HYBRID INSERTION HEURISTICS FOR VEHICLE ROUTING PROBLEM WITH SOFT TIME WINDOWS

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1. Introduction

Industrial growth and expanding employment opportunities have led to the urban-oriented economic development in many countries. Demand of transportation, both in terms of passengers as well as for freight is also increasing in and around these big urban conurbations. A high proportion of total goods movement occurs within cities[1)], and most of this movement is based on road transport. Traffic congestion, noise, vibrations, generation of NOx, SPM, CO_2 and other environmental problems, crashes and loading and unloading on the street side are typical problems caused by the road-based freight transport in urban areas.

Such freight movement related problems has magnified the need for research in the field of city logistics. The Vehicle Routing and scheduling Problem with Time Windows (VRPTW) can be used as a tool for evaluating many city logistics schemes. For example, the VRPTW could be used in the analysis of cooperative delivery systems and ideal location of logistics terminals, which belong to infrastructure planning and management problems in city logistics. Depending on the nature of the time windows, the VRPTW is further expanded to the Vehicle Routing and scheduling Problem with Hard Time Windows (VRPHTW) and the Vehicle Routing and scheduling Problem with Soft Time Windows (VRPSTW).

Most of the exact optimization research has been directed towards the hard time windows variant, though it lacks the practicality found in real life problems. Furthermore, these exact techniques allow waiting at no penalty cost when a vehicle arrives earlier than the start time of service. This results in more waiting time as compared to the case when waiting is penalized[2)]. On the other hand, soft time windows are often encountered in practical freight transport, yet mostly heuristics (approximate) solutions are used for the VRPSTW in city logistics related research.

Recently column generation techniques have been used efficiently for the VRPHTW, successfully solving large size problems as well as significantly reducing the required computational efforts. This could be desirable to develop exact solution approaches for the soft time windows variant as well. However, complex soft time windows constraints and time dependent costs have been the greatest barriers in the way of these efforts.

This paper presents a hybrid solution technique for the VRPSTW embedding a heuristics solution technique (Insertion heuristics) within the exact solution techniques (Column Generation) in order to improve the solution quality and to reduce computational times. The basic idea is to replace the NP-hard subproblem of column generation scheme i.e. the Elementary Shortest Path Problem with Resource Constraint (ESPPRC) by an insertion heuristics, which can efficiently handle complex soft time windows constraints. The shadow prices generated by column generation method are utilized to find the reduced costs, which are then used in the insertion heuristics to find negative reduced cost routes (columns) for the master problem.

LITERATURE REVIEW

Many researchers have used heuristic techniques for the soft time windows environment with the idea to reduce the number of vehicles or overall solution cost[3)]. Balakrishnan[4)] described three simple heuristics for the VRPSTW based on the nearest neighbour, Clarke-Wright savings and space-time rules. A detailed overview of the VRPSTW and its solution techniques can be found in Taniguchi *et al.*, [1)].

The Dantzig-Wolfe decomposition of the VRPTW results in the set partitioning master problem and an Elementary Shortest Path Problem with Resource Constraints (ESPPRC) as its subproblem[5)]. However, many researchers have worked with various shortest path relaxations as subproblems to solve the VRPHTW[6), 7)]. Instead of the NP-hard ESPPRC, this study employs an insertion heuristic subproblem to solve the Dantzig-Wolfe decomposition of the VRPSTW. Insertion heuristics are one of the earliest route-building heuristics for VRPTW[8)]; these have also been used in the initialization procedure for route improvement heuristics and for metaheuristics developed for the VRPTW.

Set partitioning linear optimization has been employed in many heuristics for the VRPSTW, in past. For example, Rochat and Taillard[9)] used a heuristics approach by first generating many candidate routes using intensified and

* Key words: logistics planning, Logistics, hybrid heuristics, soft time windows
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diversified tabu search and then used set partitioning linear programming. Calvete *et al.*[10] exploits goal programming to enumerate all the feasible routes and then used a set partitioning problem to solve the VRP with soft time windows with heterogeneous fleet and multi objectives. Alvarenga *et al.*[11] used a specialized genetic algorithm for the VRPHTW to generate routes to be optimized with set partitioning formulation at the end of the algorithm.

All the above-cited references used the set partitioning linear programming after enumerating some or all possible candidate routes for the VRPSTW, while this study utilizes the useful dual information, i.e. shadow prices, obtained every time a set partitioning linear program is solved. The set partitioning master problem and the heuristic subproblem are solved in cycles. The shadow prices are generated in master problem, which guide the optimization in the heuristic subproblem that in return provides the routes with negative reduced cost to augment the set partitioning linear program. A similar hybridization approach is developed by Qureshi *et al.*[12] considering genetic algorithms as the heuristics subproblem. An excellent review of the heuristic methods applied to the VRPHTW and the VRPSTW can be found in Braysy and Gendreau[13), 14)].

2. Model Formulation

The VRPSTW is defined on a directed graph $G = (V, A)$. The vertex set V includes the depot vertex 0 and the set of customers $C = \{1, 2, . . ., n\}$. The set K represents the set of identical vehicles with capacity q stationed at the depot. The arc set A consists of all feasible arcs (i, j), $i, j \in V$. A cost c_{ij} and a time t_{ij} is associated with each arc $(i, j) \in A$. The time t_{ij} includes the travel time on arc (i, j) and service time at vertex i, while a fixed vehicle utilization cost is added to all outgoing arcs from the depot. With every vertex of V associates a demand d_i where $d_0 = 0$, and a time window $[a_i, b_i]$, which represents the earliest and the latest possible service start times.

This study incorporates the soft time windows constraint by extending the latest and earliest possible arrival time a_i to a_i' and b_i to b_i' as shown in Figure 1. The maximum penalty is considered equivalent to the cost of a dedicated single vehicle route only serving the concerned vertex. Taking c_l and c_e as the unit late arrival penalty cost and the unit early arrival penalty cost, respectively, a_i' and b_i' can be defined as:

$$a_i' = \max\left[0, a_i - \frac{(c_{0i} + c_{i0})}{c_e}\right] \tag{1}$$

$$b_i' = \min\left[b_0 - t_{i0},\ \ b_i + \frac{(c_{0i} + c_{i0})}{c_l}\right] \tag{2}$$

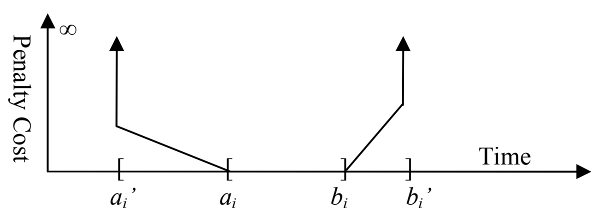

Figure 1 Penalty cost function of the VRPSTW

Even though arrival is allowed earlier than the start of time windows, a vehicle has to wait until a_i to start the service. This waiting time is considered as the early arrival penalty, as c_e is usually set equal to vehicle operating cost (VOC).

The VRPSTW can be mathematically formulated as:

$$\min \sum_{k \in K} \sum_{(i,j) \in A} c'_{ij} x_{ijk} \tag{3}$$

subject to

$$\sum_{k \in K} \sum_{j \in V} x_{ijk} = 1 \qquad \forall\, i \in C \tag{4}$$

$$\sum_{i \in C} d_i \sum_{j \in V} x_{ijk} \le q \qquad \forall\, k \in K \tag{5}$$

$$\sum_{j \in V} x_{0jk} = 1 \qquad \forall\, k \in K \tag{6}$$

$$\sum_{i \in V} x_{ihk} - \sum_{j \in V} x_{hjk} = 0 \qquad \forall\, h \in C, \qquad \forall\, k \in K \tag{7}$$

$$\sum_{i \in V} x_{i0k} = 1 \qquad \forall\, k \in K \tag{8}$$

$$a_i' \le s_{ik}' \le b_i' \qquad \forall\, i \in V, \qquad \forall\, k \in K \tag{9}$$

$$a_i \le s_{ik} \le b_i' \qquad \forall\, i \in V, \qquad \forall\, k \in K \tag{10}$$

$$s_{ik} + t_{ij} - s_{jk} \le \left(1 - x_{ijk}\right) M_{ijk} \qquad \forall\, (i, j) \in A, \qquad \forall\, k \in K \tag{11}$$

$$x_{ijk} \in \{0, 1\} \qquad \forall\, (i, j) \in A, \qquad \forall\, k \in K \tag{12}$$

$$c'_{ijk} = \begin{cases} c_{ij}, \text{if } \mathrm{a_j} \le s'_{jk} \le b_j \\ c_{ij} + c_l(s'_{jk} - b_j), \text{if } b_j < s'_{jk} \le b'_j \\ c_{ij} + c_e(a_j - s'_{jk}), \text{if } a'_j \le s'_{jk} < a_j \end{cases} \tag{13}$$

The model contains two decision variables s_{jk}' that determine the arrival time at a vertex $j \in C$ by a vehicle $k \in K$ as well as the travel cost of the arc (i, j) (Eq. (13)), and x_{ijk} that determine whether the arc (i, j) is used in the solution ($x_{ijk} = 1$) or not ($x_{ijk} = 0$). M_{ijk} is a large constant. Objective function (3) minimises the total cost including the fixed vehicle utilisation cost and the travel cost on the arcs as well as the penalty costs. Constraint (4) ensures that every customer must be serviced only once, while constraint (5) is the capacity constraint. Constraints (6), (7) and (8) are flow conservation constraints. Constraint (9) specifies the relaxed time windows for VRPSTW and restricts the arrival time at all vertices to be within their relaxed time windows [a_i', b_i']; whereas, constraint (10) restricts the service start time within [a_i, b_i']. Constraint (11) specifies that if a vehicle travels from i to j, service at j cannot start earlier than that at i. Finally, constraint (12) shows the integrality constraint.

3. Hybrid Insertion Heuristics (HIH)

Column generation or Dantzig-Wolfe decomposition provides a flexible framework that can accommodate complex constraints and time dependent costs[15]. It decomposes the VRPSTW problem (3)-(12) into a set partitioning master problem and an Elementary Shortest Path Problem with Resource Constraints (ESPPRC) as its subproblem. The subproblem gives feasible shortest paths subjected to constraints (5)-(12). This study uses a heuristic subproblem instead of the ESPPRC, since at present no exact ESPPRC algorithm is able to incorporate complex soft time windows constraint and time dependent costs. On the other hand, heuristics such as insertion heuristics can easily deal with complex cost structures.

The master problem, which now consists of selecting a set of feasible paths of minimum cost generated in the heuristics subproblem, is mathematically described as:

$$\min \sum_{p \in P} c_p y_p \tag{14}$$

subject to

$$\sum_{p \in P} a_{ip} y_p = 1 \qquad \forall i \in C \tag{15}$$

$$y_p \in \{0, 1\} \qquad \forall p \in P \tag{16}$$

Where the set of all feasible paths is shown by P, y_p takes a value of 1 if the path $p \in P$ is selected and 0 otherwise. The cost of the path p is denoted by c_p, and a_{ip} represents the number of times path p serves customer i. In the actual application the set covering master problem is solved by replacing constraint (15) by (17), as the linear programming relaxation of set covering type master problem is more stable than the set partitioning type[10].

$$\sum_{p \in P} a_{ip} y_p \ge 1 \qquad \forall i \in C \tag{17}$$

4. Insertion Heuristics Subproblem

Usually the ESPPRC subproblem is solved for the Dantzig-Wolfe decomposition by removing the assignment constraint (4) and considering homogenous vehicles, thus finding routes for single vehicle only. The insertion heuristics subproblem is solved using the same constraint set as VRPSTW formulation that not only finds the shortest routes but assigns customers to the vehicles. The only difference is the use of the reduced cost $\overline{c'_{ij}}$ found by the Eq. (18), using the dual variables (prices) (π_i) generated in the master problem.

$$\overline{c'_{ij}} = c'_{ij} - \pi_i \qquad \forall\, i \in V \tag{18}$$

A modified version of the Stochastic Push Forward Insertion Heuristic (SPFIH)[11] is used as the subproblem, which can deal with the soft time windows. It is a sequential heuristic where the route for one vehicle is completed before starting the route for another vehicle. The first customer is chosen randomly while the remaining are inserted by minimizing the reduced cost of the route. When no such new insertion is possible due to violation of either capacity or the relaxed time windows constraint, a new route is initialized with a random customer, which is not yet served.

Let $(i_0, i_1, i_2, \ldots, i_m)$ be the current route with $i_0 = i_m = 0$. The service start time s_{i_r}, waiting time w_{i_r} and late arrival time l_{i_r} are known for $0 \leq r \leq m$. Insertion of a customer vertex u between i_{p-1} and i_p, causes a *push forward* (PF_{i_p}) in the schedule at the customer i_p that may change the values of s_{i_r}, w_{i_r} and l_{i_r}, $p \leq r \leq m-1$. Defining the new service start times $s_{i_r}^{new}$, $p \leq r \leq m-1$, due to insertion of the customer u, the conditions $s_u \leq b'_u$ and $s_{i_r}^{new} + PF_{i_r} \leq b'_{i_r}$ provide the feasibility criteria for a feasible insertion position of the customer u. Similar to Solomon[8], the best feasible insertion place is determined using Eq. (19) for each unrouted customer u. As in this study, the insertion heuristics is used as the subproblem; reduced costs are used to find the insertion cost of the customer u between i_{p-1} and i_p (Eq. (20)). Furthermore, an additional term is added to consider the changes in early and late arrival penalties for the customers: i_r, $p+1 \leq r \leq m-1$. Finally, the best customer u^* to be inserted in the route, is obtained using Eq. (21).

$$c_1(i(u), u, j(u)) = \min[c_1(i_{p-1}, u, i_p)],\; p = 1, \ldots, m \tag{19}$$

$$c_1(i_{p-1}, u, i_p) = \overline{c'_{i_{p-1},u}} + \overline{c'_{u,i_p}} - \overline{c'_{i_{p-1},i_p}} + \sum_{r=p+1}^{m-1}(c_e(w_{i_r}^{new} - w_{i_r}) + c_l(l_{i_r}^{new} - l_{i_r})) \tag{20}$$

$$c_2(i(u^*), u^*, j(u^*)) = \min_u[c_1(i(u), u, j(u))] \tag{21}$$

5. Expected Results

Mostly metaheuristics such as Genetic Algorithms (GA) are used to solve the VRPSTW. Results of the Hybrid Insertion Heuristics (HIH) will be compared with the results of a GA heuristic, to evaluate its performance in terms of solution quality and computation time requirements. The computational results on benchmark instances and conclusions will be presented at the conference.

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