

# 交通混雑外部性に対する容積率及び都市境界規制 Regulation on building sizes and city sizes in cities with traffic congestion

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現在世界各国で交通混雑が問題となっており、実施の困難な混雑税以外の政策を考えることが必要である。本研究は密度規制、特に容積率 (FAR) と都市境界 (UGB) の最適規制が混雑税の代替・補完政策となりうるかを厚生分析により検討する。分析の結果、UGB 規制だけでは社会的厚生は混雑税による増加と比較して約 8%の上昇率に留まることを示す。一方、FAR と UGB 規制を両方課した場合は混雑税と比較して社会的厚生が 70~80%上昇する。したがって、FAR と UGB 規制の併用は混雑税の代替・補完政策となりうることを示す。

**Key words** : 交通混雑, 外部性, 容積率規制 (FAR)、都市境界 (UGB)

## 1. Introduction

Today, many cities in both developed and developing countries suffer from severe traffic congestion. Traffic congestion wastes a massive amount of time and fuel, besides causing environmental problems. The first-best policy against the congestion externality is to impose a congestion toll but such policy is infeasible in the real-world practice because it necessarily entails enormous implementation and administration costs.

Instead of congestion pricing, a city can relieve traffic congestion by changing the spatial distribution of residences through enforcement of population density regulations. Among density regulations, lot size zoning, floor area ratio regulation, and urban growth boundary regulation are more commonly practiced. Population density regulations have been widely studied, such as in Brueckner(2007). In particular Wheaton (1998) shows that regulating density achieves a result similar to the congestion pricing.

However no previous studies examined the FAR regulation in the presence of the UGB regulation. Thus the purpose of the present paper is to provide a quantitative evaluation of the FAR and UGB

regulation as a second-best policy in a congested city.

For such purpose, this paper numerically simulates the magnitude of welfare gain under (i) optimal FAR regulation with no UGB regulation, and (ii) simultaneous imposition of FAR regulation and UGB regulation, by comparing with that gained under the first-best congestion-toll policy.

## 2. The model

The model essentially follows Brueckner (2007). The city is assumed to be circular and is symmetric along any radial axis. The residential area in the city expands from  $x=1$  at the edge of the central business district (CBD) to  $x=z$  at the urban boundary.

### (1) Household Behavior

Residents are assumed to have a quasi-linear utility function which depends on the housing square footage  $q$  and numerical composite goods  $c$  that include all consumer goods except floor space, and is given by

$$u(c, q) = c + \alpha \ln q \quad \text{s.t.} \quad y = t + c + pq, \quad (1)$$

where  $y$  is income,  $p$  is the price per square foot of housing, and  $t(x)$  is commuting cost at distance  $x$

from the CBD. Because many households can reside in a single building, floor rent  $p$  equals the maximum floor rent bid by a household as a result of the competition among residents. Substituting the resulting demand functions back into the utility function, equating the result to a parametric utility level  $u$ , and solving for  $p$  and  $q$  yields

$$p = \alpha \exp(-\kappa), q = \exp(\kappa) \quad (2)$$

where,  $\kappa = (u - y + t + \alpha) / \alpha$ .

## (2) Developers' Behavior

When FAR regulation is not imposed, the developers are assumed to be perfectly competitive, and, therefore, are price-takers. The sum of developers' net profit from total floor space supply in the city, denoted  $\pi$  is given by

$$\pi = p\theta S^\beta - S - r, \quad (3)$$

where  $F(S) = \theta S^\beta$  is the intensive form of the production function  $S$  is the capital-to-land ratio and  $r$  is per unit of land and the price of capital is normalized at unity.

Next population density, denoted  $D$ , equals housing square feet per unit of land divided by square feet per dwelling, and is expressed as

$$D = F(S) / q. \quad (4)$$

However, under the FAR regulation, the floor space supply,  $F$ , is set exogenously which implies that the developers cannot maximize profit with respect to  $F$ . Under the FAR regulation, land rent and population density are expressed, respectively, as

$$r \equiv r(y - t(x), u, F(x)) \quad (5)$$

$$D = f(x) / q(x) \equiv D(y - t(x), u, F(x)) \quad (6)$$

## (3) Commuting Cost: the External Factor

It is assumed that automobiles are the only mode of commuting and the commuting cost is incurred only when commuting to and from the CBD boundary. At distance  $x$  from the CBD, traffic volume, denoted  $N(x)$ , is given by

$$N(x) = \int_x^z 2\pi s(1 - \rho) D ds, \quad (7)$$

where  $z$  is the distance from the CBD to the urban boundary, and  $1 - \rho$  of the land at each distance is available for housing.

We adopt the following commonly used functional form of the commuting cost to cross the ring at  $x$  when commuting towards the CBD

$$T(x) = \eta + \delta [N(x) / 2\pi x \rho]^\gamma. \quad (8)$$

When an additional commuter joins traffic at  $x$ , the resultant change in congestion cost is given by  $dT(x) / dN(x)$ , which when multiplied by  $n(x)$  gives the total externality caused by unpriced congestion, expressed as

$$n(x) \frac{dT(x)}{dN(x)} = \gamma \delta \left( \frac{n(x)}{2\pi x \rho} \right)^\gamma \equiv \tau(x) \quad (9)$$

where  $\tau(x)$  equals congestion toll at  $x$  that fully internalizes congestion externality. The total commuting cost from  $x$ , inclusive of the congestion toll and direct costs per km, denoted  $t(x)$ , is given by

$$t(x) = \int_1^x [T(s) + \tau(s)] ds. \quad (10)$$

When no toll is levied,  $\tau(s)$  in (7) is set to zero.

## (4) Welfare Function

Finally, social welfare, denoted  $W$ , which is composed of the total household utility, differential rents and total revenue from the congestion toll, is given by

$$W = \bar{N}u + \int_1^z 2\pi(1 - \rho)x(r - r_a)dx + \int_1^z N(x)\tau(x), \quad (11)$$

where  $\bar{N}$  is the total household and  $r_a$  is agricultural rent.

## 3. Numerical results

### (1) Setting the parameters

This section presents several numerical examples, each of which involves a comparison of five equilibria, viz. (i) the laissez-faire equilibrium, (ii) the equilibrium under the congestion-toll regime, (iii) the equilibrium with an optimally chosen UGB regulation, (iv) the equilibrium with an optimally chosen FAR regulation, and (v) the equilibrium with an optimally chosen FAR and UGB regulation.

To simulate different levels of congestion externality, we set five sets of the combination of  $\gamma$  and  $\delta$ , which are referred to as Examples 1-3

Example 1:  $\gamma = 1.25, \delta = 0.001$ , < 1<sup>st</sup>>;

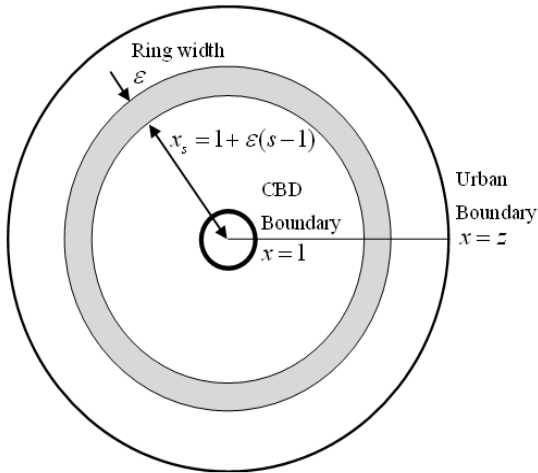
Example 2:  $\gamma = 1.20, \delta = 0.001$ , < 3<sup>rd</sup>>;

Example 3:  $\gamma = 1.4, \delta = 0.0015$ , < 2<sup>nd</sup>>;

where < > denotes each example's ranking with regard to the resultant total congestion externality level in decreasing order.

We set parameters as follows. The total number of households  $N$  is set at 100,000. The income per household is set at \$40,000 as in Brueckner (2007). The housing parameter  $\alpha$  in the quasi-linear utility function  $v(c, q) = c + \alpha \ln q$  is set at 8000, implying 20 percent of the income of \$40,000. Next, setting  $\rho = 0.2$  as in Brueckner (2007), 20 percent of the land in each ring is allocated for roads. Agricultural land rent  $r_a$  is set at \$150,000 per square km. The parameter  $\beta$  in the housing production function  $\theta S^\beta$  is set at 0.85, and the multiplicative factor  $\theta$  is set at 0.0001 as in Brueckner (2007). Moreover we divide the city

into narrow, discrete rings with an equal width denoted  $\varepsilon$ . The ring width  $\varepsilon$  is set at 0.5 km (see Fig. 1 where  $s \geq 1$  denotes the ring index such that for the CBD edge,  $s = 1$ ).



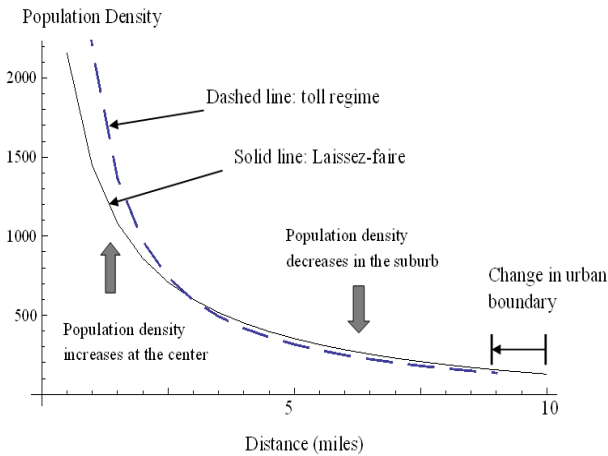
**Fig. 1 The model**

The intercept parameter  $\eta$  in the commuting-cost function (8) expresses the travel cost incurred while driving through one ring (0.5 km) in the case of no congestion. Setting the number of trips to the CBD as 225 round trips per year, average speed as 30 km/hour, travel cost including travel time as US\$30/hour and one worker per household,  $\eta$  is set at US\$225.

**(2) Numerical results**

The results of numerical simulation are presented in Table 1.

Results under the toll regime are shown in the second row in Table 1. Utility is 6621 which is less than each of no toll regime for exploit congestion tax from a resident. Also, population density rises by more than two-fold at the center (see Table 1), followed by a similar increase in the building size and in land rent (see Fig. 2).

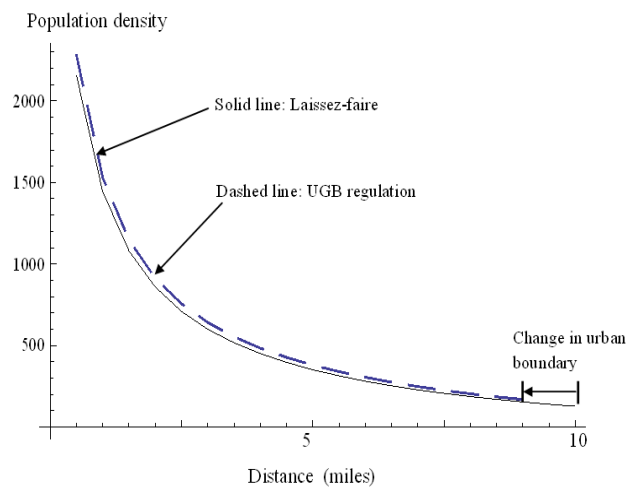


**Fig. 2 Population density distribution under toll regime**

	z	social welfare(10^8)	gain	u
<b>1.(<math>\gamma=1.25, \delta=0.001</math>)</b>				
Laissez-faire	20	9.07		8332
Toll regime	18	9.17	100.0	6621
optimal UGB	18	9.08	8.5	8265
optimal FAR	19	9.14	62.6	8389
optimal FAR and UGB	18	9.14	71.9	8318
<b>2.(<math>\gamma=1.20, \delta=0.001</math>)</b>				
Laissez-faire	20	9.73		8987
Toll regime	19	9.76	100.0	7885
optimal UGB	18	9.73	0.7	8904
optimal FAR	20	9.75	80.5	9007
optimal FAR and UGB	19	9.76	84.2	8967
<b>3.(<math>\gamma=1.4, \delta=0.00015</math>)</b>				
Laissez-faire	20	9.46		8722
Toll regime	19	9.51	100.0	7158
optimal UGB	18	9.47	8.8	8642
optimal FAR	20	9.50	80.6	8755
optimal FAR and UGB	19	9.50	84.2	8716

**Table 1 numerical results**

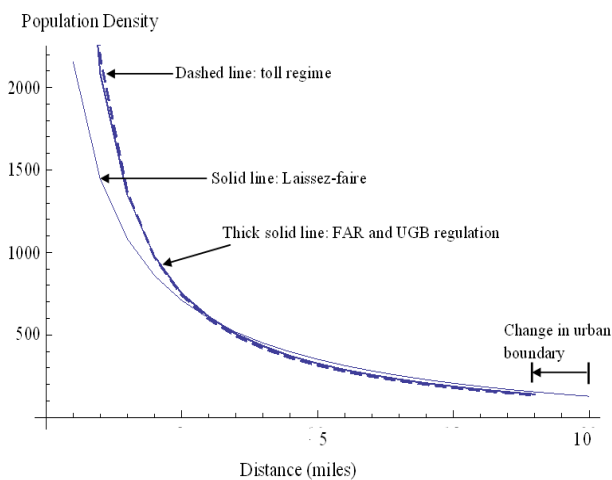
Results for the UGB regulation are shown in the third row in Table 1. Because population density increases slightly throughout the city, including in the suburb, the traffic congestion does not decrease significantly (see Fig. 3). Therefore the UGB regulation produces only a small welfare gain relative to the laissez-faire case as same as Brueckner(2007)



**Fig. 3 Population density distribution under UGB regulation**

Results for the FAR regulation are shown in the fourth row. under the optimal FAR regulation, the welfare gain 62.6% compared to toll regime, and is therefore a useful substitute for the first-best toll regime. Moreover, under the FAR and UGB regulation, the resultant welfare gain is about 72% of the toll regime, which is more than under the FAR regulation.

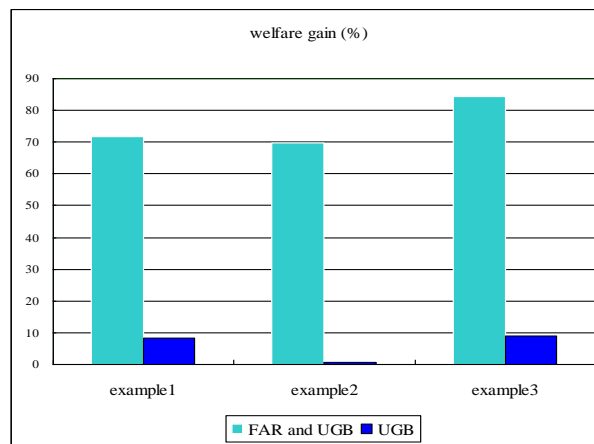
The population distribution pattern under the FAR and UGB regulation is almost in resemblance with that achieved under the toll regime as shown in Fig. 4. For that reason, the mitigation of traffic congestion under the FAR and UGB regulation proves to be significant.



**Fig. 4 population density distribution of toll regime and FAR regulation**

Including two further examples, the welfare gain, which is our main focus, is highest under the toll regime among all policies under consideration. This is not surprising given that the toll regime is a first-best policy. On the contrary, the optimal UGB policy produces only a small fraction – at best 8.8 percent (Example 3) – of the welfare gain achieved under the toll regime.

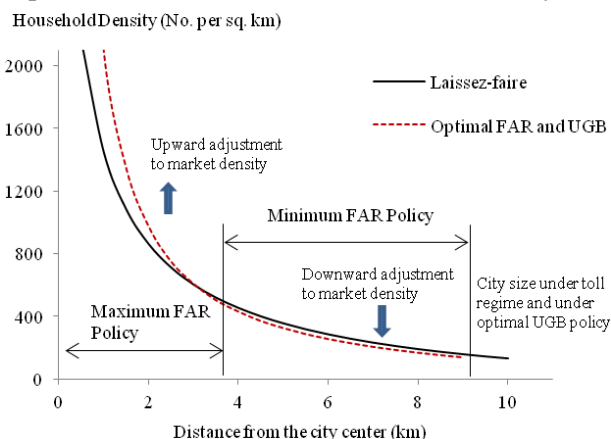
This justifies Brueckner (2007)’s conclusion that the UGB policy is a poor substitute for the toll regime. However, the welfare gain under the optimal FAR policy is impressive – and even more so if UGB is also in place. The welfare gain under the optimal FAR policy with UGB accounts for about 70 to 84 percent of that achieved under the toll regime in our three examples whereas even without UGB regulation, the optimal FAR policy still yields 63 to over 80 percent of the welfare gain under the toll regime



**Fig. 5 Social welfare gain under the FAR and UGB regulation as a percentage of the gain under the toll regime**

### (3) optimal regulation

Fig. 6 shows how the city should enforce FAR regulation. The optimal FAR regulation raises population density at the center and decreases suburban population density. Thus the central locations must be regulated under the “minimum FAR regulation”, which regulates the total floor space to be greater than the market equilibrium floor space. Likewise, “maximum FAR regulation”, which regulates the total floor space to be smaller than the market equilibrium floor space, should be imposed in locations nearer the urban boundary.



**Fig. 6 Optimal FAR and UGB regulations in a closed monocentric city**

Moreover, FAR regulation generates deadweight loss, which is given by  $DL$ , for optimal conditions,  $DL < 0$  in the central locations and  $DL > 0$  in the suburban locations. For this reason The combination of maximum FAR policy at the central locations with minimum FAR policy at the farthest locations is more efficient than the enforcement of only maximum FAR regulation (Bertaud, A., Brueckner, J.K., (2005)) in order to minimize total deadweight loss which is the cost of reducing

negative externality through the FAR policy.

The qualitative result related to FAR regulation hold irrespective of whether UGB regulation is imposed or not. Regarding UGB regulation, under simultaneous imposition of FAR and UGB regulation, the optimal city is more compact than the market city. See Kono et al. (2010) for further explanation on the feasibility of the “minimum FAR regulation and maximum FAR regulation”.

Finally, in a monocentric city model similar to that treated in this paper, Wheaton (1998) shows that population density always requires upward adjustment, which can be achieved through maximum lot size zoning throughout the city. However, our results provide a different outcome: although the result achieved by Wheaton (1998) holds at the central locations, at locations nearer the urban boundary, population density should be lower than the market population density. Accordingly, minimum FAR regulation that raises the population density cannot achieve a socially optimal solution on its own: maximum FAR regulation at urban boundary locations, which decreases population density, is also necessary.

The difference in the optimal regulations between the FAR regulation and lot size zoning arises from the fact that the FAR regulation, even if it is optimal, is a second-best measure against traffic congestion externality due to the indirect adjustment of population density. In other words, the FAR regulation can only control the total floor space of a building but not the per-capita floor space consumption. On the other hand, lot size zoning treated by Wheaton (1998) is the first-best policy against traffic congestion externality; unlike the FAR regulation, the lot size zoning directly regulates population density without generating any deadweight loss. The comparison of results achieved by Wheaton (1998) and this paper is summarized in Table 2.

Study <sup>o</sup>	Model <sup>o</sup>	Population Density Regulation <sup>o</sup> near the CBD <sup>o</sup>	Population Density Regulation <sup>o</sup> near the Urban Boundary <sup>o</sup>
This paper <sup>o</sup>	Closed City <sup>o</sup>	Minimum FAR regulation <sup>o</sup>	Maximum FAR regulation <sup>o</sup>
Wheaton (1998) <sup>o</sup>	Closed and Open Cities <sup>o</sup>	Maximum lot size zoning <sup>o</sup>	No regulation <sup>o</sup>

**Table 2 Optimal regulation in this paper and Wheaton (1998)**

#### 4. Concluding remarks

This paper carries out a numerical analysis on the FAR policy against unpriced congestion in the presence or absence of a UGB policy. Whereas we concur with Brueckner (2007)’s conclusion that the UGB policy is a poor substitute for the toll regime, our results also show that the welfare gain under the optimal FAR policy with a suitably chosen UGB makes up a significant fraction of that achieved under the toll regime. Thus, we establish that the optimal FAR policy with a UGB as an effective substitute for a Second-best toll policy. Moreover, even without a UGB, we find an impressive welfare gain under the optimal FAR policy alone.

Next, in the case of FAR regulation, the optimal FAR regulation raises population density at the center and decreases suburban population density. In other words, a monocentric closed city requires minimum FAR regulation at the center and maximum FAR regulation in the suburb to produce the upward adjustment to the market population density near the CBD and the downward adjustment near the UGB respectively

#### Acknowledgments

This research was supported by grants from the Ministry of Education, Culture, Sports, Science and Technology which are gratefully acknowledged. Despite assistance from many sources, any errors in the paper remain the sole responsibility of the authors.

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