REVIEW OF BENEFIT EVALUATION MODELS FOR TRAVEL TIME RELIABILITY IMPROVEMENTS

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1. Background and Objective

The main objective of this paper is to review econometric models that can be used to forecast travel time reliability improvements for cost-benefit-analysis. Several countries have decided or are considering to include the cost of travel time reliability in their cost-benefit analysis, including the US, UK, Netherlands, and Sweden. From the existing transportation literatures on the valuation of travel time reliability, there are basically two existing approaches; the mean-variance approach which assumes that every traveler has a priori estimates of the mean and variance of the travel time between OD pair, and the scheduling approach in which a particular departure time choice is a function of 'schedule delay early', and 'schedule delay late', related to a 'Preferred Arrival Time' (PAT). In this paper, recent research progresses as well as practical implementations at several countries are summarized and compared.

2. Traveler Behavior Assumptions

To evaluate the value of travel time reliability we should first discuss the relationship between travel time (un)certainty and travel behavior. From the existing transportation literature on the valuation of reliability, there are basically two existing assumptions and others are related to them:

<u>(1) Mean-variance approach</u>

The mean–variance approach is used to estimate an individual's travel choice behavior. Brastow and Jucker[1] assumes that every traveler from A to B has a priori estimates of the mean and variance of the travel time and the objective of each traveler, k, is to minimize:

$$E(T_P) + \lambda_k V(T_P),\ P \in P_{AB},$$

where λ_k :parameter represents the degree to which the variance of travel time is undesirable to traveler k,
T_p :the expected travel time from A to B on path p,
$V(T_p)$:the variance of travel time on path p,
P_{AB} :the set of all paths from A to B.

Applications of this approach can be found in Hollander[2] and Noland and Polak[3]. Some contributions (de Palma and Picard[4]; Abdel-Aty, Kitamura and Jovanis[5]; Jackson and Jucker[6]) have replaced the variance with standard deviation or with another measure of scale, an interquantile range (Brownstone and Small[7]; Lam and Small[8]; Small et al.[9]) have instead defined reliability as the range between, e.g., the 0.5 and the 0.9 quantiles of the distribution of durations.

<u>(2) Scheduling approach</u>

In the scheduling approach, benefits estimates stem from concepts of early departure and late arrival[10]. Small[10] derives departure time choice in a deterministic context. Later work by Noland and Small[11] developed these ideas in the context of travel time variability. A particular departure time choice is a function of four components; travel time, 'schedule delay early' (*SDE*), 'schedule delay late' (*SDL*), and a 'lateness' dummy variable (*L*) that is set to unity if schedule delay late is non-zero. The latter three components are conditioned by the notion of a 'Preferred Arrival Time' (*PAT*), as follows:

$$U = \alpha T + \beta SDE + \gamma SDL + \delta L$$

Where: L *is* a dummy variable set to one if $SDL > 0$, otherwise zero.

Travel time variability affects individuals' utility to the extent that the arrival time at a destination is affected. The individual holds preferences for being early or late, compared to a preferred arrival time (*PAT*), see Small[10], and Noland and Small[11].

$$E[U] = \alpha\, E[T] + \beta\, E[SDE] + \gamma\, E[SDL] + \delta\, E[L]$$

Where: $E[L]$ is the probability of lateness.

Keywords: Travel time reliability, Travel time variability, reliability ratio, cost-benefit-analysis.
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(3) Travel time budget approach

Lo, Luo, and Siu[12] introduced travel time budget approach. The travel time budget is defined based on the probability requirement of arrivals within the travel time budget, whereas the earlier approaches measure the relative value or weight between the mean and standard deviation of travel time in route choice decisions. The travel time budget is defined as:

[Travel Time Budget] = [Expected Travel Time] + [Travel Time Margin].

Travelers may depart earlier to allow for additional time, or add a travel time margin to the expected trip time, in order to avoid late arrivals. The travel time budget associated with route p , b_p can be expressed as:

$$b_p = E(T_p) + \lambda\sigma_{Tp}$$

where T_p : the random variable of travel time on route p

$E(T_p)$: the mean of T_p,

σ_{Tp} : the standard deviation of T_p.

The parameter λ is related to the requirement on punctual arrivals. For trips that have a high penalty on lateness, one expects that travelers would reserve a relatively large travel time budget, or equivalently, a high value of λ. Formally, the value of λ is related to the probability that a trip completes within the travel time budget, written as:

$$P\{T_p \leq b_p = E(T_p) + \lambda\sigma_{Tp}\} = \rho$$

where ρ : the probability that the actual trip time is within the travel time budget.

(within budget time reliability (WBTR))

Siu, and Lo[13] assumed that the route travel time T is a random variable, whose probability density and cumulative distribution functions are f and F, respectively. It is then up to the traveler to choose a travel time budget b such that his schedule utility is optimal while satisfying his requirements. Then the linear disutility function is:

$$U(b) = \alpha T + \beta SDE + \gamma SDL$$

To solve for the optimal travel time budget by the MEU approach, they[13] find b^* which maximizes $E[U(b)]$. Since travelers dislike early and late arrivals, all the coefficients of the utility function are negative, i.e. $\alpha, \beta, \gamma < 0$.

$$E[U(b)] = \alpha\int_0^\infty tf(t)dt + \beta\int_0^b (b-t)f(t)dt + \gamma\int_b^\infty (t-b)f(t)dt = \alpha E(T) + \beta\int_0^b (b-t)f(t)dt - \gamma\int_b^\infty (t-b)f(t)dt$$

Maximizing $E[U(b)]$ is:

$$b = F^{-1}\left(\frac{\gamma}{\beta+\gamma}\right) = F^{-1}\left(\frac{1}{1+\beta/\gamma}\right)$$

where:

$$0 < \frac{1}{1+\beta/\gamma} < 1 \Leftrightarrow 0 < \frac{\beta}{\gamma} < \infty \cdot$$

They[13] concluded that the exact travel time budget depends on the specific travel time distribution. They[13] applied this approach for the normal distribution as well as the triangular distribution.

(4) Unified approach

The mean–standard deviation approach and the scheduling approach were unified by Fosgerau & Karlström[14], who showed how the standard deviation could be adjusted using a "mean lateness" parameter. Using a simple formulation of scheduling utility, they[14] show that the maximal expected utility is linear in the mean and standard deviation of trip duration, regardless of the form of the standardized distribution of trip durations. They[14] derive the expected cost for a general distribution of trip durations and obtain the simple result that the optimal departure time as well as the optimal expected cost depends linearly on the mean and standard deviation of the distribution of trip durations, provided the standardized distribution is fixed. They[14] noted that H is the mean lateness in standardized travel time referred to as "mean lateness factor" can be calculated as follows:

$$H\left(\Phi, \frac{\beta}{\beta+\gamma}\right) = \int_{\frac{\gamma}{\beta+\gamma}}^{1} \Phi^{-1}(s)ds$$

The expected utility of an agent who faces a given distribution of trip durations and who optimally chooses his departure time can be written as:

$$EU^* = \alpha\mu + (\beta+\gamma)H\left(\Phi, \frac{\beta}{\beta+\gamma}\right)\sigma$$

Based on that unified approach, Franklin[15] developed a predictive model of mean lateness. They present an estimated statistical model of "absolute" mean lateness, which is stated in terms of minutes and therefore more interpretable than the alternative "standardized" mean lateness. They[15] presented a new function for absolute mean lateness, K:

$$K\left(f(\sigma), \frac{\lambda}{\nu}\right) = \int_{1-\frac{\lambda}{\nu}}^{1} F^{-1}(s/\sigma)ds$$

Fosgerau & Karlström[14] state T in terms of a fully standardized random variable X, whose standard deviation is 1, and then multiply the drawn value of X by travel time distribution's standard deviation term, σ. On the other hand, in Franklin[15] the integral is taken over the unstandardized travel time distribution. Hence, the standard deviation of travel times, σ, is absorbed into the integral itself. They relate the two to each other as follows:

$$K\left(f(\sigma),\frac{\lambda}{\nu}\right)=\sigma\cdot H\left(\phi,\frac{\lambda}{\nu}\right)$$

The expected utility function that depends on the preference parameters (λ,ω,ν), the mean travel time μ, and the absolute mean lateness K:

$$EU^{*}=(\lambda+\omega)\mu+\nu K\left(f(\sigma),\frac{\lambda}{\nu}\right)$$

Where: $\omega = \alpha - \beta$, $\lambda = \beta$, and $\nu = \beta + \gamma$.

They[15] also estimated a statistical model of mean lateness is to provide a means of forecasting the value of the absolute mean lateness under uncertain future scenarios, subject to various transport policies and traffic conditions. The dependent variable is the log of the ratio between absolute mean lateness and free flow travel time, i.e. $K_i' = \log(K^i/t^i_{ff})$, where K^i is the absolute mean lateness and t^i_{ff} is the free flow travel time for segment i. Model estimation was performed using ordinary least squares regression. Also Batley et al.[16] represent the random variable T by a combination of the mean and variance, using Z as a standardized random variable with mean = 0 and variance = 1. They write $T = \mu + Z\sigma$ and write $\Phi(Z)$ as the distribution of the standardized variable. It can then be shown that the optimum departure time is given by:

$$PAT-\left(\mu+\sigma\Phi^{-1}\left(1-\frac{\beta}{\beta+\gamma}\right)\right)$$

Where Φ^{-1} is the inverse cumulative distribution of Z. Substituting this into the utility function, it can be shown that they[16] obtain an optimum utility equivalent to:

$$\alpha\mu+\sigma.\left\{(\beta+\gamma).\int^{1}\left(1-\frac{\beta}{\beta+\gamma}\right).\Phi^{-1}(y).dy\right\}$$

Hence, the reliability ratio $\tilde{\tilde{\rho}}$ can be written as:

$$\tilde{\tilde{\rho}}=\left\{\frac{\beta+\gamma}{\alpha}.\int^{1}\left(1-\frac{\beta}{\beta+\gamma}\right).\Phi^{-1}(y).dy\right\}$$

3. Benefits of Travel Time Reliability Improvements

The previous studies have derived empirical values for the parameters of the mean-variance approach and the scheduling approach based on traveler's individual perception. These valuation methods are hardly used due to the lack of knowledge on how to predict and value travel time variability and the lack of appropriate data. As an alternative, the value of time has been studied as the change in the mean of travel time distribution while the value of reliability was based on the value of the change in the variance, standard deviation or the range between quantiles of travel time distribution. Consequently, several researches try to develop models that estimate travel time variations. These models can be used to estimate travel time reliability for cost-benefit-analysis. However, studies that provide quantitative models forecasting the value of travel time reliability improvements are still rather scarce. Future researches should attempt to develop such models. The following sub-section reviews the researchers conducted in this area:

(1) Estimation of Value of Time (VOT) and Value of Reliability (VOR)

Lam, and Small[8] measured VOT and VOR using data on actual behavior of commuters on state route 91 in California in a real pricing context by observing travelers who face a choice between a free congested lanes and a variably tolled express lanes. The data collected from mail surveys and the distribution of travel time is measured using loop detector data. The models represent travel time by its median, and unreliability by the difference between the 90th percentile and the median. They assume that for traveler n chooses route i the utility function is:

$$U_{in}=V_i(t_{in},v_{in},c_{in},x_n)+\varepsilon_{in}$$

where t, v, and c are the travel time, travel time variability, and cost. And x is a vector of observable socio-economic or other characteristics (including time of day and car occupancy). Then a bionomial logit model was estimated. The VOT and VOR are defined as:

$$VOT_n=(\partial V/\partial t_n)/(\partial V/\partial c_n),\quad VOR_n=(\partial V/\partial v_n)/(\partial V/\partial c_n)$$

Liu et al.[17] presented a mixed logit formulation of route choice behavior on the same route (state route 91) as a function of travel time, reliability, and cost. They[17] calculated the time-dependent VOT, VOR and degree of risk aversion (DORA), whose variations in different periods reveal the relationship between commuters' route choices and their departure times, as well as the variation of their preferences with departure time. They[17] presented time-dependent disutility function as:

$$U_{np}(t) = \beta_n'(t) x_{np}(t) + \varepsilon_{np}$$

where $U_{np}(t)$: the total disutility of path *p* at time *t* for traveler *n*,

$x_{np}(t) = \lfloor T_p(t), R_p(t), C_p(t) \rfloor_n$: the cost vector of path *p* at time *t* for traveler *n,*

$T_p(t)$: the travel time of path *p* at time *t*,

$R_p(t)$: the variability of path *p* at time *t*,

$C_p(t)$: the out-of-pocket monetary cost of path *p* at time *t*,

$\beta_n'(t) = \left[\beta_p^T(t), \beta_p^R(t), \beta_p^C(t)\right]$: the aversion parameters vector of traveler *n*,

ε_{np} : unobserved extreme random value for traveler *n* using path *p*.

The term ε_{np} assumed to be identically and independently distribution across all travelers and routes, captures the person-varying differences between true disutility value $U_{np}(t)$ and deterministic disutility calculated by the given linear function:

$$V_{np}(t) = \beta_n'(t) x_{np}(t)$$

The VOT and VOR are time dependent and defined by:

$$VOT_n(t) = \frac{\partial U_{np}(t) / \partial T_p(t)}{\partial U_{np}(t) / \partial C_p(t)} = \frac{\beta_n^T(t)}{\beta_n^C(t)}, \; VOR_n(t) = \frac{\partial U_{np}(t) / \partial R_p(t)}{\partial U_{np}(t) / \partial C_p(t)} = \frac{\beta_n^R(t)}{\beta_n^C(t)}$$

And the degree of risk aversion (DORA) is defined by:

$$DORA_n(t) = \frac{VOR_n(t)}{VOT_n(t)} = \frac{\beta_n^R(t)}{\beta_n^T(t)}$$

They[17] assume that the coefficients to travel time, its reliability and cost satisfy normal distributions. The probability that traveler n will depart at time t and choose route p, conditioned on $\beta(t)$ is given by:

$$L_{np}(\beta(t); t) = \frac{e^{\beta'(t) x_{np}(t)}}{\sum_{\forall q \in Q^{rs}} e^{\beta'(t) x_{nq}(t)}}$$

The unconditional probability is the integral of $L_{np}(\beta(t); t)$ over the distribution of all possible values of $\beta(t)$, i.e.,

$$p_{np}(t) = \int \frac{e^{\beta'(t) x_{np}(t)}}{\sum_{\forall q \in Q^{rs}} e^{\beta'(t) x_{nq}(t)}} \cdot f(\beta(t)) d\beta(t)$$

The Monte Carlo or Quasi-Monte Carlo (QMC) simulation was used to integrate out mixed logit probability by discretizing the density function of the coefficient variable $\beta(t)$. Then the unbiased Monte Carlo simulation to the mixed logit model is generated by averaging the MNL values over a set of samples which are drawn from the conditional density function. Brownstone and Small[7] compared results from evaluations of two recent road pricing demonstrations in two projects in southern California, combines pricing with priority for high-occupancy vehicles in the form of ''High Occupancy/Toll'' (HOT) lanes. In this scheme, a set of express lanes on an otherwise free and congested road offers high-quality service to people who are willing to pay a time varying toll and/or who ride in carpools. This paper[7] reviewed and compared results on VOT, and VOR from data sets taken from the two HOT-lane projects in southern California. The models assume that a traveler *i* faces an actual or hypothetical choice at time *t* among alternatives *j*. The alternatives include commuting lanes (toll or free) and possibly other travel features like carpooling, time of day, or acquisition of an electronic transponder. Using notation adapted from Small et al.[18], the traveler chooses the option that maximizes a random utility function:

$$U_{itj} = \theta_{ij} + \beta_i X_{itj} + \varepsilon_{itj}$$

Variables included in X_{itj} measure the toll C_{itj}, travel-time T_{itj}, and (un)reliability R_{itj}. The values of travel time and reliability are defined as:

$$VOT_i = (\partial Uitj / \partial T_{itj}) / (\partial U_{itj} / \partial C_{itj}), \; VOR_i = (\partial Uitj / \partial R_{itj}) / (\partial U_{itj} / \partial C_{itj})$$

(2) Estimation of Value of Travel Time Variations

Eliasson[19] uses an econometric model and finds a non-linear relationship between the relative standard deviation of travel time (standard deviation divided by travel time) and the relative increase in travel time (travel time divided by free-flow time) on urban roads. He used two similar measures: the standard deviation of travel time (σ) and the difference between the 90- and 10-percentile (*s*), scaled by the factor 2.56. If the travel time is normally distributed, the measures will coincide, and as long as it is symmetrically distributed they will only differ by a scale factor. He[19] concluded that the travel time distributions are skewed for moderate levels of congestion, while it tends to be symmetrically distributed for high levels of congestion. He[19] used (*s*) as a measure of the variability rather than the standard deviation. The relationship between the travel time variability (measured by *s*) and the explanatory variables can be written approximately as:

$$S = const * traveltime^{1.2} * \sqrt{\frac{traveltime}{free\ flow\ traveltime} - 1}$$

The constant depends on the length of the link, whether the queues are building up or dissipating, and on the speed limit. Based on that paper[19], Eliasson[20] presents a cost–benefit analysis of the Stockholm congestion charging system. The data sources are travel time and traffic flow measurements made before and after the charges were introduced. He[20] valued travel time variability as:

0.9 * (value of time) * (standard deviation of travel time)

A paper that is similar in its focus as the previous one (Eliasson[19]) has been written by Peer et al.[21]. They[21] developed an econometric regression analysis model based on travel time data of 146 highway links in the Netherlands that can be used to predict travel time variability for cost-benefit-analysis (CBA). Peer et. al.[21] standardized mean travel time by subtracting free flow time from the mean travel times in which the calculations yield a delay in minutes. They[21] developed different models including delay and Volume-capacity-ratio (VCR) as explanatory variables. They[21] found that a nonlinear multiplicative model of the following structure performs best in terms of predictive power:

$$Stdev = \beta_1 * Delay^{\beta_2} * VCR^{\beta_3}$$

They[21] used the nonlinear regression that does not take into account the flow-capacity-ratio to determine costs. A hypothetical VOT of 10c/minute and a VOR of 8c/minute are assumed (implying a reliability ratio= VOR/VOT=0.8). For each point in the sample they calculated the costs (*C*) a driver faces by the formula:

$$C = 10 * Delay + 8 * Stdev.$$

Transport Analysis Guidance (TAG)[22] modified the models which were developed for the London Congestion Charging using additional data collected in Leeds (2003) with these improvements reported in Arup (2004)[23]. The form of model developed forecasts the Standard Deviation of Journey Time from Journey Time (*t*) and Distance (*d*) for each origin to destination flow. Under the further assumptions that distances and free-flow speeds do not change as a result of the scheme, the change in journey time variability (represented by $\Delta\sigma_{ij}$) is given by:

$$\Delta\sigma_{ij} = 0.0018\left(t_{ij2}^{2.02} - t_{ij1}^{2.02}\right)d_{ij}^{-1.41}$$

Where: t_{ij1} and t_{ij2} are the journey times before and after the change for the journey from *i* to *j* (seconds)

$\Delta\sigma_{ij}$ is the change in standard deviation of journey time for the journey from *i* to *j* (seconds)

d_{ij} is the journey distance from *i* to *j* (km)

The reliability benefit is therefore calculated using:

$$Benefit = -\sum_{ij} \Delta\sigma_{ij} * \left(\frac{T_{ij2} + T_{ij1}}{2}\right) * VOR$$

Where: T_{ij1} and T_{ij2} are number of trips before and after the change.

Gilliam et al.[24] developed methods for estimating and forecasting travel time variability using data collected from Global Positioning System (GPS) tracker equipped vehicles by means of regression models. They[24] relate a coefficient of variation of travel times CV (defined as standard deviation divided by mean travel time) to a congestion index CI (actual travel time divided by free flow time). The data used is based on travel time data for 34 routes (up to 12 km long) in England for a period of three years. Multi-link journeys along the route were confirmed by 'average link CV and average link CI' (in which the relationship between CV and CI for a route is to consider the average link CV and average link CI for each timeslot was examined). Using average relationship performs 95% confidence, the average relationship found was:

$$CV_t = 0.16\, CI_t^{1.02}\, d^{-0.39}$$

Where: CV_t, CI_t= Coefficient of variation and congestion index in timeslot *t*, *d* = Distance (in kilometers)

4. Conclusion

This paper reviews and compares the statistical models concerning valuation of travel time reliability to provide useful opportunities for appraising travel time reliability improvements. Traveler's behavior toward travel time reliability costs has been investigated in the mean-variance approach which assumes that every traveler has a priori estimates of the mean and variance of the travel time between OD pair, and the scheduling approach in which a particular departure time choice is a function of 'schedule delay early', and 'schedule delay late', related to a 'Preferred Arrival Time' (PAT). The value of time has been studied as the change in the mean of travel time distribution while the value of reliability was based on the value of the change in the variance, standard deviation or the range between quantiles of travel time distribution. Consequently, several researches try to develop models that estimate travel time variations. However, studies that provide quantitative models forecasting the value of travel time reliability improvements are still rather scarce. Future researches should attempt to develop such models.

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