AN ANALYTICAL APPROACH FOR DESCRIBING DISTRIBUTION OF INTERSECTION ENTERNING TRAFFIC FLOW ON A GRID ROAD NETWORK

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1. Introduction

In this paper, we try to provide a simple analytical approach to describe distribution of intersection entering traffics on a grid road network. The approach is a continuous model with explicit structure and free of the need of a detailed *OD* (Origin-Destination) table.

For the purpose of proper road network planning and controls, it is very important to quantify the traffic distribution on a road network. Currently the most widely used approaches can be roughly divided into two types: the static traffic assignment models and the dynamic traffic simulation models, either macroscopic or microscopic ones. However, the traditional models demands very detailed *OD* information, with inevitable complicated, black-box type of algorithms. As the result, the users often reach to the outputs without knowing the underlying logic behind. The traditional models are all link oriented, except the microscopic analysis, intersections play a trivial role (if any) in the traditional models, despite the fact that intersections rather than basic sections of road links, decide the performance of urban road networks in modern cities.

In this paper we assume the passing probability of an intersection equals to the utilization of the intersection in the routes between *OD* pairs within a road network. Under this assumption, an explicit analytical model is developed to describe the distribution of traffic within a grid network by a single pair of *OD*, firstly a discrete model and followed by a continuous model that can be subjected to the method of integral in the future. Finally we use a proved commercial simulation model to validate the proposed approach. Our approach can be regarded as a simplified estimation method of traffic distribution and this work is also helpful for the researches on the road network density. Through our research we do not consider about the influence of congestion since our objective is limited to the planning stage only.

2. An Discrete Intersection Utilizing Model of a Single OD Pair

A road network consists of both links (roads) and nodes (intersections). For most modern urban road networks, the common knowledge is that the smoothness of traffic movement is more likely decided by intersections, rather than basic road sections. This research concentrates on the intersections by assuming all *OD* trips are all between them instead of the traditional zones. Similarly, routes between any *ODs* are

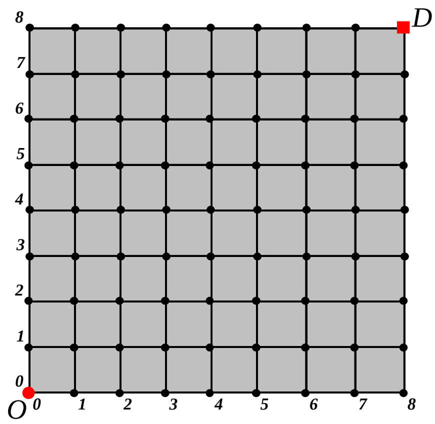

Figure 1: A Road Network Model

*Keywords: road network theory, route choice model

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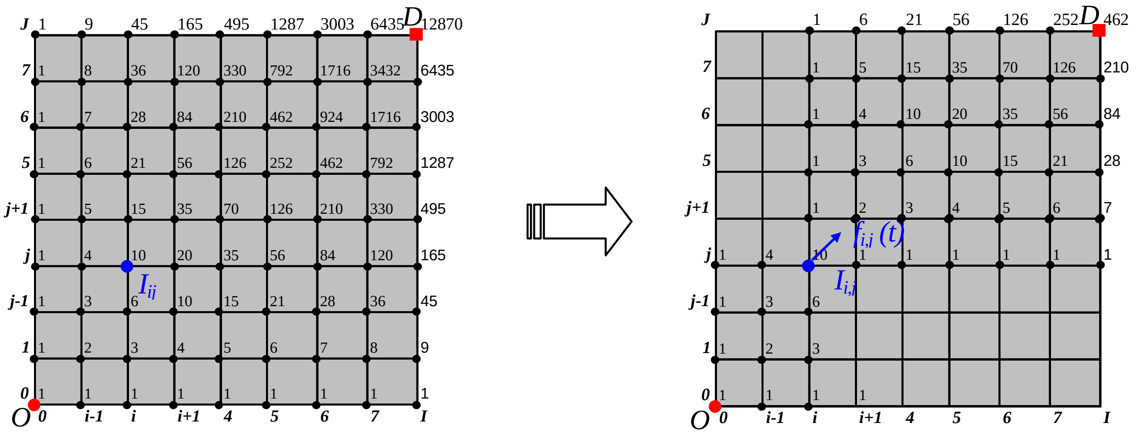


a) $R_o(i,j)$, route numbers from O b) Estimating Directional Traffics

Figure 2: An Intersection Choice Model

also described by a sequence of intersections, rather than links.

(1) A Road Network Model

A road network consists of intersections and roads connected them is shown in Fig. 1. In this paper, we assume the roads (solid lines) as of high-hierarchical with a uniform background of low-hierarchical roads (grey shade), as suggested by H. Lehmanh[1].

(2) A Discrete Intersection Choice Model for Estimating Directional Intersection Entering Traffics

As stated earlier, in this paper, we assume the percentage of traffic crossing a certain intersection equals to the utilizing probability of the intersection in the routes between *O* and *D*.

a) Number of routes from *O*

Route choice behavior is defined by choices of intersections in this paper to estimate directional intersection entering traffics. For a network of *IJ* with *O* and *D* at two opposite corners, the route numbers from *O* to all the intersections are shown in Figure 2-a). If we define *R(O,I)* is the number of routes from *O* to the intersection $I_{i,j}$, we can have the equation 1.

$$R(O,I_{ij}) = \frac{[(i-0)+(j-0))]!}{(i-0)!(j-0)!} = \frac{(i+j)!}{i!\,j!} \tag{1}$$

b) Estimating Directional Traffics

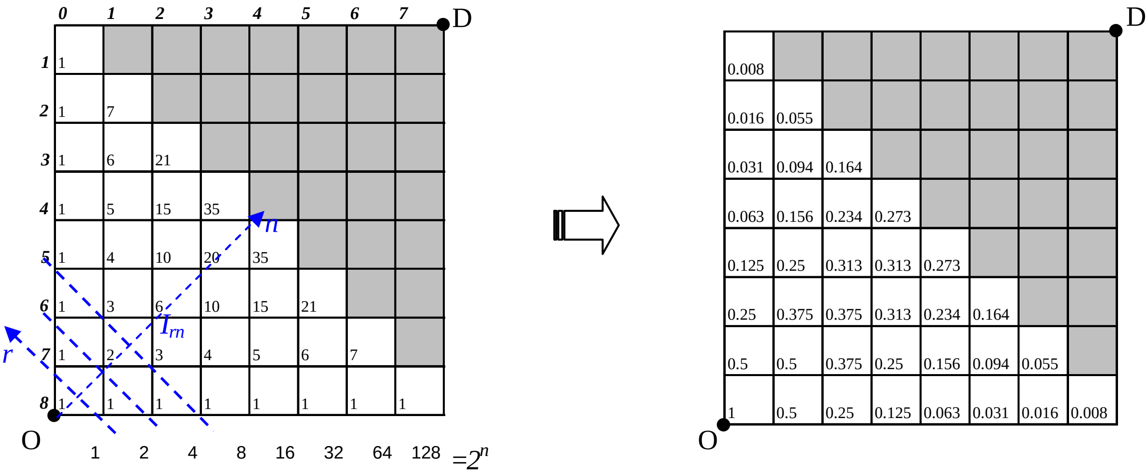
As shown in Figure 2-b), at the intersection $I_{i,j}$, the total entering traffic at time *t*, $f_{ij}(t)$, can be estimated by the following equation (2).

$$f_{ij}(t) = \rho_{OD}(t)\frac{R(O,I_{ij})\cdot R(I_{ij},D)}{R(O,D)} = \rho_{OD}(t)[\frac{(i+j)!}{i!\,j!}\cdot\frac{(I-i+J-j)!}{(I-i)!(J-j)!}/\frac{(I+J)!}{I!\,J!}] = \rho_{OD}(t)\cdot p_{ij} \tag{2}$$

where *R(Iij, D)* is number of routes from *Iij* to *D*, *R(O,D)* is the total number of routes from *O* to *D*, and $\rho_{OD}(t)$ is the trip generating/attracting rate between *O* and *D*.

- **Definition of Intersection Utilizing Probability (p_{ij})**,: here we define p_{ij} as the crossing probability from O to D and it can be calculated by the equation (3). Obviously, pij depends on the network itself thus independent of the demand on the network or the OD pattern.

$$p_{ij} = \frac{R(O,I_{ij})\cdot R(I_{ij},D)}{R(O,D)} = \frac{(i+j)!}{i!\,j!}\cdot\frac{(I-i+J-j)!}{(I-i)!(J-j)!}/\frac{(I+J)!}{I!\,J!} \tag{3}$$



a) Number of Routes b) Utilizing Probability

Figure 3: Intersection Utilizing Probability of a Symmetrical Network

3. Development of a Continuous Intersection Utilizing Model

In this section, we try to develop a continuous model on the basis of the discrete model by assuming a symmetrical road network with infinite road density. A symmetrical grid network is shown in Figure 4, the number of routes and the utilizing probability can be easily estimated by using the earlier model.

As for the new *r*-*n* coordinate system, when the traffic from O to D crossing the blue lines, p_{rn} the utilizing probability of an intersection I_{rn} on these blue lines can be calculated by the following equation (4) since it consist of a perfect binomial distribution.

$$p_{rn} = \frac{R(O, I_{rn})}{\sum_{r=0}^{r<=n} R(O, I_{rn})} = \frac{\begin{pmatrix} n \\ r \end{pmatrix}}{\sum_{r=0}^{r<=n} \begin{pmatrix} n \\ r \end{pmatrix}} = \begin{pmatrix} n \\ r \end{pmatrix} / 2^n = \frac{n!}{r!(n-r)!} (\frac{1}{2})^n \tag{4}$$

where $R(O, I_{rn})$ is the number of routes from O to I_{rn}.

As $n \to \infty$, by assuming $x = \frac{r - n/2}{\sqrt{n}/2}$, applying the Stirling Equation of $n! \cong \sqrt{2\pi} e^{-n} n^{n+1/2}$, an appropriate continuous equation can be obtained[2)].

$$p(r) \cong \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2}} \tag{5}$$

where $\sigma = \sqrt{n}/2$

4. Validation of the Proposed Model

In order to validate the proposed models, a 9 by 9 grid road network is constructed under a simulation platform of SOUND/4U©. The simulation platform is based on a macroscopic model developed under the cooperation between the Industrial Science Institute, University of Tokyo and i-Transport Lab. Co., Ltd (ITL).

For the purpose of comparison, by assuming $\rho_{OD}(t) = 10000$, intersection entering traffics of the grid network are estimated by both equation (4) and the simulation model. The results can be found in the table 1 and table 2.

(1) Estimation Results

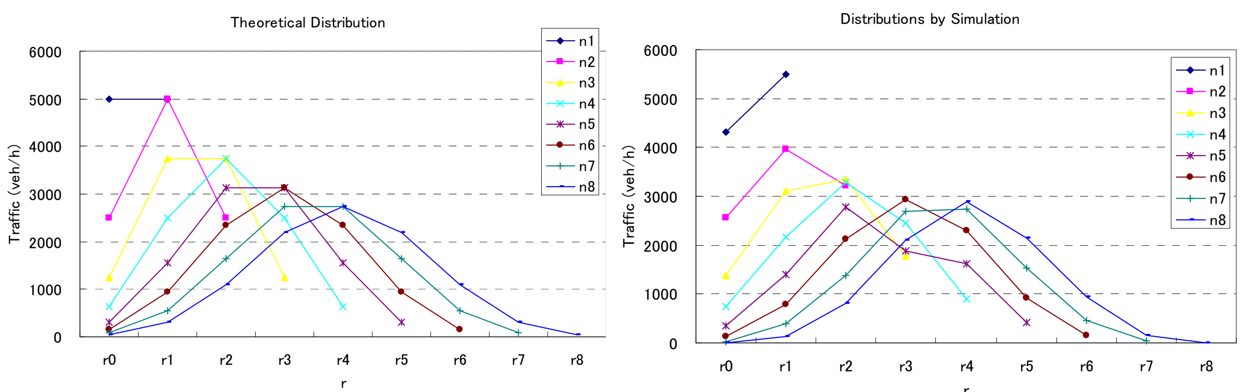
Table 1: Estimation Results of the Proposed Models

n \ r	0	1	2	3	4	5	6	7	8	Sum
0	10000									10000
1	5000	5000								10000
2	2500	5000	2500							10000
3	1250	3750	3750	1250						10000
4	625	2500	3750	2500	625					10000
5	313	1563	3125	3125	1563	313				10000
6	156	938	2344	3125	2344	938	156			10000
7	78	547	1641	2734	2734	1641	547	78		10000
8	39	313	1094	2188	2734	2188	1094	313	39	10000

Table 2: Estimation Results of SOUND

n \ r	0	1	2	3	4	5	6	7	8	Sum
0	9906									9906
1	4324	5491								9815
2	2553	3961	3209							9723
3	1386	3119	3344	1782						9631
4	735	2164	3295	2442	904					9540
5	343	1399	2778	1893	1627	408				8448
6	122	793	2128	2937	2301	919	156			9356
7	32	385	1381	2693	2727	1538	462	47		9265
8	6	121	811	2099	2893	2146	939	151	7	9166

(2) Traffic Distribution



a) Distribution by Proposed Model

b) Distribution by Simulation

Figure 4: Comparison of Distributions of Intersection Entering Traffics

For simulation results, since many vehicles stay on the roads when the calculation finished, the further the intersections, the fewer the intersection entering traffics are estimated. General speaking, the proposed model can give out an acceptable estimation result.

5. Concluding Remarks

In this research, an analytical approach is proposed to describe intersection entering traffic distributions on a grid network. Both a discrete and a continuous model is developed and the estimation results are validated by a mature commercial simulation platform. The proposed approach is full analytical with explicit structure; its influence on both the academic and practical fields is expected.

In future, the author will concentrate on two parts: 1) the case of a full OD table and 2) the case of non grid networks. The major purpose of the future research will also be insist on, to describe a network using the network parameters only.

Acknowledgements

The SOUND/4U Simulation platform kindly provided by the i-Transport Lab. Co., Ltd (ITL) is highly appreciated. Thank you very much, Mr. Hanabusa, for all the kindly support! The kindly suggestions and encouragement from Prof. Nakamura, Prof. Okamura and other members of the transportation lab at Yokohama National University is also highly appreciated.

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