A POISSON HIDDEN MARKOV MODEL FOR DETERIORATION PREDICTION OF ROAD ASSET SYSTEM

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1. Introduction

Shortages in annual budget allocation for road asset management system often limit managers in making decision to implement a full scale visual inspection. Hence, lacking of monitoring data is quite a prevailing risk in infrastructure asset management practices. In many cases, road authorities have only information on numbers of potholes occurrences, while information on other pavement distress is absent. However, managers need to make decision not only with potholes repair but also with overall surface treatment. In this paper, we introduce Poisson hidden Markov model to solve such limitations. The model discusses the probabilistic and functional relationship between Markov transition probabilities among condition states and the Poisson process of each states, which is linked with the occurrence of potholes. Markov transition probability is considered as hidden information. We use the estimated Poisson parameters to trigger the Markov transition probability.

It has been widely known that Markov chain model has its advantage that it generalizes the deterioration process of pavement management system (PMS) in the transition pattern among condition states. Condition states of roads are performance indexes, which take into account various important performance measurement values. For example, condition states like Pavement Condition Index (PCI) in America or Management Criteria Index (MCI) in Japan are weighted values in discrete numbers of pavement distress (cracking, rut, roughness, etc) [1)2)]. Furthermore, Markov models can be used to address uncertainty under the absence of historical data as the probability of observing future state depends only on the probability of observed condition states at the present. To date, the development both on academic research and practical application with Markov models for PMS can be seen with many competitive PMS software packages.

However, there is a fact that, management and maintenance of PMS frequently face local matters that are not fully discussed in Markov models. Since condition states of pavement are weighted values of several important pavement distresses. Other pavement distress might not be considered together with condition states. One of the prominent pavement distress, which is separately studied beside aggregate condition states, is the occurrence of potholes. Engineers and researcher believe that pothole occurrence is, in some extend, a local matter, and thus requires different management approach in comparison with Markov chain models. Keeping abreast of this problem, our paper is thus to consider pothole management and application of Markov chain model, with significant assumption that condition state transition probabilities are in close relation with numbers of potholes appearing on the road surface.

Further regarding the management of pothole in PMS, up to present, most of research has applied the stochastic model with Poisson distribution [3)]. This type of model is called "Poisson hazard model". In the models, Poisson parameters are estimated based on observed numbers of pothole, which are recorded over a long monitoring and inspection period. The existing approach with employing Poisson hazard model in PMS still remains with ad-hoc and limitations as Poisson process itself is regarded as a memoryless process and neglecting the conditional dependency on Markov transition probability [4)].

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* Keywords: Pothole management, Markov Hazard Model, Poisson hidden Markov model, Bayesian Statistics, Markov Chain Monte Carlo

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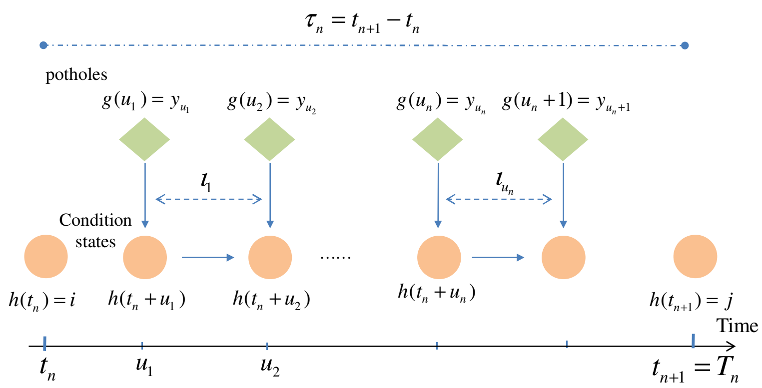

Figure 1: Conditional dependence of Poisson parameters on Markov process.

There exists a high probability that the occurrence of potholes and the progress of transition pattern between condition states are in strong correlation to each other. If it is of the true assumption, it certainly opens up an opportunity for us to formulate a new type of model for pavement management under constraint of incomplete monitoring data. For example, in many situations, our monitoring data comprises of only records on pothole over the years, while information on other pavement distresses are in poor, insufficient, or missing.

In this paper, we propose an analytical model called ``Poisson hidden Markov model", which has not been discussed so far in the literature of infrastructure management. The Poisson process is attached with each condition state in Markov model. Our assumption of the model is presented in section 2. Section 3 details model's formulation. Estimation method is discussed in section 4. The later sections show results of empirical study and conclusion remark.

2. Pothole occurrence and deterioration pattern

Fig. 1 describes the natural deterioration process of a pavement section. As we can see from the figure. After the opening of a new road, condition states $i(i = 1, \cdots I)$ get worse throughout the operation time, so does with numbers of potholes. In this respect, it is possible to state that numbers of potholes are more or less conditional dependent on the transition pattern of condition state i. Correlation between potholes occurence and transition of pavement distresses have been proved and documented.

In actual management practices, there are many cases that visual inspections record only pavement distress and convert its values into discrete condition states. However, in the same period, numbers of pothole are missing. In another case, we have a rich numbers of potholes, but not having any information on levels of condition states. In another word, information on the transition of condition states is partially observed or unobserved.

Our study focuses on the case that monitoring data of pavement distress is missing. Under that circumstance, simulation with multi-state Markov hazard model is not possible. What in rich quantity of data-set are numbers of potholes occurred in road sections, which were recorded over a period of time. Given the assumption that occurrence of pothole follows Poisson process. The underlying transition probability, which probabilistically determines the arrival rate of potholes occurrence, is followed with Markov process. The Markov process is thus considered as hidden property of deterioration process. An illustration of the Poisson hidden Markov model can be also referred to Fig. 1.

In Fig. 1, transition of condition states $h(t_n = i)$ and $h(t_{n+1} = j)$, $(i, j = 1, \cdots, I)$ of a road section at respective time t_n and t_{n+1} follow Markov chain process with transition probability π_{ij}. This transition probability is observable as the result of frequent visual inspection. However, within the period $\tau_n = t_{n+1} - t_n$, condition state of a road section could not be recorded. Within the period τ_n, it is supposed that condition state should be in the range $[i, j]$ if there has been no M&R action implemented. In short, information with underlying Markov process in the period τ_n is not observable. Instead, we can observe numbers of potholes $g(u_n) = y_{u_n}, (u_n = 0, \cdots, T_n - 1)$. The potholes occurrence is believed to follow Poisson process.

Following sections describe Markov transition probability π_{ij} and the Poisson hidden Markov model.

3. Model formulation

(1) Markov transition probability

In the paper of Tsuda et al (2005) [5], the author proposed a profound research methodology to illustrate the deterioration process by means of Markov transition probability among discrete condition states. In this paper, we employ a general mathematical form for Markov transition probability π_{ij} from his paper. Details of calculation and estimation should be referred to original paper.

$$\pi^{ij}(z) = \sum_{k=i}^{j} \prod_{m=i}^{k-1} \frac{\theta_m}{\theta_m - \theta_k} \prod_{m=k}^{j-1} \frac{\theta_m}{\theta_{m+1} - \theta_k} exp(-\theta_k z), \tag{1}$$

Where θ is hazard rate for respective condition state i and z is time interval between two inspections. General model is given in the paper of Tsuda et al (2005) [5] . The methodology given in his paper is considered as a backbone and profound research in stochastic modeling and application on the field of infrastructure asset management.

(2) Poisson hidden Markov process

Information regarding the condition state of a road section can be determined by evaluating inspection data observed at respective time $t_n(\mathrm{n} = 1,2,\cdots)$. However, as shown in Fig. 1, it is certain that condition state of a road section at time $u_n, (u_n = 1,\cdots, T_n - 1)$ within the local period τ_n is unidentified. In another word, condition state $h(t_n + u_n) = l_{u_n}$ is hidden.

We pay attention on the local period $\tau_n = [t_n, t_{n+1})$, where having the observed information $h(t_n) = i, h(t_{n+1}) = j$ at $t_n, \text{and } t_{n+1}$. Within this period, M&R has not been carried out. The only observable information is numbers of potholes $g(u_n) = y_{u_n}$ at respective internal time $(u_n = 0,\cdots, T_n - 1)$. The occurrence of pothole is assumed as a random process with Poisson distribution, in which, arrival rate $\mu(l_{u_n}) > 0$ is defined in following equation.

$$\mu(l_{u_n}) = z\alpha^{\ l_{u_n}}, \tag{2}$$

Where, $z = (z_1,\cdots, z_P)$ is vector of explanatory variable and $\alpha^{\ l_{u_n}} = (\alpha_1^{l_{u_n}},\cdots,\alpha_P^{l_{u_n}})'$ is vector of unknown parameters, which can be written as $\alpha = (\alpha^{\ 1},\cdots,\alpha^{\ I-1})$. The sign $'$ indicates transformation of vector and P shows the number of explanatory variables. It is important to note that the arrival rate $\mu(l_{u_n})$ is averagely defined only in the period $[u_n, u_n + 1)$. The condition state l_{u_n} is assumed to be constant in the period ι_{u_n}. This assumption is the crux of our model as it allows applying Poisson process appropriately. Reason is due to the fact that Poisson arrival rate depends greatly on the length of observation time. With this assumption, we can describe the conditional probability $\pi_{u_n}(y_{u_n}|l_{u_n})$ to which, the numbers of potholes y_{u_n} occurring in the period ι_{u_n} is defined as follows:

$$\pi_{u_n}(y_{u_n}|l_{u_n}) = \ Prob\ [g(u_n) = y_{u_n}|h(t_n + u_n) = l_{u_n}] = exp\{-\mu(l_{u_n})\}\frac{\{\mu(l_{u_n})\}^{y_{u_n}}}{y_{u_n}!}, \tag{3}$$

where y_{u_n} is numbers of potholes accumulated till time u_n . Equation (3) is also satisfied with condition $\sum_{y_n=0}^{\infty} \pi_n(y_{u_n}|l_{u_n}) = 1$. In addition, we can defined the conditional state probability $\rho_{u_n}(l_{u_n}|i)$, which represents the event that condition state becomes l_{u_n} at time $\mathrm{u_n}$ given $h(t_n) = i$ at t_n.

$$\rho_{u_n}(l_{u_n}|i) = \ Prob\ \{h(t_n + u_n) = l_{u_n}|h(t_n) = i\} = p^{il_{u_n}}(u_n), \tag{4}$$

We further define the probability $\tilde{\pi}_{u_n}(y_{u_n})$, to which y_{u_n} (numbers of potholes) appeared in the period ι_{u_n}.

$$\tilde{\pi}_{u_n}(y_{u_n}) = \sum_{l_{u_n}=i}^{j} \pi_{u_n}(y_{u_n}|l_{u_n})\rho(l_{u_n}|i) \tag{5}$$

The conditional state probability $h(t_n) = i$ is expressed in observed value vector $\bar{y}_n = (\bar{y}_0,\cdots,\bar{y}_{T_n})$ within the interval τ_n. As a result, probabilistically, we can propose likelihood $\mathcal{L}(\xi_n, \theta\ )$ to represent the occurrence of all cummulative events. Vector of observed values is $\bar{\xi}_n = \{\bar{y}_n, \bar{\iota}, \bar{j}\}$ and $\theta = (\ \alpha\ , \beta\ )$ is vector of unknown parameters, the sign $^{-}$ denotes the measureable information.

The likelihood function $\mathcal{L}(\bar{\xi}_n, \theta\ )$ will be estimated under the conditions: 1) condition state $h(t_n) = i$ is observed at t_n, 2) observed values vector $\bar{y}_n$, representing numbers of pothole occurrence, are measured in respective time $u_n(u_n = 0,\cdots, T_n - 1)$ within duration $[t_n, t_{n+1})$, 3) condition state $h(t_{n+1} = j)$ is observed at t_{n+1}. Following equations recurrently describe the likelihood function $\mathcal{L}(\bar{\xi}_n,\theta)$.

$$\mathcal{L}(\bar{\xi}_n,\theta) = \pi(\bar{y}_0|i)\sum_{l_1=i}^{j} p^{il_1}\ell_1(l_1) \tag{6-a}$$

$$\ell_{u_n}(l_{u_n}) = \pi(\bar{y}_{u_n}|l_{u_n})\sum_{l_{u_n}+1=l_{u_n}}^{j} p^{l_{u_n}l_{u_n}+1}\ell_{u_n}(l_{u_n}+1) \quad (1 \le u_n \le T_n - 1) \tag{6-b}$$

$$\ell_{T_n-1}(l_{T_n-1}) = \pi(\bar{y}_{T_n-1}|l_{T_n-1})p^{l_{T_n-1}j} \tag{6-c}$$

4. Estimation Methodology

(1) Monitoring data

In general, the entire monitoring data on target road system consists of K numbers of road sections. Healthy status of individual road section k will be inspected frequently to reveal its condition state and number of pothole accumulated. It is assumed that at time t_n, condition state $h(t_n^k)$ is detected on road section k. If the observed condition state is over a pre-determined standard limit state for M&R, an immediate M&R should be applied to renew its performance. In this respect, we can introduce a new starting point s_0^k right after each $M\&R$ at $(t^k = 0,\cdots)$, for each road section $k(k = 1,\cdots,K)$. In short, it is understood that M&R is executed in $t_n^k(n = 1,\cdots,N^k)$, with N^k representing the sequent numbers of inspections. In addition, monitoring data consists also the numbers of potholes appeared in each period $\tau_n^k = [t_n^k, t_{n+1}^k)(n = 0,\cdots,N^k - 1)$ at $u_n^k(u_n^k = 0,\cdots,T_n^k - 1)$. The observed value vector of potholes is $\bar{y}_n^k = (\bar{y}_0,\cdots,\bar{y}_{T_n^k-1})$. Briefly, data vector $\bar{\xi}_n^k = \{\bar{y}_n^k, \bar{h}(t_n^k), \bar{h}(t_{n+1}^k)\}$ is defined in each period τ_n^k of section k. The entire data set can be denoted as $\bar{\Xi} = \{\bar{\xi}_n^k : n = 0,\cdots,N^k, k = 1,\cdots,K\}$. Likelihood $\mathcal{L}(\bar{\xi}_n^k,\theta)$, concerning measurable data $\bar{\xi}_n^k$, has been defined in equations (6-a) - (6-c). Hence, the joint probability (or likelihood), taking account of the entire data set Ξ, can be ultimately formulated as follows:

$$\mathcal{L}(\bar{\Xi},\theta) = \prod_{k=1}^{K}\prod_{n=1}^{N}\mathcal{L}(\bar{\xi}_n^k,\theta). \tag{7}$$

To this point, it is suffice to say that solving the crux of Poisson hidden Markov model returns in the problem of estimating parameter vector $\hat{\theta}$ that maximizes the likelihood function in equation (7).

Likelihood function in equations (6-a) - (6-c) of Poisson hidden Markov model is regarded as a nonlinear polynomial function, with hyper-parameters (high-order parameters). It has been proved that solving a set of likelihood function like (6-a) - (6-c) on real data by using conventional likelihood maximization method will encounter troublesome, complexity, and costly efforts. Reasons are due to the fact that, numerical solution with likelihood maximization approach on nonlinear polynomial function has tendency to ends up with local optimal values of parameters and repeated truncates. Efforts in coping with these problems can be referred to the application of Bayesian inference statistic [6)]. In this paper, we propose Bayesian estimation and MCMC method as a handy approach to obtain optimal values of model's parameters.

(2) Complete likelihood function

As earlier mention, in the period $[t_n^k, t_{n+1}^k)$, condition state $\bar{\iota}_n^k, \bar{\jmath}_{n+1}^k$ are measurable. The only hidden information is condition state at time $t_n^k + u_n$ (see Fig. 1). To disclose the hidden condition state, we describe it in the form of hidden variable vector $\boldsymbol{m}_n^k = (m_1^k,\cdots,m_{T_n^k-1}^k)$, which must satisfy following conditions in the situation that no M&R has been carried out.

$$\bar{\iota}_n^k \le m_1^k \le \cdots \le m_{T_n^k-1}^k \le \bar{\jmath}_{n+1}^k \tag{8}$$

True information on the values of $\boldsymbol{m}_n^k$ cannot visually measured. However, for the convenience of estimation, we assume it to have a fixed value as $\widetilde{\boldsymbol{m}}_n^k = (\widetilde{m}_1^k,\cdots,\widetilde{m}_{T_n-1}^k)$. In addition, following dummy variable is introduced to supplement the satisfaction of hidden condition state.

$$\delta_{s_{u_n}}^k = \begin{cases} 1 & \widetilde{m}_{u_n}^k = s_{u_n}^k \\ 0 & \widetilde{m}_{u_n}^k \ne s_{u_n}^k \end{cases} \quad (s_{u_n}^k = \bar{\iota}_n^k,\cdots,\bar{\jmath}_n^k; u_n = 1,\cdots,T_n - 1) \tag{9}$$

Suffice it to say that if visual observation disclose $\widetilde{\boldsymbol{m}}_n^k$ as true value, and according to Dempster et al (1997) [7)], we can rewrite likelihood function in (6-a) - (6-c) as follows:

$$\overline{\mathcal{L}}_n^k(\tilde{\boldsymbol{m}}_n^k, \bar{\xi}_n^k, \theta) = \prod_{u_n=0}^{T_n-1} \sum_{s_{u_n}^k=\bar{\imath}_n^k}^{\bar{J}_n^k} \pi_{u_n^k}(\bar{y}_{u_n}|s_{u_n}^k)^{\delta_{s_{u_n}^k}} \{p^{s_{u_n}^k s_{u_n+1}^k}\}^{\delta_{s_{u_n}^k}} = \prod_{u_n=0}^{T_n-1} \pi^k(\bar{y}_{u_n}|\tilde{m}_{u_n}^k) p^{\tilde{m}_{u_n}^k \tilde{m}_{u_n+1}^k} \quad (10)$$

Equation (10) is considered as complete (or full) conditional posterior distribution [8]. with a finer explicit form than that in the likelihood equations (6-a) - (6-c). Nevertheless, a difficulty remains at this point is how to assign a realistic value for hidden variable $\boldsymbol{m}$ since it is unobservable.

It is noted that condition (8) is satisfied as long as there is no M&R in the period τ_n. In view of probability distribution, hidden variable $\boldsymbol{m}$ can be derived by applying the full conditional posterior distribution in Bayesian inference. In which, the prior probability distribution in Bayesian estimation is assumed as follows:

$$Prob\,\{m_{u_n}^k = m|\, \tilde{\boldsymbol{m}}_{-u_n}^k\} = \frac{\overline{\mathcal{L}}(\tilde{\boldsymbol{m}}_{-u_n}^k, \bar{\xi}_n^k, \theta)}{\sum_{m=\tilde{m}_{u_n-1}^k}^{\tilde{m}_{u_n+1}^k} \overline{\mathcal{L}}(\tilde{\boldsymbol{m}}_{-u_n}^{m,k}, \bar{\xi}_n^k, \theta)} = \frac{\pi^k(\bar{y}_{u_n}^k|m) p^{m\tilde{m}_{u_n+1}^k}}{\sum_{m=\tilde{m}_{u_n^k-1}}^{\tilde{m}_{u_n^k+1}} \pi^k(\bar{y}_{u_n}^k|m) p^{m\tilde{m}_{u_n+1}^k}}, \quad (11)$$

where $m_{u_n}^k = m(m \in \{\tilde{m}_{u_n-1}^k, \cdots, \tilde{m}_{u_n+1}^k\})$, $\tilde{\boldsymbol{m}}_{-u_n}^k = (\tilde{m}_1^k, \cdots, \tilde{m}_{u_n-1}^k, \tilde{m}_{u_n+1}^k, \cdots, \tilde{m}_{T_n}^k)$, and $\tilde{m}_{-u_n}^{m,k} = (\tilde{m}_1^k, \cdots, \tilde{m}_{u_n-1}^k, m, \tilde{m}_{u_n+1}^k, \cdots, \tilde{m}_{T_n}^k)$.

It is clear at this point that if the posterior probability distribution of hidden variable $m_{u_n}^k \in \{\tilde{m}_{u_n-1}^k, \cdots, \tilde{m}_{u_n+1}^k\}$ at time u_n^k is measurable, transition probability $\pi(\bar{y}_{u_n}^k|m)$ and probability distribution function $p^{m\tilde{m}_{u_n+1}^k}$ $(u_n = 0, \cdots, T_n - 1; n = 1, \cdots, N, k = 1, \cdots, K)$ can be ultimately estimated. It is also noted that the posterior probability distribution of hidden variable $m_{u_n}^k \in \{\tilde{m}_{u_n-1}^k, \cdots, \tilde{m}_{u_n+1}^k\}$ is conditionally depended on the observed value of $\tilde{\boldsymbol{m}}_{-u_n}^k$.

To solve the likelihood equation (10), it is required to estimate the value of hidden variable $\boldsymbol{m}$. As a result, the main task is to estimate the unknown parameters α and β, which are embedded in the transition probability functions. In fact, there is no possibility to seek for the posterior distribution of all hidden variables. Thus, MCMC simulation is recommended to use in randomly generating the hidden variable $\boldsymbol{m}$.

5. Empirical Study

We conduct an empirical study using a set of monitoring data on potholes in Japanese national road system. Data was recorded during the 3 years period from 2007 to 2009. Total numbers of road sections are 236. Each road section represents for an average sectional length of 100 meters. Fig. 2 displays the numbers of potholes in monthly basis. Though, we discuss the application of Bayesian and MCMC in previous section, due to limitation of time, this empirical study shows results of applying Maximum likelihood estimation approach for equation (6-a)-(6-c) with forward and backward algorithm [9]. Results of empirical study alternately display in Table 1, Table 2, and Figure 3. Outcomes of empirical study using Maximum likelihood are only for comparision and benchmarking with the expected results of applying Bayesian and MCMC methods, which will be scheduled to publish in the later version of this manuscript.

In table 1, condition state from 1 to 5 is defined by means of crack values. If crack value is less than 15%, condition state is set to equal to 1. Condition state 5 means that percentage of cracking is over 60% and it is considered as absorbing condition state. The Markov transition probability is given in Table 1. It is used to estimate the hazard function and draw the deterioration curve in Figure 2. By observing the deterioration curves, we can relatively understand the time expectancy of each condition states. For example, condition state 1 has a life span about 20 years. It is quite long in comparision with results of previous studies on Japanese research papers. However, the ending time of the entire life span is about 40 years, which seems a relevant results. The difference of results could be due to the assumption of condition states over the ranking values of crack. With this findings, it suggest us to further investigate in empirical study, eliminating bias in condition state assumption, and collecting sufficient enough numbers of monitoring data.

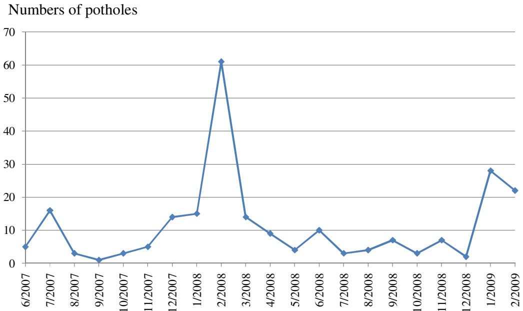

Figure 2: Time series data of pothole occurrence

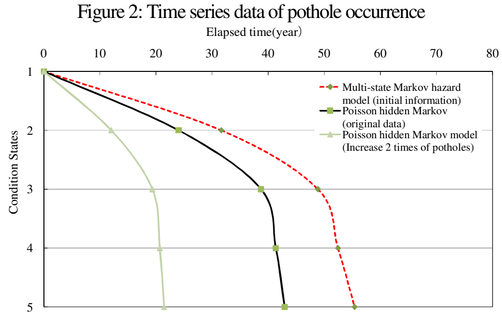

Figure 3: Comparision of deterioratoin curves.

Table 1: Markov transition probability of targeted road

Condition	Condition state				
State	1	2	3	4	5
1	0.8824	0.1176	0.0	0.0	0.0
2	0.0	0.8823	0.1177	0.0	0.0
3	0.0	0.0	0.9473	0.0525	0.0002
4	0.0	0.0	0.0	0.9998	0.0002
5	0.0	0.0	0.0	0.0	1.0

Table 2: Results of statistical test for Poisson hidden Markov model.

Month	Potholes	Month	Pothole	Month	Pothole
7-Jun	5 (6.21E-02)	8-Jan	15 (1.58E+00)	8-Aug	4 (-2.00E+00)
7-Jul	16 (1.85E+00)	8-Feb	61 (NA)	8-Sep	7 (-8.71E-01)
7-Aug	3 (-2.42E+00)	8-Mar	14 (1.30E+00)	8-Oct	3 (-2.42E+00)
7-Sep	1 (-3.41E+00)	8-Apr	9 (-2.00E-01)	8-Nov	7 (-8.71E-01)
7-Oct	3 (-2.42E+00)	8-May	4 (-2.00E+00)	8-Dec	2 (-3.01E+00)
7-Nov	5 (-1.60E+00)	8-Jun	10 (1.18E-01)	9-Jan	28 (4.75E-01)
7-Dec	14 (-1.30E+00)	8-Jul	3 (-2.42E+00)	9-Feb	22 (3.54E-02)
Loglikelihood			-144.4761		

6. Conclusion and Recommendation

This paper has presented a new approach to estimate the Markov transition probability of pavement under shortage of monitoring data using a Poisson hidden Markov hazard model. The underlying Markov transition probability of targeted road sections is considered as hidden information. In order to estimate the hidden Markov transition probability, we consider its condition states to link with occurrence of potholes, which is randomly followed Poisson process. Thus, given the observed numbers of potholes as monitoring data in a specific operation period, we can use Poisson hidden Markov model to determine the underlying Markov transition probability. Empirical study was conducted on a set of potholdes records using Maximum likelihood approach. Results are only for demonstration of how the model can be applied in the real world. For a sound and final results, we will conduct and present in the upcoming full version of this paper results of empirial study applying Bayesian estimation and MCMC method ing searching for global optimal values of model's parameters.

References

1) Shahin, M, Pavement Management for Airports, Roads, and Parking Lots, Springer, 2005.
2) Nam, L.T, Thao, N.D, Kaito, K, and Kobayashi, K. "A Benchmarking Approach to Pavement Management: Lessons from Vietnam", Infrastructure Planning Review, JSCE 26 (1), 1001-112.
3) Madanat, S, and Ibrahim, W.H.W, "Poisson Regression Models of Infrastructure Transition Probabilities. Journal of Transportation Engineering, ASCE 121, 267-272, 1995.
4) Tanaka, T, Nam, L.T, Kaito, K, and Kobayashi, K, "Probabilistic Analysis of Underground Pipelines for Optimal Renewal Time". Infrastructure Planning Review, JSCE 26 (1), 123-132.
5) Tsuda, Y. Kaito, K. Aoki, K and Kobayashi, K, Y. "Estimating Markovian transition probabilities for bridge deterioration forecasting," JSCE Journal (in Japanese), no. 801/I-73, October 2005.
6) Titterington, D, Smith, A, and Makov, U, "Statistical Analysis of Finite Mixture Distributions". John Wiley and Sons, 1985.
7) Dempster, A, Laird, N.M, and Rubin, D.B, "Maximum Likelihood from Incomplete Data via the EM Algorithm". Journal of the Royal Statistical Society, Series B-Vol.39, 1-38, 1997.
8) Dani, G, and Hedibert, F.L, "Markov Chain Monte Carlo-Stochastic Simulation for Bayesian Inference". Chapman and Hall, CRC, 2006.
9) Zucchini, W, MacDonald, L.L, "Hidden Markov Models for Time Series". Taylor and Francis, 2009.