COMPARISON OF CORDON AND OPTIMAL LOCATION ROAD PRICING USING GENETIC ALGORITHM

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1. INTRODUCTION
As economies develop and technology improves, the availability of vehicles increases and car ownership in urban areas rises. As a result, many cities have been suffering from heavy traffic congestion in recent years. The sharp increase in car ownership has led to a series of problems in metropolitan areas, mostly relating to congestion, safety, and environmental issues. Researchers have looked into many technology and policy alternatives to deal with this situation. As one method of traffic control, road pricing had attracted much attention. London, Singapore, and Stockholm have all implemented various forms of congestion pricing and obtained effective results. Through road pricing, changes have been achieved in traffic demand, traffic destination, departure times, and vehicle routing, while unnecessary trips have been reduced and the overall operating conditions of the road network have improved greatly.

The idea of road pricing derives from the viewpoint that road users are actually paying less than the cost (Marginal Cost) that should be paid. By implementing a charging method, traffic conditions move closer to the optimal state; this kind of pricing is called first-best pricing. Under the assumption that all links can be charged, first-best road pricing can be solved by system optimization assignment. In fact, however, first-best pricing is not practically appealing, because of high operational costs and poor public acceptance. As a result, there are no first-best pricing schemes have been implemented up to now.

In recent years, more practical methods based on second-best road pricing have received considerable attention. Yang and Huang (1997) investigated the principle of marginal cost pricing and presented a general model of link congestion tolls with elastic demand. This model can be used not only in first-best pricing but also in alternative second-best pricing schemes. Verhoef (2002) proposed a method for determining second-best toll levels in a study in which only a subset of all links can be charged. Shepherd et al. (2004) showed that a genetic algorithm (GA) approach can be used for large-scale networks, but the paper presented only a mini network case with few links. Sumalee et al. (2005) developed a GA-based method and compared judgmental and optimal road pricing cordons. Zhang et al. (2004) and Sumalee (2004) presented an investigation of cordon-based second-best congestion pricing, with both papers focusing on the design of the cordon using GA. Zhang et al. (2004) looked into variable demand equilibrium under second-best pricing with a parameterized pricing scheme. Santos (2004) simulated cordons in eight towns and estimated the impacts of schemes using SATURN (Simulation and Assignment of Traffic to Urban Road Networks); the paper compared the benefits of single cordon pricing and double cordon pricing. Maruyama et al. (2007) compared cordon and area-based road pricing using an elastic demand model. Kanamori et al. (2008) used a semi-dynamic model of stochastic user equilibrium (UE) to evaluate the effect of cordon road pricing. In most cases, this research work focused mainly on numerical case studies using small-scale networks.

In fact, for a given toll level on a certain network, it is unknown where charges should be imposed to obtain the optimal effect. Cordon pricing for a central business district is a practical approach but it might not be an optimal one. The objective of this study is to compare cordon pricing at a certain toll level with a system of optimal toll points obtained using GA-based bi-level programming. The study is based on the full Nagoya metropolitan area road network. In order to compare the effect of cordon pricing with optimal toll points, a cordon pricing scheme with a uniform toll level and two optimized schemes with various link sets are examined.

The paper is organized as follows. In the next section, we will discuss the bi-level programming model for second-best road pricing and explain how a GA is used to solve the problem. Details of the GA coding method will be given, since the approach is different to other work mentioned above. In section 3, the Nagoya metropolitan area case study will be described, including a comparison between the cordon pricing scheme and the optimal toll scheme. Section 4 presents the results of the case study and offers an analysis, and our final conclusions are given in section 5.

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2. MODEL FORMULATION

Tolls charged at optimal points can be regarded as link congestion tolls in the network, so the problem can be depicted as a bi-level second-best pricing model that is subject to standard traffic equilibrium. The problem can be solved using a GA, an algorithm that has proven very useful in solving large-scale network questions. In this study, because optimal toll points and levels are determined simultaneously, the method of coding the GA is a little different from the general method used for optimal points with a uniform toll level. Details of the differences are given later in this section.

2.1 Upper level

The upper level model can be evaluated to maximize the social welfare gained from the tolled network.

\[ \text{Max } SW = UB - SC \]  

Here user benefit (UB) is evaluated by the following formulation:

\[ UB = \sum_{w \in W} \int_{x=0}^{d_w} D_w^{-1}(x) \, dx \]  

where \( W \) is the OD pair, \( W \) is the set of OD pairs, \( d_w \) is the demand between OD pair \( w \), and \( D_w^{-1}(x) \) is the Inverse Demand function, whose value depends on the travel cost. And the social cost (SC) is formulated as:

\[ SC = \sum_{a \in A} v_a t_a(v_a) \]  

where \( a \) is the link, \( A \) is the set of links, \( v_a \) is the flow on link \( a \) belonging to links set \( A \), and \( t_a(v_a) \) is the travel cost on link \( a \), which is a function of link flow. Then the social welfare can be expressed as following formulation:

\[ SW = UB - SC = \sum_{w \in W} \int_{x=0}^{d_w} D_w^{-1}(x) \, dx - \sum_{a \in A} v_a t_a(v_a) \]  

The demand function is set as:

\[ D_{rs}(C_{rs}) = D_{rs}^0 \exp \left[ \sigma (1.0 - \frac{C_{rs}}{C_{rs}^0}) \right] \]  

where \( D_{rs}^0 \) is the demand for minimum generalized travel cost during assignment, \( C_{rs}^0 \) is the minimum generalized travel cost, and \( \sigma \) is the price elasticity of demand.

2.2 Lower level

The lower level model is the user equilibrium with variable demand:

\[ \text{Min } Z(x,d) = \sum_a \int_0^{v_a} t_a(\omega) \, d\omega - \sum_{rs} \int_0^{d_{rs}} D_{rs}^{-1}(\omega) \, d\omega \]  

Subject to

\[ f_k^{rs} \geq 0 \]  

\[ d_{rs} \leq \bar{d}_{rs} \]  

\[ d_{rs} = \sum_k f_k^{rs} \]  

In this formulation, \( f_k^{rs} \) is the flow on the \( k \)th path between OD pair \( rs \), \( d_{rs} \) is the demand between the OD pair, \( \bar{d}_{rs} \) is the upper bound of demand between origin \( r \) and destination \( s \), \( D_{rs}^{-1}(x) \) is the Inverse Demand function, whose value depends on the travel costs. (Because the algorithm for solving the variable demand user equilibrium problem is very complex, it cannot be depicted here in details. Full details of the algorithm for user equilibrium with variable demand can be found in Urban Transportation Network, Chapter 6, Sheffi, 1985.) The form of travel cost is evaluated as:

\[ t_a(x_a) = t_a \left[ 1 + \alpha (v/c)^b \right] + \text{toll} / \text{VOT} \]  

In this formulation, tolls are set in the uniform level. With respect to a certain link, once this link was selected as a toll link, the monetary cost will be a certain amount, and details will be given in the following section.

In order to make the problem practicable, we choose several subsets of all links in the network as candidate toll links; only links in these subsets can be selected for tolls. These subsets are determined empirically and we place links into various combinations as road pricing schemes. When a link is selected for one combination, it means the link is charged.
For a practical network, the total number of combinations is huge, even if are selected only from a subset. Going through all combinations using an optimal result finding procedure would be very difficult. A genetic algorithm (GA) is able to determine the optimal result because of its favorable procedure of natural selection. The GA procedure used is outlined below.

**Step 1.** (Initialize population) Randomly select links from the subset and randomly generate toll levels

**Step 2.** (Fitness evaluation) Employ the model to obtain the optimum scheme from combination of toll links

**Step 3.** (Selection) Select schemes with higher values of social welfare for reproduction

**Step 4.** (Crossover and mutation) Pair the populations and randomly exchange the toll links between pairs. Randomly change some toll links and toll levels

**Step 5.** (Verification of halt criterion) If the convergence criterion has not been satisfied, go to Step 2 and continue with the next generation; otherwise stop the program

The detailed GA process used in this study is shown in the figure 1. During the initialization procedure, information about the network and its subsets for 3 cases, OD data, and model parameters are all read by the program, and the first generation of chromosomes (schemes) are generated randomly. The fitness of all schemes is evaluated in the next procedure, where lower-level programming plays the core role. With respect to a certain scheme, there will be an assignment of variable demand user equilibrium and after this assignment the social welfare, user benefit, and social cost will be calculated and stored. Through the procedure of selection, crossover and mutation, chromosome of higher fitness get more chance to survive the next generation, the procedure move to circles. At last when the termination criteria is satisfied the GA procedure will end and output results, which consists of combination of optimal points, the social welfare etc.

![Figure 1: GA procedure](image1.png)

Figure 2 shows the detailed structure of one chromosome for inner cordon, where one chromosome represents a toll scheme, a combination of toll links. The length of each chromosome is the same number as the cordon toll links number, as for the inner cordon, the length is 20, for the middle cordon the length is 42 and for outer cordon it is 50. Every chromosome consists of two rows, the first row is the gene index row and the second row represents the toll link index. For example, in figure 2 the first entry in the second row there is a number of 231, which means that the link with ID 231 in the subset is selected as the toll link.

![Figure 2: Structure of chromosome](image2.png)

**NAGOYA METROPOLITAN AREA NETWORK CASE STUDY**

As a case study, the model was applied to the Nagoya metropolitan area network. This network comprises 4209 one-direction links and there are at total of 279 zones in the city area. Personal trip (PT) data collected in 1996 is used to simulate traffic flows on the network. The unit of traffic flow placed on the network is the standard Passenger Car Unit (PCU) and we assume that the value of vehicle driver time is 83.4 Japanese Yen per minute. In this case study, we set only a uniform toll level of 300 Japanese Yen, it means once a link is selected as toll link, it is tolled 300 Japanese Yen.
In order to compare optimized toll points with cordon pricing, we introduce three cases, as detailed in the paragraphs that follow.

**Case A.** **Cordon pricing.** In this case we choose the 3 cordon areas of Nagoya central urban area. The small cordon is central business district of Nagoya city, the large cordon is the central urban area of Nagoya and the one between these 2 cordons is the middle cordon, because when vehicles enter these 3 areas there are respectively 20, 50 and 42 possible toll links, we then choose schemes with the same number of links for comparison in the next 2 cases.

**Case B.** **Optimal toll points for artery road subset pricing.** The subset in this case is selected from among the artery roads in the central urban area of the Nagoya road network. There are roads with at least 4 lanes in the Nagoya city area and that are not expressway links or existing toll links. The total link number of this subset is 1198, this means there are 1198 potential links can be chosen as the toll links.

**Case C.** **Optimal toll points for central urban area road subset pricing.** The subset in this case is selected from the central urban area of the Nagoya road network. It is easy to know that the subset of Case B is a subset of Case C and in Case C also the links in the subset exclude expressway links and existing toll links. The total link number of this subset is 2096.

Figure 3 shows the location and range of the three subsets taken from the road network of Nagoya city. The top-left image shows the location and range of cases in the Nagoya road network, all of the cases are carried out in the Nagoya central metropolitan area. The Case A subset is the set of links within the 3 cordon lines of the top-right image, the red line is inner cordon, the pink one is the middle cordon and the brown one is outer cordon. All links in the combination are one-direction links and vehicles are charged only upon entering the cordon. The Case B subset consists of the bold blue links in the left-lower image. These links are artery roads in the central city area with at least 4 lanes, Expressway links and existing toll links are excluded. The Case C subset consists of the bold green links in the right-lower image. All links in the central city area are included, while as with Case B the expressway links and existing toll links are excluded too.

![Figure 3: Cases of the study](image)

**RESULT OF CASE STUDY**

With the introduction of road pricing schemes on the network, traffic flows are affected in various ways: most drivers try to change their route, some adjust their departure time or move to another mode of transport, such as subway. But from the point of traffic conditions on the network as a whole, it moves toward the system optimum. In other words, the network becomes less congested and users of the network benefit.

Results were obtained for the three cases by applying the GA-based bi-level programming model. As anticipated, the results showed that the cases with optimal toll points and levels achieved better results than the cordon pricing case. In figure 4, we can see that for all of the cordons Case A results in the lowest value of social welfare; both Cases B and C
yield better values of social welfare than cordon pricing. This comparison demonstrates that, although cordon pricing schemes are more feasible and easier to realize, schemes based on optimal toll points are more attractive as regards social welfare.

Table 1 shows the detailed result in terms of number. The result of different toll link numbers share the same trend in 3 cases, all results of Case B and Case C are better than cordon pricing cases with the same toll link numbers. With respect to link numbers, in the case study it shows that with more toll links the social welfare tend to be higher, which can be forecasted, but the values of social welfare in 20 links scheme in Case B and Case C are better than the cordon scheme of 42 and 50 links, which can be concluded easily that the cordon pricing scheme leads inefficiency in expenditures necessary for the implementation of road pricing. From the result of the case study, we can also see that the value of Case B is only a little lower than the value in Case C, so it can also conclude that the optimal location schemes can get satisfied affection being implemented on artery road subset network instead of total urban area road subset network.

With respect to Case B and Case C, the values is very close and we assumed that it can also use less links to achieve similar effectiveness, and we did compute more optimal schemes with less toll links for Cases C, the toll link numbers are set from 8 to 20. Figure 5 shows the calculated results, the trend of different toll link number is increasing and it shows that the peak values are around 16-18 links.

Figure 6 shows the example of optimal result for Case C of 20 links with the same number of inner cordon. The figure shows the combination of optimal links in the network, with the toll links in the scheme marked in bold red style. In this figure, the green lines are artery roads subset of central metropolitan area. It is clear that most of the toll links tend to be located in the central part of the urban network or on arterial corridors, and non-artery links can hardly be chosen as toll links. In this optimal result, the whole 20 toll links are all artery links, this is common that we can see the

<table>
<thead>
<tr>
<th>Toll link numbers</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>2.0468112E+08</td>
<td>2.0630126E+08</td>
<td>2.0638543E+08</td>
</tr>
<tr>
<td>42</td>
<td>2.0531886E+08</td>
<td>2.0646006E+08</td>
<td>2.0652708E+08</td>
</tr>
<tr>
<td>50</td>
<td>2.0540568E+08</td>
<td>2.0656883E+08</td>
<td>2.0663627E+08</td>
</tr>
</tbody>
</table>

Figure 4: Comparison of the three cases

Figure 5: Comparison between cases of different numbers of toll links

Figure 6: Example of optimal result for Case C of 20 links with the same number of inner cordon.
results of Case B and Case C in all of different number toll links schemes appear very similar.

Figure 6: Results for Case C

CONCLUSION

In this study, we develop a bi-level model based on a genetic algorithm to determine optimal combination of toll points. As a case study, the model was applied to the actual large-scale road network of the Nagoya metropolitan area. In order to compare the effectiveness of the method with other policy options, we introduced three cases for the case study: the cordon pricing case, the optimal combination of toll points case with tolls on central city area artery roads only, and the optimal combination of toll points pricing case with tolls on any road in the central urban area road network. This comparison between the cordon pricing case and the two optimal toll points cases demonstrated that the latter cases performed better than the former one, although cordon pricing schemes are more feasible and easier to realize, schemes based on optimal toll points are more attractive as regards social welfare.

REFERENCES