Public Infrastructure Investment in the presence of Perfect Labor Mobility

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The central and local governments’ incentives to invest in public infrastructure and compete for subsidies are determined by the trade-off between political benefits and economic costs, the latter depending on the extent of decentralization of fiscal authority. However, there has been a lack of attention to the role of labor mobility across jurisdictions, known to be particularly important when governments (center and local) cannot impose strict control on inter-regional migration. To analyze how the local governments determine their investment strategies to maximize the sum of welfare in the region in response to the possible policy regimes: fiscal centralization, complete decentralization and partial decentralization, I endogenize the relationship between the production labor inputs and public infrastructure provided by the central and local governments. I show that a simple model with labor mobility across regions suggests the opposite interpretation of the conventional understanding of decentralization: complete fiscal decentralization and partial fiscal decentralization are not necessarily the best in terms of economic efficiency in the sense at least one local government or both local governments invest more in public infrastructure than under complete fiscal centralization. This implies that the central government is still needed to get involved in the entire process of decision making in public infrastructure provision.

Key Words: Infrastructure Planning, Fiscal Decentralization, Comparative Statics

1. Introduction

National and Regional economic performance depends heavily on public infrastructure such as highways, airports and seaports. Generally, such publicly funded infrastructure is supposed to streamline regional economic activities.

Even under the current tight fiscal conditions, the Japanese government is criticized for not providing for the public infrastructure sufficiently enough1). There is much controversy regarding how both the central and local governments can respond to the expectations, especially regarding role allocation between the central government and the local governments. Some lawmakers claim that decentralization must be promoted for efficient infrastructure development2). It is generally thought that a decentralized infrastructure management has several advantages over the centralized one: the local government is better-informed about the needs of the local communities and relevant socio-economic conditions. Centralized infrastructure management, however, is still essential for avoiding worthless and overlapping investments.

Previous studies have suggested that the governments’ incentives to invest and compete for subsidies are determined by the trade-off between political benefits and economic costs, the latter depending on the extent of decentralization of fiscal authority. For example, Yingyi Qian and Gerard Roland3) analyzed the link between fiscal competition and government incentives for bailouts in the context of the organization of government, and showed that fiscal competition among local governments increases the opportunity costs of bailout under factor mobility. David Wildasin4) analyzed the extent to which the local expenditures are affected by a bailout policy of the central government where positive externalities associated with provision of public goods exist. He examined the conditions under which the central government does or does not intervene in local expenditures. However, an insufficient number of studies with free labor mobility has been done in this area. Most of the relevant studies have assumed that the total population size of regions is fixed and remains constant. However,
in reality, people are likely to seek better job opportuni-
ties and move elsewhere even if already employed. Popula-
tion movement has a significant impact on the fiscal policy of the government: on the revenue side, it affects the amount of generated tax revenues such as income tax, property tax and the like; and on the expenditure side, it affects the amount of public goods such as education, social security and the like. Such population movement is generally triggered by the relocation of industries and businesses seeking better locations where they can benefit from well-provided infrastructure.

The question is how the governments’ incentives to provide public infrastructure are affected by the fiscal contracts between the central and local governments when the mobility of laborers is fully incorporated into the analysis.

This paper departs from previous work in this respect. I endogenize the relationship between the size of the population of the regions, in addition to public infrastructure provided by the governments to analyze how local governments determine their investment strategies to maximize the sum of welfare in the region in response to the policy regimes: fiscal centralization, complete decentralization and partial decentralization. With labor mobility, this paper shows that at least one local government or both local governments tend to invest more in public infrastructure under both the complete decentralization regime and the partial decentralization regime than under centralization regime.

Under the assumptions that labor is perfect mobile across the regions within a country and immobile across countries and there is no bail-out policy, I find that complete fiscal decentralization and partial fiscal decentralization are not necessarily the best in terms of economic efficiency in the sense at least one local government or both local governments invest more in public infrastructure than under complete fiscal centralization. Numerical simulation results suggest that careful parameterization of the economy is very important to determine whether decentralization leads to a “race to the bottom”. In context of public infrastructure management, a “race to the bottom” means that the competition of the two local governments would lead to undermine social welfare of regions and the whole country by overinvestment in public infrastructure, and may result in an investment equilibrium far away from the first best state. These imply that competition between governments is not necessarily all-around good thing and that the involvement of the central government is still needed throughout the entire process of decision making in public infrastructure provision.

2. The Basic Model

Consider a three-tiered transition economy consisting of: (i) a central government, that allocates central government budget of the purpose of regional redistribution and the provision of a national public good; (ii) two local governments, each endowed with the autonomy to perform the tasks of tax collection towards the provision of a local public good such as education, social welfare and so on, and the provision of a local public infrastructure such as highways, seaports and airports; (iii) private enterprises and households. Private enterprises serve the dual role of providing employment opportunities to private households and tax revenues to central and local governments.

In what follows, I lay out the specifics of the decision problem of the agents in three-tiers of transition economy, and examine how the central government’s planning problem trickles down to affect the total public infrastructure investment.

(1) Production Function

Private enterprises located in region $i$ carry out production activities in a competitive environment, taking as input of production household labor inputs, $L_i$ with (an endogenously determined) real wage $w_i$. Private enterprises generate revenue according to a quadratic production function $(g(I_i)a_i - \frac{b}{2}L_i)L_i$, which is twice continuously differentiable and strictly concave with respect to $L_i$. $I_i$ denotes the public infrastructure that makes private enterprises more productive. $I_i$ is financed by the central government or local government. $g(I_i)$ is also concave function.

Finally, $a_i$ denotes endowed locational constant representing other factors that makes enterprises more or less productive such as education, geography and the like, and $b$ denotes constant regarding production household labor inputs which is common to all the regions.

The profit maximization problem of the representative private enterprise is given by:

$$\pi_i = \max_{L_i} (g(I_i)a_i - \frac{b}{2}L_i)L_i - w_iL_i - T_i, \quad (1)$$

where where $\pi$ is maximized profit of the enterprise in region-i and $T_i$ is tax imposed by a local government as lump sum.

The first order condition associated with the
profit maximization problem (1) is given by
\[ g(I_i)a_i - bL_i = w_i. \]  
(2)

To simplify discussion in following sections, throughout the paper I will define \( g(I_i) \) as:
\[ g(I_i) = (1 + I_i)\delta, \]  
(3)
where \( g(I_i) \) is continuously differentiable and strictly concave, and thus \( 0 \leq \delta < 1 \). Since we have \( g(0) = 1 \), we have always positive output regardless the value of \( I_i \).

(2) Labor Mobility

I assume that local residents can move freely between the two regions to take advantage of better job opportunities and higher wages without any cost of relocation. I assume perfect labor mobility and thus, local residents belonging to the region-\( j \), if \( w_i > w_j \), would move their residential places from region-\( j \) to region-\( i \), and vice versa. Hence, migration of labor would finally stop and wage ratio would be equalized between the two regions. Furthermore, take note that all the population must be employed in labor market of the country, and the labor market must be cleared.

In labor market of the county, we must have:
\[ w_i = g(I_i)a_i - bL_i = g(I_j)a_j - bL_j = w_j, \]  
(4)
\[ L_i + L_j = L. \]  
(5)
From (3), (4) and (5), we have:
\[ L_i(I_i, I_j) = \frac{L}{2} + a_i(1 + I_i)\delta - a_j(1 + I_j)\delta \]  
(6)
Since we must have \( L_i > 0 \ \forall \ i \), we must have also
\[ |a_i(1 + I_i)\delta - a_j(1 + I_j)\delta| < bL. \]  
(7)

(3) Social Welfare Function

The objective of each local government to maximize the sum of the welfare of the third-tier participants. To do so, the two local governments independently choose \( I_i \) and \( I_j \) taking the other government’s choice as given.

The social welfare of the region-\( i \) is given as follows:
\[ W_i(I_i, I_j) = \max_{I_i} \pi_i + wL_i + P_i \]  
\[ = \max_{I_i} (g(I_i)a_i - b)I_i - I_i, \]  
(8)
with first order condition:
\[ g'(I_i)a_iL_i + (g(I_i)a_i - bL_i)\frac{\partial L_i}{\partial I_i} = 1. \]  
(9)
where \( a_i \) is region-specific parameter (given). \( P_i \) denotes the amount of local public goods other than public infrastructure, i.e., education, social security and the like. Since real \( w \) is common to the two regions, the welfare of households depends on local public good \( P_i \) as well as sum of laborers’ incomes \( wL_i \) and enterprises’ profit \( \pi_i \). In addition, we must have \( P_i = T_i - I_i \) due to the local governments’ budget constraint when there is neither lump sum tax paid to the central government nor subsidies distributed by the central government.\(^1\)

From the equation (3) and (6), the equation (9) is equivalent to:
\[ g'(I_i)a_i\left[\frac{L_i b + g(I_i)a_i}{2b}\right] - 1 = 0, \]  
(10)
and from (3), equation (10) can be rearranged as follows by dividing \( g'(I_i)a_i \):
\[ \frac{L_i}{2} + a_i(1 + I_i)\delta - \frac{a_j(1 + I_j)\delta}{2b} = \frac{I_i^{1-\delta}}{\delta a_i}. \]  
(11)
where \( I_i = 1 + I_j \).

It can be verified from equation (10) that an optimal investment \( I_i \) increases as \( a_i \) increases for small enough \( \delta \). This is because both \( g'(I_i)L_i(I_i, I_j) \) and \( g'(I_i)\delta \) decrease in \( I_i \) if \( \delta < \frac{1}{2} \). It can be also verified that \( I_i \) decreases as \( a_j \) increases if \( \delta < \frac{1}{2} \) for the same reason. In other words, as region specific factor \( a_i \) increases, i.e., improves education, public investment in the same region also increases while the other region’s investment decreases.

In addition, it should be noted that an optimal investment \( I_i \) for the social welfare maximization problem is given by function of \( I_j \) and vice versa. Thus, optimal public investment is determined through behaviors of the two local governments in public investment.

Allowed to provide public infrastructure directly by using lump sum tax paid by local governments, the central government chooses the amount of investment, \( G_i \), so that the sum of the nation’s social welfare, \( \sum_i W_i \), can be maximized.

(4) Other Assumptions

In order to acquire a unique solution for the social welfare maximization problem (8), following two assumptions must be satisfied.

a) Elimination of Corner Solution

Every region has public infrastructure \( I_i > 0 \) for production activities. In order to eliminate
\(^1\) Budget constraint must be \( P_i = T_i - \tau_i - I_i \) when there is lump sum tax or subsidies, \( \tau_i \) (if the sign is positive, there is lump sum tax paid to the central government).
order condition is

$$\frac{\partial W_i}{\partial I_i} = 0 = \frac{\delta a_i \{a_i - a_j (1 + I_j)^{\delta} \} + b (a_i - 4)}{4b} > 0$$  \tag{12}$$

b) Concavity of $W_i(I_i, I_j)$

Social welfare function of the region (8) must be concave with respect to both $I_i$ and $I_j$. Second order condition is

$$\frac{\partial^2 W_i}{\partial I_i^2} < 0. \tag{13}$$

Later, I will discuss the sum of the nation’s social welfare $\sum W_i$. I will assume the central government maximizes $\sum W_i$ by choosing both $I_i$ and $I_j$ simultaneously under centralization regime. In order to have a unique solution $(I_i, I_j)$, I also assume:

$$\frac{\partial^2 W_i}{\partial I_i^2} < 0, \frac{\partial^2 W_i}{\partial I_j^2} < 0, \left( \frac{\partial^2 W_i}{\partial I_i \partial I_j} \right)^2 > 0. \tag{14}$$

These two assumptions imply that $b$ and $\bar{L}$ must be large enough.

3. The Centralization Case

I begin by analyzing the centralization case as a benchmark. The centralization means that the central government makes all decisions regarding public infrastructure investment and the local governments have no way to modify the investment plan decided by the central government.

(1) Social Welfare Function

The objective of the central government, taking as given the region-specific parameter $a_i$ and the region-common parameter $b$, is to maximize the sum of the welfare of the whole country. To simplify the discussion, the local governments surrender $T_G$, as lump sum tax to the central government, and provide local public goods $P_i$ other than public infrastructure by using remaining $T_i - \tau_i$. The central government allocates its revenue to local public infrastructure $I_i$ within its budget.

The social welfare maximization problem of the central government is described as follows.

$$W_C(I_i, I_j) = \max_{I_i, I_j} (\pi_i + \pi_j) + w \bar{L} + P_i + P_j;$$

Subject to $P_i \leq T_i - \tau_i, i = 1, 2,$ and $I_1 + I_2 \leq \tau_1 + \tau_2$. \tag{15}$$

The social welfare maximization problem given the equation (15) is modified as:

$$W_C(I_i, I_j) = \max_{I_i, I_j} \sum_{i=1}^{2} [(g(I_i)a_i - b L_i) L_i - I_i] \tag{16}$$

The first order conditions associated with the social welfare maximization problem (16) is given by

$$g'(I_i)a_i L_i = 1, i = 1, 2. \tag{17}$$

Since $W_C(I_i, I_j)$ is concave function in both $I_i$ and $I_j$, the second order condition is satisfied. Furthermore, since $L_i$ is a function of the two decision variable $I_i$ and $I_j$ from the equation (6), the implicit solutions to the first order conditions (17) give the optimal public infrastructure investment $I_i$ and $I_j$. Equation (17) provides the following two implications: (i) a solution to (17) denoting $I_i^C$, increases as region-specific parameter $a_i$ increases; (ii) $I_i^C$ decreases as the other region’s parameter $a_j$ increases; and (iii) $I_i^C$ is less than $I_i^D$ denoting the solution to equation (9), the last one being implied by the fact that $(g(I_i)a_i - b L_i) \frac{a_i}{\partial x} > 0$ and $g'(I_i)a_i L_i$ is decreasing in $I_i$.

4. The Complete Decentralization Case

Consider the complete decentralization case where the local governments hold absolute powers in determining all public infrastructure investment policies. Under the complete decentralization regime, the local governments provide their public infrastructure by their own funds and the central government no longer intervene with them.

(1) Investment Equilibrium Under the Complete Decentralization Regime

Given equation (10), we have already obtained necessary conditions for determining an optimal investment. By concavity of $W_i(I_i, I_j)$, the equation is supposed to be necessary and sufficient conditions for the social welfare maximization problem.

A unique investment equilibrium between the two local governments is determined by Nash Equilibrium of the two local governments’ investment behaviors. This is given by the intersection of two curves in figure (1).

In the context of the two local governments model, $I_1$ and $I_2$ simultaneously solve

$$g'(I_i) a_i \left( \frac{L_i (I_i^*, I_j^*)}{2b} + g(I_i^*)a_i \right) = 0, i = 1, 2, \tag{18}$$

Let $x^* = \arg \max f(x)$ subject to $x \geq 0$. If $x^* = 0$, then we must have $\frac{\partial f(x^*)}{\partial x} \leq 0$ and $\frac{\partial f(x^*)}{\partial x^2} x^* = 0$ for necessary condition.
and Ψ
2
the case with symmetric property of the problem.

Hence, we can obtain a unique investment equilib-rium between the two local governments,(I
1
, I
2
), as a intersection of the two best response curves.

(2) Estimation of Total Investment under the Complete Decentralization

I now examine the extent to which local public infrastructure is provided under the complete decentralization regime. To see this, I compare the investment equilibria of the complete decentralization and the centralization case.

Let I
iD
i
 denote the optimal public infrastructure investment for the region-i government under the decentralization regime. Also, let I
iC
i
 denotes the optimal public infrastructure investment for the region-i government under the centralization regime.

Let Ψ
i
(I
i
) denote the left-hand side of the equation (9). Hence,

Ψ
i
(I
i
) := g(I
i
)a
i
L
i
 + (g(I
i
)a
i
 − bL
i
) ∂L
i
 ∂I
i
 (19)

and Ψ
i
(I
i
) = 1 gives the optimal I
i
, given an opponent’s strategy I
j
.

Now I summarize this observation in the following lemma and give a proof:

Lemma 1 .
Under complete decentralized public infrastructure provision regime, a rich region (with larger a
i
) invests more in public infrastructure than a poor region (with smaller a
i
) does.

Proof
Without loss of generality, suppose a
1
 ≥ a
2
 and a
1
 = a
2
 + a’ with a’ ≥ 0. First of all, I consider the case with a’ = 0. Then, we must have I
1
 = I
2
 by symmetric property of the problem.

Now I suppose that a’ > 0. Ψ
1
(I
1
, I
2
; a’) = 1 and Ψ
2
(I
1
, I
2
; a’) = 1 give an equilibria, (I
1
, I
2
) under complete decentralized public infrastructure provision regime. Totally differentiating

Ψ
i
(I
1
, I
2
; a’) ∀i, then we have:

\[
\frac{dI_1}{dI_2} = \frac{1}{\Delta} \begin{bmatrix}
\frac{\partial \Psi_2}{\partial I_2} & -\frac{\partial \Psi_1}{\partial I_2} \\
-\frac{\partial \Psi_2}{\partial I_1} & \frac{\partial \Psi_1}{\partial I_1}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial \Psi_1}{\partial a'} da' \\
\frac{\partial \Psi_2}{\partial a'} da'
\end{bmatrix}
\] (20)

where

\[
\Delta = \frac{\partial \Psi_1}{\partial I_1} \frac{\partial \Psi_2}{\partial I_2} - \frac{\partial \Psi_2}{\partial I_1} \frac{\partial \Psi_1}{\partial I_2} > 0. \tag{21}
\]

This is because if Δ < 0, then I would not find any equilibria which satisfies Ψ
i
(I
1
, I
2
) = 1 ∀i as figured in Figure 2.

In order to analyze how an investment equilibria (I
1
, I
2
) changes according to changes in the value of a’, signs of \( \frac{\partial \Psi_i(I_1, I_2; a')}{\partial a} \) ∀i, j and \( \frac{\partial \Psi_i(I_1, I_2; a')}{\partial a} \) ∀i should be examined.

By concavity of W
i
(I
i
, I
j
), we have \( \frac{\partial \Psi_i(I_1, I_2; a')}{\partial I_i} < 0 \) ∀i, and \( \frac{\partial \Psi_i(I_1, I_2; a')}{\partial I_j} < 0 \).

Also, by simple calculation, I have \( \frac{\partial \Psi_i(I_1, I_2; a')}{\partial a} > 0 \) and \( \frac{\partial \Psi_i(I_1, I_2; a')}{\partial a} < 0 \), since \( a_i(1 + I_1^*) \delta < bL \) by assumptions.

From equation (20), I also have \( \frac{dI_1}{dI_2} > 0 \) and \( \frac{dI_2}{dI_1} < 0 \). Hence, I conclude if a
1
 > a
2
, then I
1
 > I
2
.

On the other hand, the investment equilibria under the centralization regime is determined by the first order necessary and sufficient condition (17). Let Φ
i
(I
i
) denotes the left-hand side of the equation (17). Hence,

Φ
i
(I
i
) := g(I
i
)a
i
L
i
(1, I
j
) (22)

and Φ
i
(I
i
) = 1 gives the optimal I
i
, given an opponent’s strategy I
j
.

Since \( w = g(I_i)a_i - bL_i(I_i, I_j) > 0 \) and \( \frac{\partial L_i(I_i, I_j)}{\partial I_i} > 0 \) holds from the two equations, (4) and (6),

Ψ
i
(I
i
) > Φ
i
(I
i
) (23)
must always hold given a same opponent’s strategy \( I_j \).

Let \( D^*_i \) and \( C^*_i \) denote an investment equilibria under complete fiscal decentralization and complete fiscal centralization, respectively. Without loss of generality, suppose \( a_1 \geq a_2 \).

Then, now we have the following four possible cases: (i) \( D^*_i < C^*_1 \) and \( D^*_i < C^*_2 \); (ii) \( D^*_i < C^*_1 \) and \( I^*_1 > I^*_2 \); (iii) \( I^*_1 > C^*_i \) and \( I^*_2 > I^*_2 \); and (iv) \( I^*_2 > I^*_1 \) and \( I^*_2 > I^*_2 \). we have, thus, arrived at the following result:

**Proposition 1**.

*Under complete decentralized public infrastructure provision regime;*

[[I]] if the two regions are homogeneous, that is, \( a_1 = a_2 \), then both regions invest more in public infrastructure (\( I_i \)) than under fiscal centralization regime.

[[II]] if the two regions are heterogeneous, that is, \( a_1 > a_2 \), then at least one region or both regions invest more in public infrastructure than under fiscal centralization regime.

**Proof**

[[I] Homogeneous Case

(i) \( D^*_1 < C^*_1 \) and \( D^*_1 < C^*_2 \)

Suppose for now that \( D^*_1 < C^*_1 \) and \( D^*_1 < C^*_2 \).

Evaluated at the investment equilibrium under the complete decentralization regime, we must have:

\[
\frac{\partial g_i(D^*_1)}{\partial I_i} < 1, i = 1, 2.
\]

At the investment equilibrium under the centralization, we must have

\[
g'(I^*_1)a_iL_i(D^*_1, I^*_1) = 1 - (g(I^*_1)a_i - bL_i(I^*_1, I^*_1)) \frac{\partial L_i(D^*_1, I^*_1)}{\partial I_i}
\]

\[
< 1, i = 1, 2.
\]

(ii) \( I^*_1 > C^*_1 \) and \( I^*_1 > C^*_2 \)

Since \( g(I_i) \) is concave, we obtain:

\[
L_i(I^*_1, I^*_2) < g'(I^*_i) \frac{\partial L_i(D^*_1, I^*_1)}{\partial I_i}
\]

\[
< L_i(C^*_1, I^*_2) \forall i
\]

Since the total population of the country is constant, equation (26) yields:

\[
L = \sum_{i=1}^{2} L_i(D^*_1, I^*_2)
\]

\[
< \sum_{i=1}^{2} L_i(C^*_1, C^*_2) = L.
\]

Equation (27) condition cannot hold neither. (ii) \( I^*_1 > C^*_1 \) and \( I^*_2 > C^*_2 \)

Due to symmetric property of the problem with \( a_1 = a_2 \), this condition never holds.

(iii) \( I^*_1 > C^*_1 \) and \( I^*_2 > C^*_2 \)

Due to symmetric property of the problem with \( a_1 = a_2 \), this condition never holds.

(iv) \( I^*_1 > C^*_1 \) and \( I^*_2 > C^*_2 \)

Since the assumptions (i), (ii), (iii) cannot hold, we must have \( I^*_1 > C^*_1 \) and \( I^*_2 > C^*_2 \).

[[II] Heterogeneous Case

(i) \( D^*_1 < C^*_1 \) and \( I^*_1 > C^*_2 \)

The proof is the same as part (i) of the homogeneous case and thus, this condition cannot hold.

![Best-response correspondences under different fiscal contracts](image)

Figure 3 illustrates the assignment of public infrastructure provided in the both regions under complete fiscal centralization regime and complete fiscal decentralization. \( \psi_i(I_j) \) and \( \phi_i(I_j) \) denote the best-response correspondence curves which give the optimal investment strategy of the region-i local government under fiscal decentralization and fiscal centralization, respectively. Higher best-response correspondences are achieved through the condition \( (g_i(a_i - bL_i)) \frac{\partial L_i}{\partial I_i} > 0 \) under the complete decentralization regime, implying, as should be expected, increases in public infrastructure in at least one region or both regions.

If the two regions are heterogeneous, then we still have the following three possibilities: (ii) \( D^*_1 < C^*_1 \) and \( I^*_2 > C^*_2 \); (iii) \( I^*_1 > C^*_1 \) and \( I^*_2 < C^*_2 \); and (iv) \( I^*_2 > I^*_1 \) and \( I^*_2 > I^*_2 \).

(3) Numerical Simulation

In order to see how the model works, I first see what determines the investment equilibria under the complete decentralization regime with the best-response correspondence curves and the indifference curves of the two local governments, and the Pareto optimal allocation under the cen-
tralization regime. Our main tool for exposition of the properties of the model is numerical simulation. In the case of two local governments model, it is possible to represent the determination of the investment in terms of a two dimension diagram.

One such diagram is illustrated in Figure 4. On the axes of this figure are the amount of public infrastructure investment $I_1$ and $I_2$ in the region-1 government and the region-2 government, respectively. The two bold solid lines indicate the best-response correspondence curves: the light one indicates $\psi_1(I_2)$ and the dark one indicates $\psi_2(I_1)$, respectively. The intersection of these two curves gives an investment equilibrium under the complete decentralization regime and satisfies $I^{D*}_1 = \psi_1(I^{D*}_2)$ and $I^{D*}_2 = \psi_2(I^{D*}_1)$ simultaneously. The two narrow solid lines indicate the indifference curves with the same social welfare of the two governments.

On the other hand, the large red dot indicates the pareto optimal allocation $(I^{C*}_1, I^{C*}_2)$ under the centralization regime. It can be readily verified from Figure 4 that the Pareto optimal allocation locates inside the two best-correspondence curves, that is, $I^{D*}_i > I^{C*}_i \ \forall i$. The bold broken line indicates the indifference curve with the same social welfare of the country as a whole. (Values of parameters underlying the figures are given in the table 1.)

In the case illustrated in Figure 4, the investment equilibrium is symmetric with each economy having an identical region-specific parameter, and providing the identical public infrastructure, respectively. As a result of numerical analysis, public infrastructure provision of the two local governments under the complete decentralization regime, is about 1.4 times as much as under the centralization regime, as Proposition 1 suggests.

The change in the region-specific parameter $a_i$ affects the investment equilibrium under complete decentralization regime. To see this, Figure 5 is constructed with a higher level of the region-specific parameter. All other parameters remain the same as Figure 4.

Comparing the best-response correspondence curves in Figure 5 with the curves in Figure 4. The best-response correspondence curve of the region-2 $\psi_2(I_1)$ shifts to the lower position while the curve $\psi_1(I_2)$ shifts to the right, as Lemma 1 suggests. As a result, the investment equilibrium given by the intersection of the two curves shifts to the lower right.

On the other hand, Figure 6 is constructed with a higher level of the region-specific parameter of region 2 than the case 2. The best-response correspondence curve for the region-2 $\psi_2(I_1)$ shifts to the upper position and the curve for the region-1 $\psi_1(I_2)$ shifts back to the left, as Lemma 1 suggests. Consequently, the investment equilibrium given by the intersection of the two curves shifts to the higher left compared with Case 2. In Case 3, public investment made by both the regions is more than case 1, as already discussed.

Case 4 provided the simulation result when the parameter $b$ was slightly increased compared with Case 2. As the parameter $b$ increases, the social welfare function is more concave. Intuitively, as the social welfare function is more concave, public investment is likely to decrease. This implies that both the region-specific parameter $a_1$ and the parameter regarding the marginal labor productivity $b$ are important for investment equilibrium analysis.

These four numerical simulations suggested that slight difference in parameters would lead a significant different result. This implies that careful parameterization of the economy is very important to evaluate the level of public investment under a certain fiscal contract between the central and local governments. Also, a simple model with perfect labor mobility across regions suggests the opposite interpretation of the conventional understanding of decentralization: complete fiscal decentralization is not necessarily the best in terms of economic efficiency in the sense at least one local government or both local governments invest more in public infrastructure than under complete fiscal centralization.

5. The Partial Decentralization Case as a Possible Solution

Up to this point, we have already seen the two extreme case: the centralization case and the complete decentralization case. As an intermediate case, I consider partial decentralization case in this section. Partial decentralization case means that both the central government and the local government are responsible for public infrastructure provision in accordance with an appropriate rule stipulating the role allocation between the two governments.

One possible rule is that the central government is responsible for providing public infrastructure at certain level and the local government provides additional public infrastructure taking into consideration of its own socio-economic situation, and strategy for economic development and budget. Public infrastructure provided directed by the central government is supposed to be determined by the social welfare of the country as
Table 1  Values of Parameters and Invest Equilibrium

<table>
<thead>
<tr>
<th>Case</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$b$</th>
<th>$I_1^C^*$</th>
<th>$I_2^C^*$</th>
<th>$I_1^D^*$</th>
<th>$I_2^D^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>4.0</td>
<td>4.0</td>
<td>4.0</td>
<td>3.30</td>
<td>3.30</td>
<td>4.51</td>
<td>4.51</td>
</tr>
<tr>
<td>Case 2</td>
<td>4.2</td>
<td>4.0</td>
<td>4.0</td>
<td>6.28</td>
<td>1.34</td>
<td>7.86</td>
<td>2.90</td>
</tr>
<tr>
<td>Case 3</td>
<td>4.2</td>
<td>4.2</td>
<td>4.0</td>
<td>3.67</td>
<td>3.67</td>
<td>5.80</td>
<td>5.80</td>
</tr>
<tr>
<td>Case 4</td>
<td>4.2</td>
<td>4.0</td>
<td>4.2</td>
<td>5.61</td>
<td>1.77</td>
<td>6.88</td>
<td>2.86</td>
</tr>
</tbody>
</table>

$\bar{\lambda} = 3.0, \delta = 0.4$

a whole. Contrary to the central government, the local governments maximize the social welfare of the region, not the country as a whole. Consequently, the local government are competing in providing public infrastructure each other, and tend to overinvest. In this section, I show this tendency and the equivalency between the complete decentralization case mentioned in Section 4 and the partial decentralization case in this sense.

Another possibility is that the local governments decide their public infrastructure investment by themselves, and the central government provide subsidies to the local governments. This subsidy is supposed to be proportional to the size of public investment made by each local government. This case is referred as “The Decentralization Case with Subsidies From the Central Government” and further discussed in the next section.

(1) Investment Equilibrium under Partial Decentralization Case

I describe the partial decentralization case as a simple extention of the basic model.

Under the partial decentralization regime, the decision problems of the three stakeholders are determined by the following three steps.

At the beginning of the stage, the central government determines a national infrastructure development plan. The central government cannot obtain relevant information regarding the respective local governments’ public infrastructure plan.

At the second stage, the local governments determines its own public infrastructure investment taking into consideration not only of the opponent investment strategy but also of the central government’s public infrastructure plan, $G_i$ and $G_j$. I assume the local governments can obtain relevant information regarding public infrastructure provided directly by the central government.

At the last stage, the private enterprises determines its own production activities.

The social welfare maximization problem of the central government, the equation (15) with the first order condition (17), is modified as follows:

$$W_{G^P} = \max_{G_i, G_j} (\pi_i + \pi_j) + w\bar{\lambda} + P_i + P_j$$

Subject to 

\[ P_i \leq T_i - \tau_i, i = 1, 2 \]

and 

\[ G_1 + G_2 \leq \tau_1 + \tau_2 \] (28)

Since the central government cannot obtain the local governments’ decision the decision problem (28) is equivalent to the problem represented by the equations (15) and (17). Hence, we must have $G_i^{P^D} = G_i^{C^*}$ for all $i$.

The decision problem for the local governments, the equation (8) is modified by as follows:
\[ W_{i}^{PD}(I_i, I_j, G_i, G_j) = \max_{I_i} \pi_i + wL_i + P_i - \tau_i \] (29)

To analyze the local government’s decision problem, let \( X_i \) denote the total public investment provided in the region \( i \). Thus, we have \( X_i = I_i + G_i \). Here, I suppose \( I_i > 0 \) since local governments cannot “remove” parts of the central government’s investment.

The decision problem (29) is rearranged as:
\[ W_{i}^{PD}(X_i, X_j) = \max_{X_i} \pi_i + wL_i + P_i - \tau_i^{PD} \] (30)

where \( G_i^{PD} \) and \( \tau_i^{PD} \) are optima determined by the central government’s decision problem (28).

Given these parameters, local governments establish their own investment policy to maximize the sum of region welfare just as under the complete decentralization case. By differentiating (28) we obtain necessary and sufficient conditions for solution to the problem. That is, \( X_i^{PD} \) solves
\[ g'(X_i^{PD})a_{i}L_i(X_i^{PD}, X_j^{PD})b + g(X_i^{PD})a_{i} = 0 \]

The equations (31) are equivalent to the first order necessary and sufficient conditions (18), respectively. We have, thus, arrived at the following result:

Proposition 2.

If the problem (31) has the solutions such \( I_i^{PD} > G_i^{PD} \forall i \), then the partial fiscal decentralization where the central and the local governments decide each public investment independently is equivalent to the complete decentralization in terms of economic efficiency. That is, \( X_i^{PD} = I_i^{PD} + G_i^{PD} = I_i^{PD} \).

This implies that the public investment provided directly by the central government has no effect other than welfare redistribution across the regions under the partial fiscal decentralization regime. The welfare redistribution is estimated by \( \Delta W_i^{PD} = G_i^{PD} - \tau_i^{PD} \), which is determined only through the central government’s public investment policy.

6. Decentralization with the Central Government’s Subsidies

In this section, I analyze another possible fiscal decentralization regime where the local governments primarily determine their public investment taking into account of the other local government decision and the subsidy from the central government in order to maximize the sum of the region welfare. I assume that the subsidies reallocated from the central government to the local governments are endogenously determined by the local governments’ decision on public infrastructure provision.

(1) Investment Equilibrium under Decentralization with Central Government Subsidies

Just as the partial decentralization case, I describe the decentralization with the central government’s subsidy as a simple extension of the basic model. The size of subsidies is determined based on the actual expenditure of the local governments for public infrastructure: it is proportionally redistributed among local governments. The local governments are not prohibited to use the subsidy for other purpose. That is, the subsidy from the central government is supposed to be used for providing both public infrastructure and public good such as welfare, education and so on. This type of subsidy is very similar to the local redistribution tax, which is transferred from the central government to local governments based on the size of region, population and expenditure in public investment and public good. Actually, local redistribution tax is one of the most important financial sources for local governments, and has around 15% share of the total local government’s revenue. Hence, it is also important to analyze the effect of this type of subsidies regarding the decision problem on public infrastructure provision.

Suppose that the central government provides subsidies \( sI_i \) to local governments and \( s(I_1 + I_2) \leq \tau_1 + \tau_2 \) where \( s \) is the subsidy ratio.

The decision problem, (8), is modified by as follows:
\[ W_{i}^{S}(I_i, I_j) = \max_{I_i} \pi_i + wL_i + sI_i + P_i - \tau_i \] (32)

with first order necessary and sufficient condition:
\[ a_{i}g'(I_i)L_i + (a_{i}g(I_i) - bL_i) \frac{\partial L_i}{\partial I_i} + s = 1 \] (33)

Now I examine the extent to which the local public infrastructure is provided under decentralization with the central government’s subsidy. To see this, I compare the size of the total investment with under the complete fiscal decentralization.

Let \( \Psi_i^{S}(I_i) \) denotes the left-hand side of the equation (33):
\[ \Psi_i^{S}(I_i^{S}) = a_{i}g'(I_i^{S})L_i + (a_{i}g(I_i^{S}) - bL_i) \frac{\partial L_i}{\partial I_i} + s \]

Note that \( \Psi_i^{S}(X_i^{S}) = 1 \) or \( \Psi_i^{D}(X_i^{D}) = 1 \) give an optimal solution \( I_i^{S} \) given the opponent’s strategy \( I_j \), and I always have \( \Psi_i^{S}(I_i) > \Psi_i^{D}(I_i) \) given
the same opponent’s strategy $I_j$. Hence, an analogous analysis, as demonstrated in section 4.2, yields the following result:

**Proposition 3.**
Under decentralization with the central government’s subsidy:

[I] if the two regions are homogeneous, that is, $a_1 = a_2$, then both regions invest more in public infrastructure ($I_i$) than under complete fiscal decentralization regime.

[II] if the two regions are heterogeneous, that is, $a_1 \neq a_2$, then at least one region or both regions invest more in public infrastructure than under complete fiscal decentralization regime.

**Proof**
The proof is completely analogous to that of Proposition 1.

[I] Homogeneous Case
(i) $I_1^{S*} < I_1^{D*}$ and $I_2^{S*} < I_2^{D*}$
For the sake of contradiction, assume that $I_1^{S*} < I_1^{D*}$ and $I_2^{S*} < I_2^{D*}$.
Since the condition $\Psi_i(I_i, I_j) > \Psi_j(I_i, I_j)$ always holds, and $\Psi_i(I_i, I_j)$ is decreasing in both $I_i$ and $I_j$, we have:

$$1 = \Psi_i(I_1^{D*}, I_2^{D*}) = \Psi_i(I_1^{S*}, I_2^{S*}) > \Psi_i(I_1^{S*}, I_2^{D*}) > \Psi_i(I_1^{D*}, I_2^{D*}).$$

This condition cannot hold.

(ii) $I_1^{S*} < I_1^{D*}$ and $I_2^{S*} > I_2^{D*}$
Due to symmetric property of the problem with $a_1 = a_2$, this condition never holds.

(iii) $I_1^{S*} > I_1^{D*}$ and $I_2^{S*} < I_2^{D*}$
Due to symmetric property of the problem with $a_1 = a_2$, this condition never holds.

(iv) $I_1^{S*} > I_1^{D*}$ and $I_2^{S*} > I_2^{D*}$
Since the assumptions (i),(ii),(iii) cannot hold, we must have $I_1^{S*} > I_1^{D*}$ and $I_2^{S*} > I_2^{D*}$.

[II] Heterogeneous Case
(i) $I_1^{S*} < I_1^{D*}$ and $I_2^{S*} < I_2^{D*}$
The proof is the same as part (i) of the homogeneous case and thus, this condition cannot hold.

Under complete fiscal decentralization regime, $\Psi_i(I_i, I_j) - 1$ is the marginal return to public investment and $\Psi_i(I_1^{D*}, I_2^{D*}) = 1$ gives an investment equilibrium. Under partial fiscal decentralization with the central government’s subsidy, however, $\Psi_i(I_1^{S*}, I_2^{S*}) = 1 - s$ gives an investment equilibrium. Intuitively, both local governments tend to invest in public infrastructure so that the marginal return to public investment is equalized to $-s$, which is definitely negative. This implies that at least one region or both regions tend to invest more in public infrastructure than under complete fiscal decentralization. In addition, as the value of $s$ increases we have larger difference in investment equilibrium between under partial fiscal decentralization with the central government’s subsidies and under complete fiscal decentralization.

If the two regions are heterogeneous, then we still have the following three possibilities: (ii)$I_1^{S*} < I_1^{D*}$ and $I_2^{S*} > I_2^{D*}$; (iii)$I_1^{S*} > I_1^{D*}$ and $I_2^{S*} < I_2^{D*}$; and (iv) $I_1^{S*} > I_1^{D*}$ and $I_2^{S*} > I_2^{D*}$.
Some readers might be interested in what variables determines the three conditions, (ii), (iii) and (iv), at an investment equilibrium. In order to discuss this issue, I perform comparative statics with respect to the parameters used in the model.

**Proof**

Totally differentiating equation (33), we have:

$$\begin{bmatrix} dI_1^* \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \frac{\partial \Psi_2(I_1, I_2)}{\partial I_2} - \frac{\partial \Psi_2(I_1, I_2)}{\partial I_1} \\ -\frac{\partial \Psi_2(I_1, I_2)}{\partial I_1} - \frac{\partial \Psi_2(I_1, I_2)}{\partial I_1} \end{bmatrix} \begin{bmatrix} -ds \\ -ds \end{bmatrix},$$

where $\Delta = \frac{\partial \Psi_2(I_1, I_2)}{\partial I_1} \frac{\partial \Psi_2(I_1, I_2)}{\partial I_2} - \frac{\partial \Psi_2(I_1, I_2)}{\partial I_1} \frac{\partial \Psi_2(I_1, I_2)}{\partial I_1}$. Note that $\Delta > 0$ as already discussed in Section 4.2.

Consequently, we have:

$$\frac{dI_1}{ds} > 0 \text{ if and only if } \frac{\partial \Psi_1^S}{\partial I_1} > \frac{\partial \Psi_1^S}{\partial I_2},$$

$$\frac{dI_2}{ds} > 0 \text{ if and only if } \frac{\partial \Psi_2^S}{\partial I_2} - \frac{\partial \Psi_2^S}{\partial I_1} > 0.\quad(36)$$

This means that shape of social welfare function determines how investment equilibrium changes in response to changes in the central government’s subsidy ratio $s$. This also implies that careful parameterization of the economy is very important to determine whether we have (ii)$I_1^{S*} < I_1^{D*}$ and $I_2^{S*} > I_2^{D*}$; (iii)$I_1^{S*} > I_1^{D*}$ and $I_2^{S*} < I_2^{D*}$; or (iv) $I_1^{S*} > I_1^{D*}$ and $I_2^{S*} > I_2^{D*}$.

In addition, we also arrived at the following conclusion: under decentralization with the central government’s subsidy if the two regions are homogeneous, that is, $a_1 = a_2$, then both regions invests more in public infrastructure ($I_i$) than under complete fiscal centralization regime. If the two regions are heterogeneous, that is, $a_1 \neq a_2$, then at least one region or both regions invest more in public infrastructure than under complete fiscal centralization regime. This follows by combining Proposition 1 and Proposition 3.
(2) Numerical Simulation

In order to see how the model works, I conducted an analogous numerical simulation as already presented in Section 4.3.

One such diagram is illustrated in Figure 8. The four bold solid lines indicate the best-response correspondence curves both under complete fiscal decentralization and under fiscal decentralization with the central government’s subsidies: the light curves indicate $\psi_1(I_2)$ and the dark curves indicates $\psi_2(I_1)$, respectively. Also, the inner curves indicates the best-response curves corresponding to complete fiscal decentralization, and the outer curves indicates the curves corresponding to decentralization with the central government’s subsidies. This is because $\psi_1(I_1, I_2) > \psi_2(I_1, I_2)$ always holds for the same $I_1$ and $I_2$; $\Psi^S(I_1, I_2)$ is decreasing in $I_1$; and consequently $\psi_1^D(I_1)$ must locate outside $\psi_2^D(I_1)$.

In the case illustrated in Figure 8, the investment equilibrium is symmetric with each economy having an identical region-specific parameter, and providing identical size of public infrastructure, respectively. (Values of parameters underlying the figures are given in the Table 2.) As a result of numerical analysis, public infrastructure provision of the two local governments under the complete decentralization regime, is totally different from both under complete fiscal centralization regime and under complete fiscal decentralization regime, as Proposition 1 and Proposition 3 suggest: Public investment under decentralization with the central’s subsidies is more than both under complete centralization and both complete decentralization, if the two regions are identical.

The change in the region-specific parameter $a_i$ affects the investment equilibrium under decentralization with the central’s subsidies. To see this, Figure 9 is constructed with a higher level of the region-specific parameter. All other parameters remain the same as Figure 8.

Comparing the best-response correspondence curves in Figure 9 with the curves in Figure 8, the best-response correspondence curve of the region-2 $\psi_2(I_1)$ shifts to the lower position while the curve $\psi_1(I_2)$ shifts to the right, as Lemma 1 suggests. As a result, the investment equilibrium given by the intersection of the two curves shifts to the lower right. In this case, we still have $I_1^{S*} > I_1^{D*} > I_2^{C*} \forall i$.

7. Conclusion

What I have shown in this paper is that a simple model with labor mobility across regions suggests the opposite interpretation of the conventional understanding of decentralization: complete fiscal decentralization and partial fiscal decentralization are not necessarily the best in terms of economic efficiency in the sense that at least one local government or both local governments invest more in public infrastructure than under complete fiscal centralization. Numerical simulation results suggest that careful parameterization of the economy is very important to determine whether decentralization leads to a “race to the bottom”. In the context of public infrastructure management, a “race to the bottom” means that the competition of the two local governments would lead to undermine social welfare of regions and the whole country by overinvestment in public infrastructure, and may result in an investment equilibrium far away from the first best state. In context of globalized economy and international trade, Nancy H. Chau and Ravi Kanbur(1999) showed that it is possible that South-South competition to export leads to a “race to the bottom” under certain circumstances. These imply that competition between governments is not necessarily all-around good thing and that the involvement of the central government is still needed throughout the entire process of decision.

### Table 2: Values of Parameters and Investment Equilibrium

<table>
<thead>
<tr>
<th>Case</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$I_1^{C*}$</th>
<th>$I_2^{C*}$</th>
<th>$I_1^{D*}$</th>
<th>$I_2^{D*}$</th>
<th>$I_1^{S*}$</th>
<th>$I_2^{S*}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>14.0</td>
<td>1.0</td>
<td>3.30</td>
<td>3.30</td>
<td>4.51</td>
<td>4.51</td>
<td>7.61</td>
<td>7.61</td>
</tr>
<tr>
<td>Case 2</td>
<td>24.2</td>
<td>1.0</td>
<td>6.28</td>
<td>1.34</td>
<td>7.86</td>
<td>2.90</td>
<td>15.02</td>
<td>2.03</td>
</tr>
</tbody>
</table>

$L = 3.0, b = 4.0, \delta = 0.4, s = 0.15$
making in public infrastructure provision.

There are obviously many ways in which the analysis could be extended. I would, however, emphasize four directions in particular.

First, the welfare of workers is in fact the same across the regions in my model since I did not take into account of the effects of public good $P_i$ since $P_i$ is cancelled out in my model. In other words, if workers could “vote with their feet” and still enjoy perfect mobility as Charles M. Tiebout\(^7\) considered, the uneven income consequences of decentralization may be lessened. This implies that it is important to incorporate the diversified preferences on public good in future research.

Second, the model excludes capital mobility: indeed, it has no capital. Not only the labor movement but also capital movement has a significant impact on regional economies. Thus, a natural step would be to add capital movement.

Finally, it is obviously important to discipline this analysis with some real numbers. Since the analysis indicates some different images of the general understanding of decentralization, it is crucial to do at least rough empirical work to explore the unbiased image of decentralization and check validity of the model.

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This paper represents my own personal views, and is not to be associated in any way with my official position in the government.

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