $r_{10}=0.3, r_{20}=0.6, R=0.72$						
0.2	0.3	0.3	0.626	0.738	0.018	0.0600
0.2	0.4	0.3	0.642	0.750	0.030	0.0750
0.3	0.5	0.3	0.657	0.760	0.040	0.0800
0.5	0.7	0.3	0.680	0.776	0.056	0.0800
Case P6: $r_{10}=0.1, r_{20}=0.9, R=0.91$						
0.1	0.9	0.1	0.971	0.974	0.064	0.0711
0.2	0.8	0.1	0.9621	0.966	0.056	0.0700
0.9	0.1	0.265	0.9	0.927	0.017	0.0170
0.95	0.05	1	0.9	1	0.090	0.0947

1) For case P1: if the two links have the same original reliability and the same little government work efficiency, they will get the same reliability increase to improve the network reliability mostly. And one link would be randomly selected to improve the reliability when the two links can get the same great government support.

2) For case P2: the link which got more government support should be strengthened to improve the network reliability when the two links have the same original reliability.

3) For case P3: the link which has a lower original reliability should be strengthened when the two links got the same little government support. It means that it is difficult to improve the more reliable link. On the contrary, the link which has a higher original reliability should be strengthened when the two links got the same especial great government support. In other words, although one link has a higher original reliability, it should be strengthened only the great government support can be provided.

4) For case P4, P5, P6: if the discrepancy of the original reliability between the two links is not obvious such as case P4, the link which has a lower original reliability should be improved when the two links got the same government support, and the link that has a higher original reliability should be improved after getting more great government support than the other one. If the discrepancy of the original reliability between the two parallel links is remarkable such as case P6, the link that has a higher original reliability should be strengthened when this link can get enough support. In contrast,

if the link that has a higher original reliability is given especial little government support, it should not be improved. Case P5 is a transition from case P4 to case P6.

5) For case P1 to case P6: the link which has a special higher reliability should be improved when it got the same great support such as case P3 or got more support than the other one such as case P5. But if the link which has a special lower reliability can get special great support such as case P6 or can get the same little support such as case P3, it should be improved. That viewpoint can satisfy the fact that it is difficult to improve the more reliable link than the less reliable link.

6. CONCLUSION

This paper presented a study for improving connectivity reliability by using probability importance and criticality importance firstly, then mainly discussed the variety of parallel network reliability by a simple sample of cost-benefit analysis and the cost-reliability function based on the government efficiency. And we can get these results from this paper:

1) Although there are many strategies to improve the network reliability according to the different reliability indices, a lower reliable link should be strengthened firstly based on the above-mentioned discussion and comparison.

2) In parallel network, the two links will get the same reliability increase when they have the same original reliability and got the same little government support, and one link should be randomly strengthened when the government support is great although the two links have the same original reliability. The link which got more government support should be improved when the two links have the same original reliability. The link which has a lower original reliability should be improved when the two links got the same little government support. The link which has a higher original reliability should be improved when the two links got the same especial great government support. If the discrepancy of the original reliability of two links is obvious, the more reliable link should be improved when this link got enough support, and the link that has a higher original reliability shouldn't be improved under the especial little government support.

3) The network reliability can be increased by the government support, but the effect of the government support is limited, and the effect is not obvious when the government support exceeds a critical point.

However, the problem of the network reliability based on the government support is very complex, and further research is required to identify the detailed government support and the actual effect of the government support.

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