HOTELLING COMPETITION AMONG SHOPPING STORES AND LOCATION OF CITY RESIDENTS[1]
商業競争が都市住民立地に与える影響に関する分析

Kei FUKUYAMA [2]

1. Introduction

In most of the contributions to Hotelling competition, the spatial distribution of customers is exogenously given. This setting is reasonable when the competing firms are small and therefore their location has negligible effect on consumers' decision on residential location. Modern city configuration, however, teaches us that shopping stores are sometimes quite large and their location affects on households' location decisions. For example, existence of the attractive suburban shopping center can be a counter-incentive for customers to live close to working CBD. This paper analyses the effects of Hotelling competition on locations of urban households, especially focusing on the emergence of small stores in the city, such as 'perishables' convenience stores in Japan.

2. The Model

(1) City configuration

A city exists in a long and narrow space with a unit width. The city has a fixed population which is normalized to 1. All city residents commute to CBD which is located at 0. There are zoning regulations in the city. The residential area which is represented by the range of $[R_1, R_2]$ is designated by the regulation and it includes the CBD point so that $R_1 \leq 0 \leq R_2$.

Under the zoning regulation, any large store is prohibited to locate within the residential area. There location, on the other hand, is designated outside of the range of $[M_1, M_2]$ $(M_1 \leq R_1 \leq 0 \leq R_2 \leq M_2)$ by the regulation. The open space of $[M_i, R_i](i = 1, 2)$ is a open space kept for the future possible expansion of the city. In the following model, this open space is assumed to be null for simplicity and $M_i = R_i$.

(2) Household

For simplicity, lot size for each household is assumed to be fixed. The total residential land is assumed to be 1. If all residents reside side-by-side, then the residential area is one continuous range along the linear space and is represented by $[s-1, s]$. All residents work at the CBD. A resident at location x is faced with the linear community cost of $|t \cdot x|$ to the CBD.

There is only one commodity, and the households have an inelastic demand for this good. The good is sold in the shopping stores in the suburban areas and therefore, they provide a shopping trip to a shopping store. The shopping cost is given as a linear function of $k \cdot y$ for a distance y. $t > k$ is assumed.

Shopping stores are prohibited to locate in the residential area, and are only allowed to locate in the suburban area of $[-\infty, M_1[$ and $]M_2, \infty]$.

(3) Shopping stores

Shopping stores locate on the either of the two suburban areas of the city. Shopping store i $(i = 1, 2)$ produces the commodity with the marginal cost c_i and chooses its location as well as its mill price, p_i. The profit of the shop i is then given as follows.

$$\Pi_i = (p_i - c_i)z_i, \ i = 1, 2 \tag{1}$$

where z_i is the number of shoppers at the shop i. without loss of generality, $c_1 \leq c_2$ is assumed.

Because of the outside location competition setting, the price and also location equilibria always exist both for simultaneous and sequential competitions if no residential relocation is assumed. In this analysis, we assume the two stage decision situation where shops decide their prices and locations simultaneously at the first stage, and then the households decide their residential location at the second stage.

3. The Suburban Store Competition

First consider the duopoly case where the two suburban stores compete while no convenience store exists in the city. It is easy to see that when both stores locate in the same outside of the residential area, there is always one shop (which locates more 'out') improves its profit by moving its location to the other side. Furthermore, there is no incentive for the shops to locate far than zoning edges of M_1 and M_2. Therefore, the equilibrium location of shops is always separation of the two shops on the both sides.

Let the shops locate on M_1 and M_2 be called shops 1 and 2, respectively. Each household must visit to one of the two suburban stores for commodity shopping. Let the location of the indifferent consumer who can

[1]Keywords: location theory, Hotelling competition, shopping cost, commuting cost, city configuration

[2]Member of JSCE, Ph.D., Graduate School of Information Sciences, Tohoku University (6-3-09 Aoba, Aramaki, Aobaku, Sendai, Japan, TEL: 022-795-4380, FAX: 022-795-4380)

buy the commodity from one of the two shops with the same cost be indicated by $\hat{x}$. The full trip cost for the resident of the corresponding location x is given as follows

$$T(x) = \begin{cases} p_2 + (x - M_1)k + t(-x) & \text{if } M_1 \le x \le 0 \\ p_2 + (x - M_1)k + tx & \text{if } 0 < x \le \hat{x} \\ p_1 + (M_1 - x)k + tx & \text{if } \hat{x} < x \le M_2 \end{cases} \tag{2}$$

The indifferent consumer's location is given by

$$p_1 + (\hat{x} - M_1)k = p_2 + (M_2 - \hat{x})k \tag{3}$$

And therefore, we have

$$\hat{x} = \frac{p_2 - p_1 + (M_1 + M_2)k}{2k} \tag{4}$$

Consider the residents' reaction to the prices first. The households' distribution $[s-1, s]$ is determined so that the rent (and therefore total travel cost) at s and $s-1$ are equal. This condition is given as

$$p_1 + (s - 1 - M_1)k - (s-1)t = p_2 + s(t-k) + M_2 k \tag{5}$$

We have the best reply correspondence of location by the households as

$$s(p_1, p_2) = \frac{1}{2} + \frac{p_1 - p_2 - k(M_1 + M_2)}{2(t-k)} \tag{6}$$

Consider the price strategies of the stores. The profit of the store i with price of p_i is then given as follows.

$$\Pi_1 = (p_1 - c_1)(\hat{x} - (s(p_1,p_2) - 1)),\ \Pi_2 = (p_2 - c_2)(s(p_1,p_2) - \hat{x}) \tag{7}$$

By substituting the equations (4) and (6) into (7), and taking second derivatives, we get $\frac{\partial^2 \Pi_i}{\partial p_i^2} = -\frac{t}{k(t-k)} < 0$ $(i = 1, 2)$. Consequently, the first order conditions for both shops lead us to the following subgame-perfect equilibrium prices under the duopoly, $(p_1^{*(duo)}, p_2^{*(duo)})$.

$$p_1^{*(duo)} = k\frac{3(t-k) + (M_1 + M_2)t}{3t} + \frac{2c_1 + c_2}{3},\ p_2^{*(duo)} = k\frac{3(t-k) - (M_1 + M_2)t}{3t} + \frac{c_1 + 2c_2}{3} \tag{8}$$

The equilibrium location of market divide, $\hat{x}^{*(duo)}$, and also the equilibrium households' location, $s^{*(duo)}$, are then obtained by substituting equilibrium prices into the equations (4) and (6), respectively.

$$\hat{x}^{*(duo)} = \frac{\Delta c + k\Delta M}{6k},\ s^{*(duo)} = \frac{1}{2} + \frac{\Delta c - k\Delta M}{6(t-k)} \tag{9}$$

where $\Delta c \equiv c_2 - c_1 (\ge 0)$ and $\Delta M \equiv M_2 + M_1$. Δc indicates the cost advantage of the store 1. Also, notice that the value of ΔM indicates the 'asymmetry of zoning'; higher value of this term means that the market is closer to the store 2 of the right hand outside location. So we get the following. Consequently, ΔM indicates location advantage of the store 1.

Proposition 1 *Under duopoly, more asymmetry of zoning area means more price difference between the shops. Under symmetric zoning, the zoning size does not affect on the prices.*

The following comparative static on prices is obtained.

$$\frac{\partial p_1^{*(duo)}}{\partial \Delta M} = -\frac{\partial p_2^{*(duo)}}{\partial \Delta M} > 0, \frac{\partial p_i^{*(duo)}}{\partial t} > 0, \frac{\partial p_1^{*(duo)}}{\partial k} \left(\begin{array}{c} \ge \\ < \end{array} \right) 0,\ \frac{\partial p_2^{*(duo)}}{\partial k} \left(\begin{array}{c} \ge \\ < \end{array} \right) 0.$$

The zoning regulation (to make the store location much far from CBD) has a positive effect on the price of the store, while it has negative effects on the other store on the other side of CBD. The decrease in the commuting cost brings about the increase in prices of the both stores. The effects of the increase in the shopping trip cost on the prices are more complicated. However, in general it tends to increase (decrease) the prices when the shopping trip cost is relative low (high) to the commuting cost.

Also, we can get the following comparative static on the households' location.

$$\frac{\partial s^{*(duo)}}{\partial t} = \frac{k\Delta M + \Delta c}{6(t-k)^2} \left(\begin{array}{c} \ge \\ < \end{array} \right) 0,\ \frac{\partial s^{*(duo)}}{\partial k} = -\frac{t\Delta M + \Delta c}{6(t-k)^2} \left(\begin{array}{c} \ge \\ < \end{array} \right) 0,\ \frac{\partial s^{*(duo)}}{\partial \Delta M} = k\frac{\partial s^{*(duo)}}{\partial \Delta c} = -\frac{k}{6(t-k)} < 0. \tag{10}$$

The increase in the zoning asymmetry leads to the asymmetry of households' location towards the other direction. Also, increase in the relative cost of one store makes households away from it. The effects of the trip costs are complex. It can be generally say, however, that when the store has *location advantage* (namely, $\Delta M > 0(\Delta M <)0$ for the store 1 (for the store 2)) and/or *cost advantage* (namely, $\Delta c < 0(\Delta c > 0)$ for the store 1 (for the store 2)), the decrease in the commuting cost makes the households' location away from the shop. On the other hand, the shopping trip cost has the reverse effect.

Finally, the following comparative static on market divide is obtained.

$$\frac{\partial \hat{x}^{*(duo)}}{\partial t} = 0, \ \frac{\partial x^{*}(duo)}{\partial k} = \frac{\Delta c}{6k^2} \geq 0, \ \frac{\partial \hat{x}^{*(duo)}}{\partial \Delta M} = \frac{1}{6} > 0, \ \frac{\partial \hat{x}^{*(duo)}}{\partial \Delta c} = \frac{1}{6k} > 0. \tag{11}$$

While the increase in the shopping trip cost has positive effect on the market divide, the change of commuting cost has no effects on it. Obviously, the increase in the relative cost and distance from CBD of one store decreases its market area.

Because the city residents have inelastic demand for the commodity and fixed income, the social welfare maximization configuration is defined as the one of minimized total travel costs. With the assumption of $\hat{x} \geq 0$, without loss of generality, we have

$$TC(s, p_1, p_2; M_1, M_2) = \int_{s-1}^{s} T(x)dt = \frac{-(s-1)}{2}(p_2 - M_2k + p_2 + ((s-1) - M_2)k - (s-1)t) \tag{12}$$

$$+\frac{n}{2}(p_2 - M_2k + \hat{x}t + (M_1 - n)k + p_1) + \frac{s-\hat{x}}{2}(\hat{x}t + (M_1 - \hat{x})k + p_1 + st + (M_1 - s)k + p_1)$$

We can easily check that $\frac{\partial^2 TC}{\partial s^2} = 2(t-k) > 0$. Therefore, the optimal residential location is given by $\frac{\partial TC}{\partial s} = 0$, or

$$s^{**} = \frac{1}{2} - \frac{k(M_1 + M_2) + (p_1 - p_2)}{2(t-k)} \tag{13}$$

We can consider so-called *the contestable market* by assuming potentially many shops to locate in this city with the same marginal cost of c. With the same logic with 'outside location game' of Gabszewicz and Thisse (1992), only one shop can locate on one outside location. Because there is always incentive for shops to locate in the vacant outside if any, eventually we can get the equilibrium consists of similar shop location that only one shop locate in each outside location of M_1 and M_2. The equilibrium prices and locations of households are obtained by $p_1 = p_2 = c$ and (6).

$$p_1^{*(cont)} = p_2^{*(cont)} = c, \ s^{*(cont)} = \frac{1}{2} - \frac{(M_1 + M_2)k}{2(t-k)} \tag{14}$$

The location of market divide is independently determined from the strategic behaviors, and is given by substituting $p_i = c$ into the equation (4) as

$$\hat{x}^{*(cont)} = \frac{M_1 + M_2}{2} \tag{15}$$

Proposition 2 *Under contestable market, the optimum location of residents is achieved.*

4. Competition with Convenience Stores

Now we introduce the convenience store company that sells the same commodity with the suburban supermarkets. Unlike the supermarkets, the convenience stores are allowed to locate within the residential area of the city. their distinction from the conventional stores or supermarkets is characterized by the 'uniform price'.

Let $[d_1, d_2]$ be indicated the range of the convenience store service. The price of the commodity at the convenience stores, indicated by p_c, must hold the following relationships with their service range and the prices by the suburban supermarkets.

$$p_c = (d_1 - M_1)k + p_1, \ p_c = (M_2 - d_2)k + p_2 \tag{16}$$

The profit of the convenience store company, Π_c, is then given by

$$\Pi_c(p_1, p_2, p_c) = (p_c - c_c)(d_2 - d_1) = \frac{(p_c - c_c)(p_1 + p_2 + (M_2 - M_1)k - 2p_c)}{k} \tag{17}$$

where c_c is the marginal cost of the convenience store company.

The profits of the two supermarkets, on the other hand, are given as follows.

$$\Pi_1 = p_1\{d_1 - (s-1)\}, \ \Pi_2 = p_2(s - d_2) \tag{18}$$

where s is the (best-reply) range of residential area of the city, and is given by the equation (6). By substituting the equations (6) and (16), the profits can be rewritten as

$$\Pi_1(p_1, p_2, p_c) = \frac{p_1(2p_ck - 2p_ct + M_1k^2 - 2M_1kt - p_1k + 2p_1t + k^2 - kt - kp_2 - M_2k^2)}{2k(k-t)} \tag{19}$$

$$\Pi_2(p_1, p_2, p_c) = \frac{p_2(-p_1k + k^2 + M_1k^2 - kt - kp_2 - M_2k^2 + 2p_ck - 2p_ct + 2M_2kt + 2p_2t)}{2k(k-t)} \tag{20}$$

The profit functions of the three stores are all concave functions of their own prices, and therefore, the profit maximization of each store leads to the following subgame perfect equilibrium.

$$p_c^* = \frac{(2t-k)\{(M_2-M_1)k+(c_1+c_2)\}+2k(t-k)+2(4t-3k)c_c}{4(3t-2k)} \tag{21}$$

$$p_1^* = \frac{k\left\{(10t+k)(t-k)+2k^2\right\}M_1+k(2t-k)(t+k)M_2+2k(4t-k)(t-k)+(2t-k)\{(7t-3k)c_1+(t+k)c_2\}+2c_c(4t-k)(t-k)}{2(3t-2k)(4t-k)} \tag{22}$$

$$p_2^* = \frac{-k\left\{(10t+k)(t-k)+2k^2\right\}M_2-k(2t-k)(t+k)M_1+2k(4t-k)(t-k)+(2t-k)\{(7t-3k)c_2+(t+k)c_1\}+2c_c(4t-k)(t-k)}{2(3t-2k)(4t-k)} \tag{23}$$

By substituting these equilibrium prices into the equation (6), we get the following equilibrium households' location.

$$s^* = \frac{1}{2} - \frac{2t-k}{2(t-k)(4t-k)}\{\Delta c + k\Delta M\} \tag{24}$$

The following comparative static on s^* is obtained.

$$\frac{\partial s^*}{\partial \Delta M} < 0, \frac{\partial s^*}{\partial t}\left(\begin{array}{c}\geq\\<\end{array}\right)0, \frac{\partial s^*}{\partial k} = \left(\begin{array}{c}\geq\\<\end{array}\right)0.$$

The increases in ΔM and Δc have same positive/negative effects on the households' location as the case without convenience stores. However, because $\frac{1}{6(t-k)} \leq \frac{2t-k}{2(t-k)(4t-k)}$ holds, the effects are more under the existence of convenience stores. Therefore we have;

Proposition 3 *The effects of the cost difference and also the zoning asymmetry between the two stores on residential location are enhanced by the entrance of convenience stores.*

The decrease in t has same complex positive/negative effects on the households' location as the case without convenience stores. However, because $\frac{1}{6(t-k)^2} \leq \frac{8t^2+-8kt+3k^2}{2(t-k)^2(4t-k)^2}$ holds, the effects are more under the existence of convenience stores. Therefore we have;

Proposition 4 *The effects of change in commuting cost on residential location are greater in competition with convenience stores than without it.*

The range of the convenience store location is given by $d \equiv d_2 - d_1$. The equilibrium range, d^* is then given by substituting the equilibrium prices into (16).

$$d^* = \frac{p_1^* + p_2^* - 2p_c^*}{k} + (M_2 - M_1) = \frac{1}{3t-2k}\left[\frac{2t-k}{2k}\{k(\Delta M) - (2c_c - \Delta c)\} + (t-k)\right] \tag{25}$$

We can get the following comparative static on d^*.

$$\frac{\partial d^*}{\partial \Delta M} > 0, \ \frac{\partial d^*}{\partial c_c} < 0, \frac{\partial d^*}{\partial c_i} > 0, \ \frac{\partial d^*}{\partial k} > 0, \ \frac{\partial d^*}{\partial t} < 0. \tag{26}$$

Comparing s^* in (24) with (9), we get

$$|s^* - \frac{1}{2}| < |s^{*(duo)} - \frac{1}{2}| \tag{27}$$

because $\frac{1}{6(t-k)} \leq \frac{2t-k}{2(t-k)(4t-k)}$ holds. Therefore, we get:

Proposition 5 *Entrance of convenience store company into the supermarket competition leads location of city residences more symmetric around the CBD.*

Due to the complicated structure of price functions of (22) and (23), the effects of existence of convenience stores on prices are not as clear as the one on residential location. However, at least the following is obtained.

Proposition 6 *Under symmetric locations of outside supermarket locations with zero marginal costs (namely, $M_1 = -M2 = -0.5$, $c_i = 0$), prices of supermarkets with convenience competition are lower than the one without the competition.*

5. Conclusion

The effects of entrance of convenience store company into the supermarket competition are clarified. Especially, the effects on households' location in the city are analyzed. It is especially shown that the convenience stores bring about the more even location of city residences.