A HIERARCHICAL HIDDEN MARKOV DETERIORATION MODEL FOR MULTI-COURSES PAVEMENT STRUCTURE EVALUATION

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1. Introduction

Road pavements are often designed with several structural courses based on the requirement of bearing capacity and characteristic of materials. For example, a typical asphalt road is composed of courses such as: sub-base, sub-grade, base, and surface course. Technically, the deterioration of one course proportionally depends on the deterioration of its adjacent course. If the surface structure is badly deteriorated, lower course has tendency to expose with high speed of deterioration. Similarly, if bearing capacity of lower course becomes weak, due to saturated water or heavy traffic volume, the durability of materials in surface course becomes loosely, as a result, resulting in cracking, pothole, etc. on pavement surface [1)].

In practice of road management, it is quite a phenomena that only surface course (s.c) is considered, with routine maintenance or frequent repairs, while attention on deterioration of base course (b.c) is negligently omitted. The problem is credited due to the fact that there is always a constraint in either monitoring technology or budget. For instance, conducting a test to reveal strength of materials and bearing capacity of lower course often requires high-tech tests such as: sample drilling, ultrasonic waves measurement, and recently with Falling Weight Deflectometer (FWD). These techniques are exerted to bear high costs. It is therefore, as a common practice, attention is largely only on visual inspection of surface course.

In many cases, there is a requirement to understand the true deterioration process of lower pavement structure for decision making on maintenance and repair (M&R), particularly with renewal of entire pavement structure. For example, carrying out routine M&R on s.c only upgrade the surface condition in a short run. If we continue only with M&R for s.c, certainly, after a period of time, the life cycle of renewed s.c becomes shorter. Reasons are due to the negative impact, resulting from deterioration of b.c. It is therefore necessary to investigate the deterioration process of lower course as an important part in Pavement Managemetn System (PMS).

With respect to deterioration process of lower course of road pavement, it is possible to apply the popular methodology with Markov chain model [2)]. In Markov chain model, there is a problem that condition states of b.c cannot be easily obtained by means of visual inspections. In many cases, true condition states of b.c are hidden information. In view of this problem, in this paper, we consider a type of Markov model called ``Hidden Markov" to uncover the deterioration process of b.c through examining the conditional relationship between b.c and s.c. Applications of the model into practice can be for determining the optimal time and right location for implementing FWD tests, thus further reduce the M&R cost in the the long-term management practices.

2. Deterioration Process and Markov Transition Probability

The deterioration process, regardless of annual M&R, can be presented in figure 1. It is supposed that a road is in service at time S_0 after construction period. Along with its operational time, we need to manage and maintain the road at respective time interval

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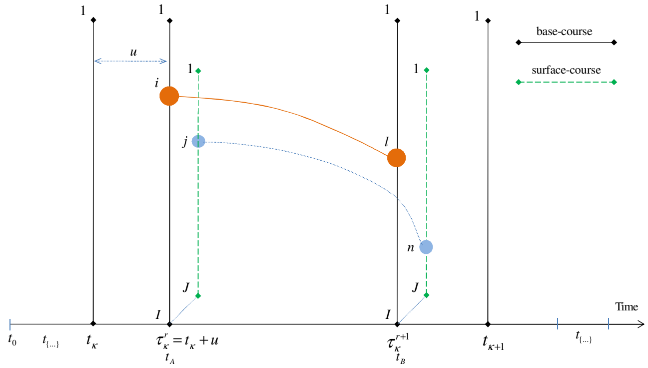
$(t = 0,1,2, \dots)$, normally in yearly basic. As a matter of practice, the frequency of M&R for s.c are more often than that for b.c. In most of the case, when M&Rs are designated for b.c, it means to renew both b.c and s.c. Thus, the M&R interval of b.c encompasses the M&R interval of s.c. In this view, intervals of visual inspection, being carried out for s.c, are also within the M&R interval of b.c. Figure 1 visualizes the difference in M&R intervals of b.c and s.c.



Note) Range of condition states of b.c and s.c are $i, l = (1, \dots, I)$ and $j, n + (1, \dots, J)$

Figure 1: Deterioration process of road pavement.

The solid vertical lines represent condition states of b.c $i, l = (1, \dots, I)$. Whilst, the dashed vertical lines represent condition states of s.c $j, n = (1, \dots, J)$.. Absorbing condition states of b.c and s.c are I and J respectively. M&Rs on b.c are alternately allocated at respective time $(t_0, t_1, \dots, t_\kappa, \dots)$. Within a period from $[t_\kappa, t_{\kappa+1})$, we can further use a local coordinate system $t_\kappa^1, \dots, t_\kappa^r, \dots t_\kappa^R$, as point in time, corresponding to M&R time of s.c. Thus, total duration in period $[t_\kappa, t_{\kappa+1})$ is a step-wise durations of $[t_\kappa, \tau_\kappa^1), \dots, [\tau_\kappa^r, \tau_\kappa^{r+1}), \dots, [\tau_\kappa^{R-1}, \tau_\kappa^R)$. The M&R period of s.c can be summarized as $[\tau_\kappa^{r-1}, \tau_\kappa^r)(r = 1, \dots, R)$, with $[\tau_0, t_\kappa)$ and $\tau_\kappa^R = T_\kappa$ (T_κ is the next M&R of b.c). The corresponding duration of the period $[\tau_\kappa^{r-1}, \tau_\kappa^r)$ is $= (z_\kappa^1, z_\kappa^R)$, with $z_\kappa^r = \tau_\kappa^r - \tau_\kappa^{r-1}$. For convenience, we denote $u = 1, \dots, T_\kappa$ in figure 1 as a point of time corresponding to z.

The transition time of condition states of b.c, from the outset, can be described within the period $[t_0, t_\kappa + \tau_\kappa^R] = [t_0, t_\kappa), [t_\kappa, t_\kappa + \tau_\kappa^1), \dots, [t_\kappa + \tau_\kappa^{R-1}, t_{\kappa+1})$. Duration of the period $[t_\kappa, \tau_\kappa^{r-1}, t_\kappa + \tau_\kappa^r)(r = 1, \dots, R)$ is expressed as $\zeta = \zeta_\kappa^1, \dots, \zeta_\kappa^R$. In another way, we can describe $\zeta_\kappa^r = z_\kappa^{r-1}(r = 2, \dots, R)$.

The probabilistic event of observation with respect to condition states of each course at any arbitrarily time t are $g^t = i(i = 1, \dots, I)$ for b.c and $h^{\tau_\kappa} = j(j = 1, \dots, J)$ for s.c can be alternatively described as follows:

(1) Markov transition probability for condition states of base course

In consideration of Markov transition probability (m.t.p), taking advantage of general mathematical formula in the paper of Tsuda et al [3], we can express the Markov transition probability of b.c p^{il} in following equation.

$$p^{il,t} = Prob[g^{t+1} = l \mid g^t = i] = \sum_{m=i}^{l} \prod_{s=i}^{m-1} \frac{\lambda^{s,t}}{\lambda^{s,t} - \lambda^{m,t}} \prod_{s=m}^{l-1} \frac{\lambda^{s,t}}{\lambda^{s+1,t} - \lambda^{m,t}} \exp(-\lambda^{m,t} z), \tag{1}$$

where the hazard rate $\lambda^{i,t}$ is in the form $\lambda^{i,t} = x^t \beta^i$. Vectors of covariates and unknown parameters are $x^t = (\hat{x}_1^t, .., \hat{x}_Q^t)$ and $\beta^i = (\hat{\beta}_1^i, .., \hat{\beta}_Q^i)$ respectively, Q is total number of covariates.

(2) Markov transition probability for condition states of surface course

The m.t.p π^{jn} of s.c is calculated within the M&R interval of b.c, with its local time interval u (refer to figure. 1).

$$\pi^{jn,u}(i) = Prob[h^{u+1} = n \mid h^u = j, g^{t_k+u} = i] = \sum_{m'=j}^{n} \prod_{s'=j}^{m'-1} \frac{\rho^{s',u}}{\rho^{s',u} - \rho^{m',u}} \prod_{s'=m'}^{n-1} \frac{\rho^{s',u}}{\rho^{s'+1,u} - \rho^{m',u}} \exp(-\rho^{n,u} \zeta) \tag{2}$$

Hazard rate $\rho^{j,u}$ is mutually dependent on a constant hazard $\lambda^{i,t}$ of the b.c. Thus, a multiplicative form can be applied for hazard rate of s.c as $\rho^{j,u}(i) = \gamma_0^i \rho^{j,u} = \lambda^i y^u \gamma^j$. Vectors of covariates and unknown parameters are $y^u = (\hat{y}^{1,u}, .., \hat{y}^{P,u})$ and $\gamma^j = (\hat{\gamma}_1^j, .., \hat{\gamma}_P^j)$ respectively. P is total number of covariates of s.c.

We assume that at time $t = 0$ (starting time of in-service period), condition states of b.c and s.c are $g_0 = 1$ and $h_0 = 1$ respectively. In another word, deterioration has not yet occurred. After a certain period of time, deterioration starts either with b.c or s.c. As long as the condition of m.t.p is satisfied, we have $P^t = \prod_{s=0}^{t-1} P^s$, with this assumption and given initial condition state probability $p_0 = (1,..,0)$. We can further calculate condition state probability distribution at time $t_k + u$ as $p^{(t_\kappa+u)} = p^0 P^{(t_\kappa+u)}$.

The m.t.p of s.c at time $t_k + u$ is partially dependent on the condition state probability of b.c $p^{i,(t_\kappa+u)}$. In another word, it is understandable that the transition probability of s.c is evaluated based on average value of condition state probability of b.c and the mixture form of Markov transition probability of s.c. From this understanding, we can derive formula for the m.t.p of s.c at time u as $\tilde{\pi}^{jn,u} = \sum_{i=1}^{I} p^{i,(t_\kappa+u)} \pi^{jn,u}(i)$, with satisfying condition $\sum_{n=1}^{J} \tilde{\pi}^{jn,u} = \sum_{i=1}^{I} p^{i,(t_\kappa+u)} \sum_{n=1}^{J} \pi^{jn,u}(i) = 1$. In summary, condition state probabilities with their Markovian characteristics as $p^t = p^0 \prod_{s=0}^{t-1} P^s$ and $\rho^t = \rho^0 \prod_{v=1}^{u-1} \tilde{\pi}^v$ are derived.

3. Hidden Markov Model and Complete Likelihood Function

Under limited budget, FWD tests can only be carried out at a certain numbers of road sections. We define a subset ψ of targeted road section $l \in \mathcal{R}$ for FWD tests, with its execution time T^l in the observation period $[0, T^l]$. Road sections without FWD tests are classified into sub-set ψ^c. With this assumption, we define likelihood functions for observational data of respective sub-sets.

(1) Likelihood function for sub-set ψ^c.

We denote $l \in \psi^c$ as arbitrary road section with its repair sequence at $t_\kappa^l (\kappa = 0, \dots, K^l)$ in period $[t_0, T^l]$. A set ψ^l is referred as time periods $[t_\kappa^l, t_{\kappa+1}^l)$ $(\kappa = 0, \dots, L^l)$, in which repairs are performed on section l. The time of inspections on the same section is $t_\kappa^l, \tau_\kappa^{1,l}, \dots, \tau_\kappa^{R,l}$ $(\kappa \in \psi^l)$. The observed condition state on road section l is expressed as $g(\tau_\kappa^{r,l})$. Thus, in the sub-set ψ^c, entire observed information can be described as $\Xi^l = \{(\tau_\kappa^{r,l}, g(\tau_\kappa^{r,l})) : (r = 0, \dots, R^\kappa, \kappa \in \psi^l)\}$. With this information, we can define the likelihood function for the event $g(t_A) = \bar{J}$ and $g(t_B) = \bar{n}$ can be defined as follows:

$$\mathcal{L}^{\bar{J}\bar{n}}(t_A, t_B) = \sum_{s=\bar{J}}^{\bar{n}} \tilde{\pi}_{\bar{J}\bar{n}}^{t_A} \ell_s(t_A + 1);\ \ell_h(t_A + u) = \sum_{s=\bar{J}}^{\bar{n}} \tilde{\pi}_{\bar{h}\bar{s}}^{t_A+u} \ell_s(t_A + u + 1); \text{and} \quad \ell_h(t_B) = \sum_{s=\bar{J}}^{\bar{n}} \tilde{\pi}_{\bar{h}\bar{s}}^{R-1} \tag{3}$$

Considering the entire observed information Ξ^l, with $l \in \psi^c$, following complete likelihood function can be defined:

$$\mathcal{L}^l = \prod_{\kappa \in \psi^l} \prod_{r=0}^{R} \mathcal{L}_{g(t_\kappa^{r,t}), g(t_\kappa^{r+1,t})} (t_\kappa^{r,t}, t_\kappa^{r+1,t}) \tag{4}$$

(2) Likelihood function for sub-set ψ .

Road section l of subset ψ with FWD test is actually included in subset ψ^c as well. We consider T^l as the time of FWD test. The condition state of sub-base revealed by FWD is $h(T^l) = \bar{\imath}^l$. Entirely, observed information on both s.c and FWD test for b.c on road section l can be summarized as $\bar{\Xi}^l = \{(\tau_\kappa^{r,l}, g(\tau_\kappa^{r,l})), (T^l, h(T^l)) : (r = 0, \dots, R^\kappa, \kappa \in \psi^l)\}$.

If we consider only the transition pattern of base course, then we can define the condition state probability of b.c at any arbitrary time $t_A + u$ as $\tilde{p}_i(t_A + u) = p_i(t_A + u) / \sum_{i=1}^{\bar{\imath}} p_i(t_A + u)$. Following the description of section 2, we can come up with the transition probability for s.c as $\tilde{\tilde{\pi}}^{jn,u} = \sum_{i=1}^{\bar{\imath}} p^{i,(t_\kappa+u)} \pi^{jn,u}(i)$. Furthermore, the likelihood function for the event $g(t_A) = \bar{J}$ and $g(t_B) = \bar{n}$ can be defined as follows:

$$\bar{\mathcal{L}}^{\bar{J}\bar{n}}(t_A, t_B) = \sum_{s=\bar{J}}^{\bar{n}} \tilde{\tilde{\pi}}_{\bar{J}\bar{n}}^{t_A} \bar{\ell}_s(t_A + 1);\ \bar{\ell}_h(t_A + u) = \sum_{s=\bar{J}}^{\bar{n}} \tilde{\tilde{\pi}}_{\bar{h}\bar{s}}^{t_A+u} \bar{\ell}_s(t_A + u + 1); \text{and} \quad \bar{\ell}_h(t_B) = \sum_{s=\bar{J}}^{\bar{n}} \tilde{\tilde{\pi}}_{\bar{h}\bar{s}}^{R-1} \tag{5}$$

In view of management, we try to keep bearing capacity of b.c above the certain level for safety reason. Thus, FWD tests are conducted with purposes that we can practically reveal the bearing capacity of b.c (or condition state). It is assumed that if bearing capacity (or condition state) of b.c happens to be under a safety level, M&R should be immediately applied on b.c. With this assumption, we need to estimate the likelihood that condition state of b.c reach to a certain point $\bar{\imath}^l$ in time T^l. The likelihood for that event can be expressed as $\tilde{\ell}(T^l) = p^{1\bar{\imath}^l}(T^l)$. As a result, a complete likelihood function concerning observed information $\bar{\Xi}^l$ can be ultimately derived as:

$$\bar{\mathcal{L}}^l = \prod_{\kappa \in \psi^l} \prod_{r=0}^{R-1} \mathcal{L}_{g(t_\kappa^{r,t}), g(t_\kappa^{r+1,t})} (t_\kappa^{r,t}, t_\kappa^{r+1,t}) / p^{1\bar{\imath}^l}(T^l) \tag{6}$$

Considering both sub-set of observed condition states on s.c and b.c $\Psi = \{\Xi^l (l \in \psi^c); \bar{\Xi}^l (l \in \psi)\}$, we can finally come up with an explicit form for likelihood function as follows:

$$\mathcal{L} = \prod_{l\in\psi^c}\mathcal{L}^l \prod_{l\in\psi}\bar{\mathcal{L}}^l \tag{7}$$

4. Estimation Methodology

In order to estimate the parameters of hidden Markov model, a feasible analytical method for solving a set of likelihood functions (4), (6), and (7) must be proposed. Conventionally, maximum likelihood estimation approach is used. However, maximum likelihood method using in mixture hazard models or hidden Markov models exhibits its limitation that it involves a high level of derivation, thus giving challenge in calculation. In addition, the obtained results can be of local optimal points, which are not considered as global optimal results. Recent stochastic development strongly suggests to apply Bayesian estimation approach and Markov Chain Monte Carlo (MCMC) simulation as an excellent methodology to estimate the parameters of Hidden Markov model[3)]. In this section, we briefly explain the analytical approach of the Hidden Markov model.

As can be seen from equation (7), the likelihood function involves two components ($\mathcal{L}^l$ and $\bar{\mathcal{L}}^l$). Each component expresses the likelihood functions for its corresponding sub-set of observed data. With respect to observed information of surface course and without considering FWD tests, likelihood function (4) is considered. In this respect, we need to estimate the unknown parameter γ in equation (2). Similarly, if we take into account the likelihood function $\bar{\mathcal{L}}^l$, we need to estimate also for unknown parameter β, which is embedded in hazard function of Markov transition probability in equation (1). In conclusion, it is suffice to say that there are two unknown parameters β and γ to be estimated. In view of Bayesian estimation approach and MCMC simulation, the model's parameters can be assumed with conjugate prior distributions. For example, multidimensional normal distribution can be used as prior distribution for either β and γ. A similar estimation approach can be referred the paper of Kobayashi et al [4)]. We only give a brief estimation procedure in figure 2.

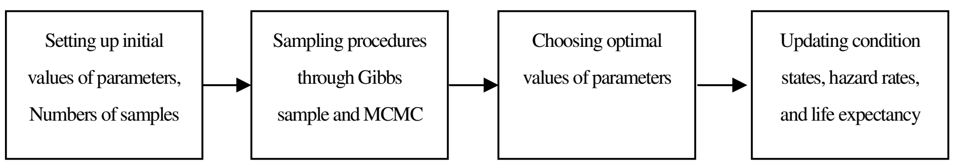


Figure 2: Deterioration process of road pavement.

5. Conclusion and Recommendation

This paper has proposed a hidden Markov model for forecasting the deterioration pattern of pavement structure. By use of the model, we can estimate the Markov transition probability of condition states for both surface course and base course. The dependency on deterioration between surface course and base course can be analytically estimated. Estimation results can be utilized for decision making process, especially, for decisions concerning with M&Rs for base course of pavement.

Estimation methodology for estimating parameters of the model takes full advantage of Bayesian updating method and MCMC simulation. In simulation, observed information from visual inspections on surface course and results of FWD tests on base course will be used as observed data. Prior distribution of model's parameters can be assumed with conjugate probabilistic distribution. It is hope that an empirical study on actual data will be conducted in a short-term upon the completion of our mission to gather sufficient visual inspections on surface course and FWD tests on base course. Furthermore, concept and methodology of our model is not only limited in PMS but also can be extended to apply in other infrastructure systems such as bridge management system, tunnel management system, etc.

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