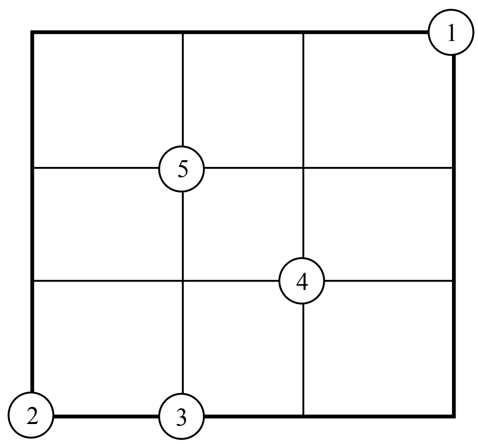
VEHICLE ROUTING PROBLEM WITH SOFT TIME WINDOWS AND PATH CHOICE

Ali Gul Qureshi**, Sideney Schreiner**, Eiichi Taniguchi*** and Tadashi Yamada****

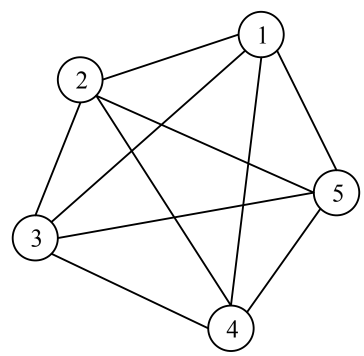
1. Introduction

The aim of city logistics is to fully optimize the urban freight movement with respect to the public and private costs and benefits including the fuel consumption and environmental costs[1]. Most of the urban freight movement is primarily based on the truck traffic, which contributes to many urban traffic related problems such as traffic congestion, parking issues, accidents and environmental problems. To counter these issues, city logistics suggests many measures and policies to reduce the number of trucks, their mileage and their idling times. The Vehicle Routing and scheduling Problem with Time Windows (VRPTW) is a typical route optimization technique employed in city logistics. It is also used to evaluate other city logistics measures relating to infrastructure planning and development (for example, optimal location of logistics terminals[2]) and to infrastructure management (such as, cooperative delivery systems[3]).

In a real logistics instance, depot and pickup/delivery points (customers) may not be directly connected with each other using a single link of the urban road network. Rather their inter-connectivity may contain various links possessing a variety of network attributes[4] and other dissimilarities such as, class of the road (highway or street, etc.), geometric aspects (width and number of lanes, etc.), tolled or un-tolled roads and municipal regulations (truck bans, etc.). Although, real urban road networks are much complex, this situation is depicted in Figure 1(a) for a grid shaped urban road network. Other node numbers except the depot (node 1) and customers (nodes 2 to 5), are not shown for the sake of clarity. On the other hand, the Vehicle Routing and scheduling Problem with Time Windows (VRPTW) is defined on a complete graph (i.e., a fully connected network), where each arc replaces a combination of actual urban road network links as shown in Figure 1(b). To fill this gap between practical and theoretic instances, a pre-processing stage of path choice between depot and customers and customer to customer is introduced. The path choice stage also faces some dilemmas such as, should it only be based on the myopic strategy of the shortest paths with respect to distance or travel time? or a network knowledge shall also be incorporated.



(a) Urban road network



(b) A VRPTW instance

Figure 1 An urban road network and its transformation in a VRPTW instance

Most of the city logistics-related research utilizes the soft time windows variant of the VRPTW (abbreviated as VRPSTW), whereby an early arrival is penalized equal to the opportunity cost lost due to waiting and a high late arrival penalty is imposed on delayed service (delivery/pickup of goods). However, most of the exact optimization research has been directed towards the Vehicle Routing and scheduling Problem with Hard Time Windows (VRPHTW), though it lacks the practicality found in real life problems because of stringent restriction on even slight delays . Furthermore, these exact techniques allow waiting at no penalty cost in VRPHTW, which results in more waiting time as compared to the cases when waiting is penalized[5] or even when waiting is allowed at no cost along with penalized late arrivals[6]. On the other hand, mostly heuristics (approximate) solutions have been used for the VRPSTW in city logistics-related research

* Key words: logistics planning, logistics, path choice, soft time windows
** Student Member of JSCE, Dr. Eng., Post Doctoral Researcher, Faculty of Engineering, Kyoto University.
C-1 Kyotodaigaku Katsura, Nishikyo, Kyoto 615-8540, Tel. 075-383-3231, Fax. 075-950-3800.
*** Fellow Member of JSCE, Dr. Eng., Faculty of Engineering, Kyoto University.
C-1 Kyotodaigaku Katsura, Nishikyo, Kyoto 615-8540, Tel. 075-383-3229, Fax. 075-950-3800.
**** Full Member of JSCE, Dr. Eng., Faculty of Engineering, Kyoto University.
C-1 Kyotodaigaku Katsura, Nishikyo, Kyoto 615-8540, Tel. 075-383-3230, Fax. 075-950-3800.

(for example see, Yamda et al.,[2], Duin et al.,[7]). This paper presents an integration of an efficient path choice model that considers the drivers' perception and various network related attributes, and an exact solution approach for the VRPSTW based on the Dantzig-Wolfe decomposition.

2. Literature Review

Usually the path choice stage is based on the shortest path algorithms considering either travel distance or travel times such as Dijkstra's algorithm[8), 9)]. Ando *et al.*,[10] used an ant colony system to obtain paths for construction of the VRPSTW instance's graph. Probably the most related work to our approach is the GIS-based decision support system "Map-Route" due to Ioannou *et al.*,[11] that considered the Euclidean distances at the first stage and solved the VRPTW. The VRPTW routes are then mapped onto the real urban road network using shortest paths between the customers. The solution is then modified using knowledge based rules, which favour the main city arteries and roads with low traffic volumes as compared to regular/narrow roads and roads with high traffic volumes. Our path choice model considers the perceived cheapest cost paths at the pre-processing stage. The model represents the drivers' perception of travel cost and shortest path based on a variety of network related attributes.

Many researchers have used heuristic techniques for the soft time windows environment with the idea to reduce the number of vehicles or overall solution cost[12]. However, this study considers an exact solution approach for the VRPSTW based on the Dantzig-Wolfe decomposition (also called column generation), which has been successfully used for the exact solutions of the VRPHTW, in past. It decomposes the VRPTW in a set partitioning master problem and an Elementary Shortest Path Problem with Resource Constraints (ESPPRC) as its subproblem[13]. While the master problem remains the same, many researchers have worked with various shortest path variations as subproblems in their column generation schemes for the VRPHTW. For instance, Desrochers et al.[14] presented the first column generation based approach for the VRPHTW, using 2-cycle elimination while solving the relaxed shortest path subproblem. Irnich and Villeneuve[15] used a relaxed shortest path subproblem with k-cycle elimination (with $k \geq 3$) in their column generation scheme for the VRPHTW. The exact solution method used to solve the VRPSTW in this paper, also relies on the set partitioning formulation of the VRPTW but a different subproblem is solved as described in the next section.

Unlike our column generation scheme that iteratively adds the feasible routes of marginal negative cost from the subproblem to the set partitioning master problem, Calvete et al.[16] exploited goal programming to enumerate all the feasible routes in the first stage and then used the set partitioning problem to solve the VRP with soft time windows with a heterogeneous fleet and multiple objectives. In a similar approach, Fagerholt[17] solved a ship-scheduling problem with soft time windows. The Traveling Salesman Problem with Capacity, Hard Time Windows and Precedence Constraint (TSP-CHTWPC) was used to enumerate all feasible routes and then their schedules were optimized using soft time windows, before using a set partitioning problem.

3. VRPSTW Model Formulation

The VRPSTW is defined on a directed graph $G = (V, A)$. The vertex set V includes the depot vertex 0 and the set of customers $C = \{1, 2, \ldots, n\}$. The set K represents the set of identical vehicles with capacity q stationed at the depot. The arc set A consists of all feasible arcs (i, j), $i, j \in V$. A cost c_{ij} and a time t_{ij} is associated with each arc $(i, j) \in A$. The time t_{ij} includes the travel time on arc (i, j) and service time at vertex i, while a fixed vehicle utilization cost is added to all outgoing arcs from the depot. With every vertex of V associates a demand d_i where $d_0 = 0$, and a time window $[a_i, b_i]$, which represents the earliest and the latest desired service start times.

This study incorporates the soft time windows constraint by extending the latest service start time b_i to b_i' as shown in Figure 2. The maximum penalty is considered equivalent to the cost of a dedicated single vehicle route only serving the concerned customer. Taking c_l as the unit late arrival penalty cost, b_i' can be defined as eq. (1); whereas the arrival time-dependent cost is formulated as eq. (2).

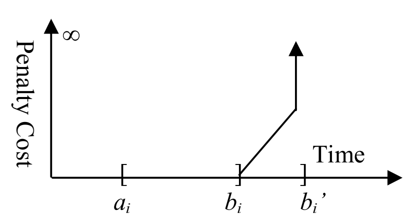


Figure 2 Penalty cost function for the VRPSTW

$$b_i' = \min\left[b_0 - t_{i0},\ \ b_i + \frac{(c_{0i} + c_{i0})}{c_l} \right] \tag{1}$$

$$c_{ijk}' = \begin{cases} c_{ij}, \text{if } s_{jk} \leq b_j \\ c_{ij} + c_l(s_{jk} - b_j), \text{if } s_{jk} > b_j \end{cases} \tag{2}$$

Using the Dantzig-Wolfe decomposition VRPSTW is formulated as

$$\min \sum_{p \in P} c_p y_p \tag{3}$$

subject to

$$\sum_{p \in P} a_{ip} y_p = 1, \qquad \forall i \in C \tag{4}$$

$$y_p \in \{0,1\} \qquad \forall p \in P \tag{5}$$

The master problem (eq.(3)- eq.(5)) consists of selecting a set of feasible paths of minimum cost generated by the Elementary Shortest Path Problem with Resource Constraints with Late Arrival Penalties (ESPPRCLAP) as the subproblem. Where, y_p takes value 1 if the path p is selected and 0 otherwise. a_{ip} represents the number of times path p serves customer i. P is the set of all feasible paths. A vector of dual variables (prices) is also generated in the master problem represented by π_i. Considering all identical vehicles and the reduced cost network, ESPPRCLAP can be formulated as:

$$\min \sum_{(i,j) \in A} \overline{c'_{ij}} x_{ij} \tag{6}$$

subject to

$$\sum_{i \in C} d_i \sum_{j \in V} x_{ij} \le q \tag{7}$$

$$\sum_{j \in V} x_{0j} = 1 \tag{8}$$

$$\sum_{i \in V} x_{ih} - \sum_{j \in V} x_{hj} = 0 \qquad \forall\, h \in C \tag{9}$$

$$\sum_{i \in V} x_{i0} = 1 \tag{10}$$

$$s_i + t_{ij} - s_j \le (1 - x_{ij}) M_{ij} \qquad \forall (i,j) \in A \tag{11}$$

$$a_i \le s_i \le b'_i \qquad \forall\, i \in V \tag{12}$$

$$x_{ij} \in \{0,1\} \qquad \forall (i,j) \in A \tag{13}$$

The model contains two decision variables x_{ij} which determines whether arc (i, j) is used in the solution ($x_{ij} = 1$) or not ($x_{ij} = 0$) and s_j that determines the service start time at vertex j by the vehicle. Objective equation (6) minimizes the reduced cost of the path. M_{ij} is a big constant. Constraint (7) is capacity constraint. Constraints (8)-(10) are flow conservation constraints. Constraint (11) is time windows constraint specifying that if a vehicle travels from i to j, service at j can not start earlier than that at i. Constraint (12) specifies the service start time at all vertices must be within their soft time windows $[a_i, b'_i]$. Finally, the integrality of the decision variables is ensured using constraint (13).

The ESPPRCLAP subproblem is solved on the same network as VRPSTW, with the arcs cost being reduced by the corresponding dual variables (prices) (eq. (14)) generated in the master problem.

$$\overline{c'_{ij}} = c'_{ij} - \pi_i \qquad \forall\, i \in V \tag{14}$$

4. Path Choice Model

The travel cost matrix c_{ij} that appears in eq. (2) and again in its reduced version in the objective function of the ESPPRCLAP (eq. (6)), is based on the path choice between the depot and customers and customers to customers. Instead of the shortest paths, merely based on the travel time or travel distance, this study considers the cheapest path concept based on the generalized cost (GC) that represents the cost perceived by the user to travel through one link. The form of the GC for the a-th link is shown in eq. (15). The first term in eq. (15), considers the travel cost of the link based on the user's value of time (VOT), and the fuel consumption, which is obtained using the average engine performance in liters per kilometer traveled, the fuel cost in monetary units per liter, and the link length in kilometers.

$$GC_a = (Cons_a + VOT \times Time_a) \times \prod_k \beta_k \tag{15}$$

Where:
β_k is the parameter for the k-th network attribute (e.g. road width).

Table 1 lists the network attributes considered in eq. (15). The corresponding parameters (β_k) were estimated for an extensive road network of Tokyo Metropolitan, consisting of 9231 nodes and 25062 links. Route duplication methodology[18] is used for the calibration of route choice model (eq. (15)) using 597 actual truck routes. The VOT was estimated as 90 yen/minute for non-container trailer trucks; estimated values of other corresponding parameters for the network attributes, are also listed in Table 1.

Table 1 Network Attributes Estimated for Road Network of Tokyo Metropolitan

	Attribute	Condition for $\beta_k = 1$	Condition for $\beta_k = 0$	Estimated values of β_k
1	Tolled road	tolled link	otherwise	0.6
2	CBD area	inside CBD area	otherwise	1.0
3	Ring Road 7	inside ring road7	otherwise	0.9
4	Number of lanes	= 4 lanes	otherwise	0.6
5	Heavy truck permission	yes	otherwise	0.8
6	Tall truck permission	yes	otherwise	0.7

5. Results and Discussion

The proposed exact solution of the VRPSTW with the route choice model, will be applied on a real life logistics instance based on the Tokyo Metropolitan road network. A comparison will be made with the traditional approach of using shortest paths. The computational results and conclusions will be presented at the conference.

References:

1) Taniguchi, E., *et al.*: City Logistics; Network Modeling and Intelligent Transport Systems, Pergamon, Oxford, 2001.
2) Yamada, T., *et al.*: Co-operative vehicle routing model with optimal location of logistics terminals, in Taniguchi, E., & Thompson, R. G., eds., City Logistics II, pp. 139-153, Institute for City Logistics, Japan, 2001.
3) Qureshi, A., & Hanaoka, S.: Analysis of the effects of cooperative delivery system in Bangkok, in Taniguchi, E., and Thompson, R.G., eds., Recent Advances in City Logistics Proceedings of the 4th International Conference on City Logistics, Langkawi, Malaysia, 2005, 59-73, Elsevier, Oxford, 2005.
4) Hyodo, T., *et al.*: Modeling of bicycle route and destination choice behavior for bicycle road network plan, Transportation Research Record, No. 1705, pp. 70-76, 2000.
5) Qureshi, A.G., *et al.*: Effects of relaxing time windows on vehicle routing and scheduling, Infrastructure Planning Review, Vol. 24, pp. 927-936, 2007.
6) Qureshi, A.G., *et al.*: A comparison of environmental impacts of hard time windows and semi soft time windows using exact routing solution, Infrastructure Planning Review, Vol. 25, pp. 451-462, 2008.
7) Duin, van.J.H.R., *et al.*: Real time simulation of auctioning and re-scheduling process in hybrid freight markets, Transportation Research Part B, Vol. 41, pp. 1050-1066, 2007.
8) Qureshi, A.G., *et al.*: Elementary shortest path problem with resource constraints and time dependent late arrival penalties, JSCE Journal Division D, Vol.63, pp. 579-590, 2007.
9) Prindezis, N., *et al.*: A business-to-business fleet management service provider for central food market enterprises, Journal of Food Engineering, Vol. 60, pp. 203–210, 2003.
10) Ando, N., *et al.*: Evaluation on route travel time with mean-variance approach (in Japanese), Proceedings of the 28th JSTE Annual Conference, pp. 173-176, 2008.
11) Ioannoue, G., *et al.*: A GIS-based decision support system for intra-city vehicle routing with time windows, The Journal of the Operational Research Society, Vol. 53, pp. 842- 854, 2002.
12) Taniguchi, E., & Thompson, R.G.: Modeling city logistics, Transportation Research Records, Vol. 1790, pp. 45-51. 2002.
13) Feillet, D., *et al.*: An exact algorithm for the elementary shortest path problem with resource constraints: Application to some vehicle routing problems, Networks, pp. 216-229, 2004.
14) Desrochers, M., *et al.*: A new optimization algorithm for the vehicle routing problem with time windows, Operations Research, Vol. 40(2), pp. 342-354, 1992.
15) Irnich, S., & Desaulniers, G.: Shortest path problems with resource constraints, in G. Desaulniers, Desrosiers, J., & Solomon M.M., eds., Column Generation, Springer, New York, pp. 33-65, 2005.
16) Calvete, H.I., *et al.*: A goal programming approach to vehicle routing problem with soft time windows, European Journal of Operational Research, Vol. 177, pp. 1720-1733, 2007.
17) Feillet, D., *et al.*: An exact algorithm for the elementary shortest path problem with resource constraints: Application to some vehicle routing problems, Networks, pp. 216-229, 2004.
18) Schreiner, S., *et al.*: A study on the influence of network attributes on the route choice behavior, Journal of the Eastern Asia Society for Transportation Studies, Vol. 7, pp. 642-653, 2007.