EXACT SOLUTION OF THE DYNAMIC VEHICLE ROUTING AND SCHEDULING PROBLEM WITH SOFT TIME WINDOWS

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1. Introduction

Industrial growth and expanding employment opportunities have led to the urban-oriented economic development in many countries. Demand of transportation, both in terms of passengers as well as for freight is also increasing in and around these big urban conurbations. A high proportion of total goods movement occurs within cities[1], and most of this movement is based on road transport. Traffic congestion, noise, vibrations, generation of NOx, SPM, CO_2 and other environmental problems, crashes and loading and unloading on the street side are typical problems caused by the road-based freight transport in urban areas.

Such freight movement related problems has magnified the need for research in the field of city logistics. The Vehicle Routing and scheduling Problem with Time Windows (VRPTW) can be used as a tool for evaluating many city logistics schemes. For example, the VRPTW could be used in the analysis of cooperative delivery systems and ideal location of logistics terminals, which belong to infrastructure planning and management problems in city logistics. Depending on the nature of the time windows, the VRPTW is further expanded to the Vehicle Routing and scheduling Problem with Hard Time Windows (VRPHTW) and the Vehicle Routing and scheduling Problem with Soft Time Windows (VRPSTW).

The classical VRPTW is defined for static input values such as fixed customer locations and static travel time. However, the traffic conditions in urban areas change with time due to varying congestion levels and incidents resulting in varying travel time on the infrastructure links. With the introduction of the Intelligent Transportation Systems (ITS) such as the Vehicle Information and Communication System (VICS) in Japan, it is possible to collect and store such dynamic travel times on a link. As far as the logistics is concerned, changes in the travel time may affect the distribution or pick-up routes of the delivery vehicles resulting in unexpected long delays if the routing is fixed and based on a static value of travel time (such as the average travel time). These traffic conditions in urban areas results in the Dynamic Vehicle Routing and scheduling Problem with Soft Time Windows (D-VRPSTW) in the field of city logistics. There exists very few heuristics approaches to solve this problem in the literature (see §2. for details); but even these heuristics techniques do not guarantee to identify the exact solution or even state the gap between the exact and the heuristically optimized solution.

Recently column generation techniques have been used efficiently for the VRPSTW[2]. This is desirable to develop an exact solution approach for the D-VRPSTW as well to exploit the rich data from ITS source and to build efficient physical distribution system. Therefore, this paper presents a column generation based exact solution technique for the D-VRPSTW, next sections would give some related previous research, general framework and the methodology of the proposed exact approach.

2. Literature Review

Many researchers have used heuristic techniques for the soft time windows environment with the idea to reduce the number of vehicles or overall solution cost[3]. Balakrishnan[4] described three simple heuristics for the VRPSTW based on the nearest neighbour, Clarke-Wright savings and space-time rules. A detailed overview of the VRPSTW and its solution techniques can be found in Taniguchi *et al.*, [1]. Recently, Qureshi *et al.*, [2] presented an column generation based exact approach for the static version of the VRPSTW; this research extends the same approach to the Dynamic Vehicle Routing and scheduling Problem with Time Windows (D-VRPSTW) case.

The Dantzig-Wolfe decomposition of the VRPTW results in the set partitioning master problem and an Elementary Shortest Path Problem with Resource Constraints (ESPPRC) as its subproblem[5]. A number of efficient exact algorithms are available based on the column generation schemes for the VRPHTW[6), 7)] as well as for the VRPSTW (as mentioned earlier). However, all these approaches are based on the static information and lack the dynamic nature of the real life logistics operations. A routing system, in which complete or a part of input information (such as number and location of customers or travel time on arcs) is not available to the decision maker at the start but it is revealed during the scheduling horizon (day of operation), and in which the decision maker *reacts* to this new information by evoking some sort of *re-optimization* mechanism is defined as the D-VRPSTW[8]. There are mainly two sources of dynamism, viz. the

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dynamic customers case, in which new service request (new customer) are called in during the day of operation, and the dynamic travel time that deals with the unexpected changes of the travel time on the network links. There exists an abundance of research on the dynamic customers case of the D-VRPTW (for example see, Larsen[9]; Chen and Xu[10]; Branchini *et al.*[11]); whereas the dynamic travel time case of the D-VRPTW has received a limited attention. In fact only two references could be found dealing with the dynamic travel times, viz. Taniguchi and Shimamoto[12], and Flieschmann *et al.*[13]; both considering the Dynamic-Vehicle Routing and scheduling Problem with Soft Time Windows (D-VRPSTW). Taniguchi and Shimamoto[12] have used a macro-simulation scheme to generate the dynamic travel time data for a theoretical test network, whereas, Flieschmann *et al.*[13] have used the data from an ITS implemented in Berlin, Germany, named as LISB, which provides the travel time data on links for every 5 minutes slot. However, both of these research works have adopted heuristics approaches to solve the D-VRPSTW. The heuristic techniques are sometimes faster and easily implemented than exact solutions, yet they do not guarantee to identify the exact solution or state how close to the exact solution a particular feasible solution is[14]. Therefore, this research proposes a column generation based exact solution approach for the D-VRPSTW with dynamic travel times to fill the existing research gap. The exact approach can be used for small to medium instances (up to 50 customers) as well as for the evaluation and calibration of the heuristics approaches.

3. General Framework

The VRPSTW is defined on a directed graph $G = (V, A)$. The vertex set V includes the depot vertex 0 and the set of customers $C = \{1, 2, . . ., n\}$. The set K represents the set of identical vehicles with capacity q stationed at the depot. The arc set A consists of all feasible arcs (i, j), $i, j \in V$. A cost c_{ij} and a time t_{ij} is associated with each arc $(i, j) \in A$. The time t_{ij} includes the travel time on arc (i, j) and service time at vertex i, while a fixed vehicle utilization cost is added to all outgoing arcs from the depot. With every vertex of V associates a demand d_i where $d_0 = 0$, and a time window $[a_i, b_i]$, which represents the earliest and the latest desired service start times.

This study incorporates the soft time windows constraint by extending the latest service start time b_i to b_i' as shown in Figure 1. Taking c_l as the unit late arrival penalty cost, the arrival time-dependent cost is formulated as eq. (1).

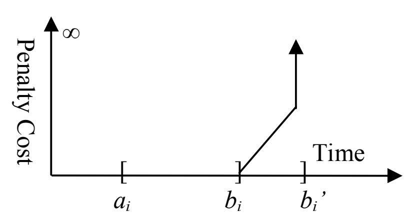


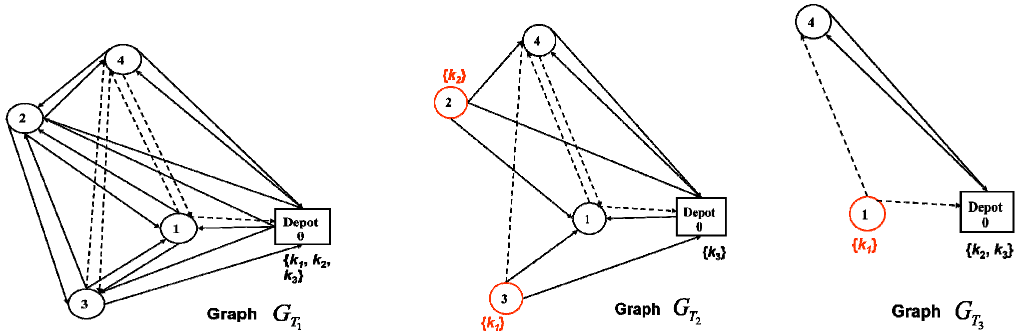
Figure 1 Penalty cost function for the VRPSTW

$$c'_{ijk} = \begin{cases} c_{ij}, \text{if } s_{jk} \le b_j \\ c_{ij} + c_l(s_{jk} - b_j), \text{if } s_{jk} > b_j \end{cases} \tag{1}$$

The proposed Dynamic Vehicle Routing and scheduling Problem with Soft Time Windows (D-VRPSTW), only considers the travel time uncertainty, therefore, all the remaining information such as customers' locations and demands are assumed to be known and fixed. The soft time windows enable delivery after the close of customer specified time windows with an associated late arrival penalty, while the waiting is allowed without any penalty (as shown in Figure 1). The D-VRPSTW would be modelled using the *rolling horizon* scheme, in which the complete scheduling horizon is divided into various time slots, each representing a time-based scenario. Thus initially, it can be defined on a complete Graph G_{T_1} for the first time slot (T_1), which consists of all customers with vehicles (k_i) stationed at the central depot (as shown in Figure 2(a)). The routes for the time slot T_1, would be planned as per the average travel times. No *divergence* is allowed once a vehicle leaves to visit a customer, i.e. the first customer on the route of an en-route vehicle is fixed. The time slots are marked with *vehicle-based events*, which means a new time slot is initiated as soon as any of the vehicle reaches the first customer on its route.

With no diversion allowed, the locations of all vehicles are forecasted and the Graph is updated (G_{T_2} (Figure 2(b))) showing vehicle locations along with all customers except those which are either serviced or are the first customers of an en-route vehicle. The arcs in G_{T_2} contain the updated travel times, therefore, routes planned in T_2 would be based on these updated travel time values. It may be noted that, nodes other than depot, which contain a vehicle resource, only have outbound links. A vehicle can abandon its remaining planned route and head back to the depot as well.

This procedures goes on till the last customer is serviced (as shown by G_{T_3} in Figure 2(c)). As the customers assigned to a particular vehicle may get changed at any of the route revision epoch (at the change of time slot), the proposed D-VRPSTW is suitable to either model a delivery system of single commodity such as heating oil, gasoline etc., or it can model any pick-up service.



(a) (b) (c)

Figure 2. Representation of the dynamism of D-VRPSTW in rolling horizon scheme

4. Methodology

The D-VRPSTW would be solved by extending a column generation approach developed for the static VRPSTW by authors[2]. Precisely, using the Dantzig-Wolfe decomposition, the D-VRPSTW would be decomposed into a set partitioning linear master problem, and in an Elementary Shortest Path Problem with Resource Constraint and Late Arrival Penalties (ESPPRCLAP)[15]. However, as the G_{T_2} (Figure 1(b)) shows, the proposed framework leads to a Multi depot-Dynamic Vehicle Routing and scheduling Problem with Soft Time Windows (MD-VRPSTW) as each node with a vehicle resource would be considered as a virtual depot. Therefore, the proposed column generation based exact approach must incorporate the multi depot aspect as well. It can be modelled by formulating a separate ESPPRCLAP for each of the virtual depot. At each column generation iteration, the set partitioning master problem would receive columns (routes/paths of negative reduced costs) from all subproblems and then it would optimize the complete problem by selecting the best set of routes covering the demands of all the customers at hand. The dual variables' values (*prices*) would be obtained as a by-product, which will be used to define new reduced cost matrices for each of the subproblems to generated new promising paths/columns, and the whole process is repeated till the subproblems fail to provide a negative reduced cost column. At this stage if the solution of the master problem LP is not integer a branch and bound tree would be explored. Therefore, at every route revision epoch, the column generation algorithm would be embedded in a branch and price algorithm. In order to track and keep the total number of vehicles in the system, a new branching scheme would be needed.

5. Results and Discussions

Changing traffic conditions and incidents cause the travel time variation in urban areas. If the operations of a logistics company is planned, by ignoring this reality, it may affect the distribution or pick-up routes resulting in unexpected long delays. On the contrary, dynamic vehicle routing and scheduling can result in better route selection, avoiding congested roads due to incidents or any other reason. This would help logistics firms in reducing their distribution/prick-up costs as well as in improving their reliability because the delays and related late arrival penalties would also be minimized. Furthermore, the amount and distribution of the environmental emissions would also be changed if the delivery/pick-up vehicles would follow updated routes, diverting from pre-defined routes containing congested roads.

Results obtained on Solomon's benchmark instances[16] would be included in the final submission and would be presented at the conference.

References:

1) Taniguchi, E., *et al.*: City Logistics; Network Modeling and Intelligent Transport Systems, Pergamon, Oxford, 2001.
2) Qureshi, A.G., *et al.*: An exact solution approach for vehicle routing and scheduling problems with soft time windows, Transportation Research Part E, Vol. 45, pp. 960-977, 2009.
3) Taniguchi, E., & Thompson, R.G.: Modeling city logistics, Transportation Research Records, Vol. 1790, pp. 45-51. 2002.
4) Balakrishnan, N.: Simple heuristics for the vehicle routing problem with soft time windows, Journal of the Operational Research Society, Vol. 44, pp. 279-287, 1993.
5) Feillet, D., *et al.*: An exact algorithm for the elementary shortest path problem with resource constraints: Application to some vehicle routing problems, Networks, pp. 216-229, 2004.
6) Desrochers, M., *et al.*: A new optimization algorithm for the vehicle routing problem with time windows, Operations Research, Vol. 40(2), pp. 342-354, 1992.
7) Irnich, S., & Desaulniers, G.: Shortest path problems with resource constraints, in G. Desaulniers, Desrosiers, J., & Solomon M.M., eds., Column Generation, Springer, New York, pp. 33-65, 2005.

8) Psaraftis, H.N.: Dynamic vehicle routing: Status and prospects, Annals of Operational Research, Vol. 61, pp. 143-164, 1995.
9) Larsen, A.: The Dynamic Vehicle Routing Problem. PhD. Thesis, LYNGBY, IMM-PHD-2000-73, IMM, 2001.
10) Chen, Z., & Xu, H.: Dynamic column generation for dynamic vehicle routing with time windows, Transportation Science Vol. 40, pp. 74-88, 2006.
11) Branchini, R. M., *et al*.: Adaptive granular local search heuristics for a dynamic vehicle routing problem, Computers & Operations Research Vol. 36, pp. 2955-2968, 2009.
12) Taniguchi, E., & Shimamoto, H.: Intelligent transportation system based dynamic vehicle routing and scheduling with variable travel time, Transportation Research Part C, Vol. 12, pp. 235-250, 2004.
13) Fleischmann, B., *et al*.: Dynamic vehicle routing based on online traffic information, Transportation Science, Vol. 38, pp. 420-433, 2004.
14) Thompson, R.G., & Van Duin, J.H.R.: Vehicle routing and scheduling", in Taniguchi, E., and Thompson, R.G., eds., Innovations in Freight Transportation, pp. 47-63, WIT Press, Southampton, 2003.
15) Qureshi, A.G., *et al*.: Elementary shortest path problem with resource constraints and time dependent late arrival penalties", JSCE Journal Division D (Doboku Gakkai Ronbunshuu D), Vol. 63, pp. 579-590, 2007.
16) Solomon, M. M.: Algorithms for the vehicle routing and scheduling problem with time windows constraints, Operations Research, Vol. 35(2), pp. 254-265, 1987.