AN ANALYSIS ON THE POSSIBLITY OF A CITY FREIGHT COOPERATIVE TRANSPORTATION SYSTEM AND THE SOLVING OF PROBLEMS USING GAME THEORY

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1. Introduction

City Cooperative Freight Transportation(COFT) system has always been considered as an effective way for solving problems brought about by increasing traffic. However, there are still a number of questions concerning on cooperation which are waiting to be overcome. As game theory becomes more and more popular, it also abstracts many researchers in the field of city logistics. However, few of them used Cooperative Game Theory and most researches emphasis on qualitative analysis or some idiographic problems such as route planning, location planning and so on. In this research, we aim to make use of Cooperative Game Theory, in-depth and adequately, as well as some corresponding mathematic models for pursuing an overall analysis of the COFT system. This will mainly involve: 1) Constructing a COFT system and designing a corresponding mathematic model, which is named characteristic function in game theory, to despite the system. 2) Providing a systematic standard in judging the rationality inside alliance's formation. There are the two following cases. 3) In case 1, the alliance is considered as profitable, so the problem of how to allocate payoff is discussed based on game theory. 4) In case 2, the alliance is not rational in the economy. However, it is necessary of founding the alliances since cooperation brings advantages to the society and the environment. Then ways in solving this problem is also discussed from the view of this theory. We hope that this study will enhance our understanding of inter-organizational relationships and decision-making behaviour, for truck logistics companies as well as the local public sector.

2. Model of the Urban Cooperative Freight Transportation System

A good design of a COFT system is certainly a good beginning for the application of cooperation. We choose a popular idea of constructing common freight DC (Distribution Centre) as our study object shown in figure 1. Left part of the figure shows an ordinary structure of city transportation before cooperation. There are several supplier factories in the exurban area, everyday some third-logistic providers transport their commodities to many retail depots inside the city. A new cooperative city logistic system is like the right part of the figure. If there are some large-scale public DCs can be built just beside the city and serves for all the nearby companies who want to cooperate, these companies can then rearrange their transportation by a more integrated view and gain the benefit of economics of scale. This is through such as improving load factors, using line haul trucks, reducing individual transportation round, and so forth. At the same time, the social and environmental conditions can also be improved. However, these companies may also face risk of paying expensive cost from such as building necessary facilities, managing and operating alliance and some other potential cooperative cost.

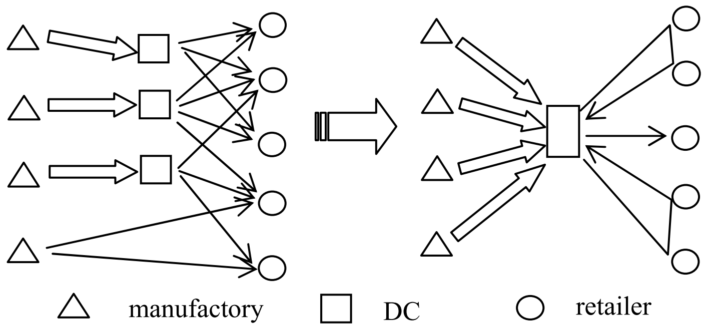

Figure 1: System change from noncooperation to cooperation

* Keyword : COFT, Game theory, Core, Shapley value, Nucleolus, ε-core

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3. Outline of Cooperative Game Theory

Usually there are three methods in analyzing the solution of the Cooperative game: the stable set, Shapely value and bargaining set. ①The stable set is given as "standards of behavior", of which the most famous one is core. For an arbitrary TU (transferable utility) game (N,v), where N is a finite or a countable player set and v is a map assigning to each $S \subset N$ a real number and $v(\phi)=0$. All the x_i satisfied $\sum_{i \in S} x_i \geq v(S)$, $S \subset N$ are all called core. The not-empty of core implicates an important signification that no individual or group has an opportunity to gain more arrangement by other ways. ②The Shapley value is different with core for that it is a way of forecasting the expected marginal amount which the player contributes to the coalition, such as the "fairness" allocation of grand coalition's gain. ③The bargaining set is obtained by considering the discussion that may actually take place during a play of the game. As one solution in the set, nucleolus has a great practical availability for us to get the value of converging the core into one optimal point. Actually, numerous real-world situations translate into models where the core is empty and thus, not applicable. Another solution named ε-core provides an alternative way to deal with empty-core scenarios for restoring efficiency with minimum amount of intervention, such as some kind of subsidy. The ε-core is defined as: min ε, subject to $\sum_{i \in N} x_i = v(N)$ and $\sum_{i \in S} x_i + \varepsilon \geq v(S)$ for all $i \in S$.

Here, ε just can be denoted as the subsidy in case of empty-core. However, it should be improved to a suitable form so as to express the right meaning. We will lay out an idea of improvement in the next sector.

Then for the concerning problems in COFT system, based on the aforementioned concepts in Cooperative Game theory, we can draw the frame of application of cooperative game to COFT system as shown in figure 2.

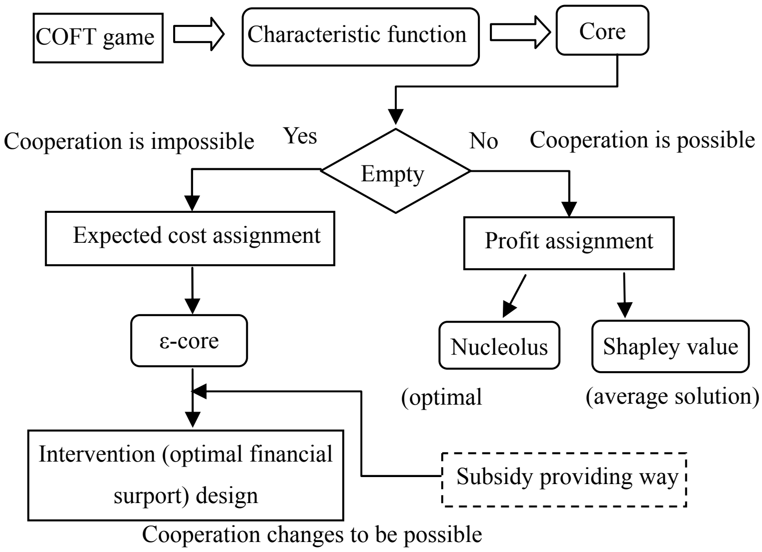

Figure 2: Frame of application of cooperative game to COFT system

4. Cooperative Game Theory associating with COFT system

(1) Characteristic Function Characteristic function specifies the consequences of the various combinations of strategy choice by all of the players in a game. It is the analysis foundation of Cooperative Game theory. Generally, the Characteristic Function is based on the cost function. In our case, logistics company i's costs are comprised by three parts before cooperation: transportation cost(TC), storage cost(SC) and fixed cost(FC) as c(i)=TC(i)+SC(i)+FC(i). Coalition S costs include one additive part of cooperative cost(OC) as c(S)=TC(S)+SC(S)+FC(S)+OC(S). c(i) can be easily obtained by some surveys while c(S) is much more complex. We use the approximative location model as our characteristic function to calculate the coalition cost in our case since it not only includes functions of selecting the optimal location of DC, the best arrangement of flow together with appropriate quantities to each flow, but also minimizes the cost which is pivotal for companies. As a system based on common DC for different industries, this model is able to satisfy two important characteristics of COFT, muti-product location and hub location.

The final objective of expressing the characteristic function, is to show the maximum cost saving for each company, or coalition. Actually, it is the difference between the sum of individual cost and one coalition's cost described as:

$$v(i)=0 \quad \text{and} \quad v(S)=\sum_{i \in S} c(i) - c(S)$$

Moreover, in contrast to the individual cost, coalition costs have some different traits, for example, a variational TC and incremental OC, TC has the trait of decreasing unit cost per ton·kilometer with the increasing scale of coalition(an example from survey shown in figure 3). This speciality is called the superadditivity in game theory and mathematically it is depicted as convex function. Economically, it is the economics of scale. OC means the cooperative cost, which mainly includes the construction cost of common DC and some accessorial cooperative cost which are at least linear increasing, even superlinear increasing. Division of these two parts in characteristic function includes great meanings. Superadditive part is one of the most interesting concepts in game theory. As definitions a game is supperadditive if: $\forall S,T \subseteq N, \quad S \cap T = \phi : v(S \cup T) \geq v(S) + v(T)$. It actually means supperadditive payoff for a larger coalition is at least as good as and perhaps better than, this part of payoff, that received as separate coalition or individuals. A simple flow game is totally stable if it is superadditive. In our case, the transportation cost is superadditive by the reason of economics of scale while the part of OC is nonsuperadditive. So, the COFT is not always a stable game. The stability of the game is mainly decided by how large the nonsuperadditive part should be. Many people think cooperation is money consuming in field of transportation, this superadditive parts of some costs are just rational contradiction of this view.

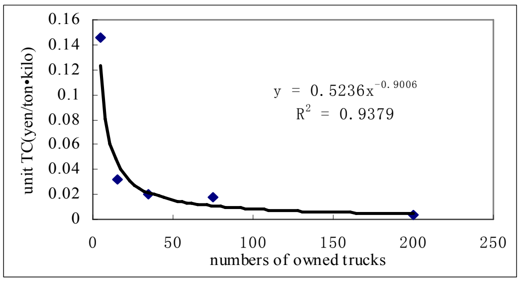

Figure 3: Logistic regression curve of transportation cost with the truck numbers owned by the COFT

(2) Core's empty and not-empty If the superadditive part of the total cost is more dominate than the progressive part for each company, every partner company can get cost saving in the alliance, the core then is not-empty. In this case, the cost saving part is big enough to cover the superfluous cost and alliance is stable. However, if there is at least one company which can not get payoff in any coalition, then we should say that this coalition is not stable, the core for the coalition must be empty. Theoretically, the nonsuperadditive parts are absolutely higher for the company.

(3) Situation of existing-core A core's not-empty indicates that there must be some payoff for the every company. Then, the consequent problem mainly focuses on how to allocate the cooperative cost, as well as the payoff, to the partner companies properly so as to keep the alliance stable. Game theory provides tool in solving these problems, which are core, the Shapely value and the nucleolus. Core is all the set of possible solutions, while the Shapley value and the nucleolus are two in them. Comparing to the nucleolus, the Shapely value is more likely to be an average allocation, while nucleolus makes the bigger one get bigger and the smaller get smaller, w implys tendency to maximize profit for some of the better subcoalitions. In application, they suit for different situations, as in our consideration. It is more likely to use the Shapley value for a comprehensive coalition, while a coalition pursuing efficient uses nucleolus.

(4) Situation of empty-core In the real world, many alliances are difficult to be formed because of its empty core. However for improving environmental conditions, sometimes it is necessary to promote these alliances to come. Based on Cooperative Game theory, if some intervention from outside, which is usually considered as financial support from public sector, can be brought to the coalition, the formation of them becomes possible. How to realize this idea, Cooperative Game Theory also provides us some useful ways, ε-core is one of them, which has two important potential usages: calculating the optimal subsidy and allocating the cost among the partner companies. Here, the following result of a case we did will help to illuminate our findings. In order to apply ε-core in obtaining the cost share for partner companies, as well as the optimal subsidy, it needs to be improved. In the case we provide the ε with a new meaning as an unit subsidy. Every partner company can obtain repeatable of this unit subsidy proportional to its size and, all companies' subsidy together is what the local public sector totally needs to provide. And the improved ε-core now is

$$\min \quad \varepsilon, \quad \text{subject to } \sum_{i \in N} x_i = v(N) \text{ and } \sum_{i \in S} x_i + n\varepsilon \geq v(S) \text{ for all } i \in S.$$

Where, n is decided by the size of the companies.

We constructed a two-cases simulation, to test our work. Case 1 is about four similar companies while Case 2 is one big and three small. Firstly, we changed one of the TC and OC part in the characteristic function and keep another immovable, because we think the TC and OC are most superadditive and nonsuperadditive part respectively in our function. Then we can draw the figure of ε's change as figure 4 and 5. Especially according to the definition of core and ε-core, the core is the whole of allocation which satisfies the condition that ε≤0, and core is empty when ε>0.

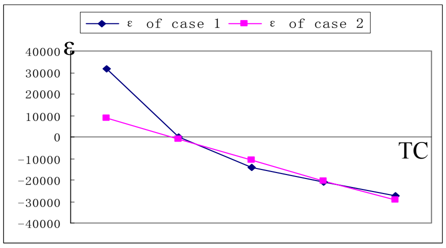

Figure 4: Change of ε with change of TC

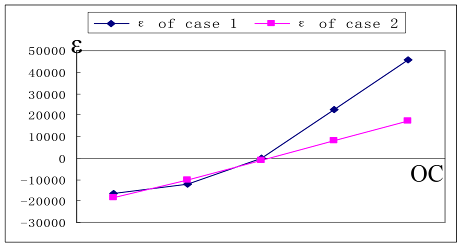

Figure 5: Change of ε with change of OC

It can be seen that the increasing of TC brings the value of ε to be less, which means that the coalition tends to earn money. The increase of the OC brings ε bigger, which means that incremental subsidy needs to be provided so as to keep coalition possible. Then the following conclusion can be drawn: if the cooperative system has a relatively high OC of establishing and managing common facilities and alliance(equivalently low TC), the core tends to be empty, whereas the core is not-empty when TC takes up a high proportion in the whole cost.

At a point of high OC, we draw the figures of companies' profit as below:

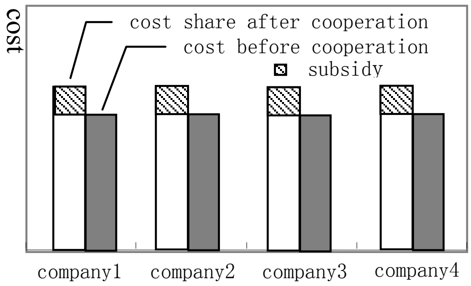

Figure 6: Companies' cost situation of case 1

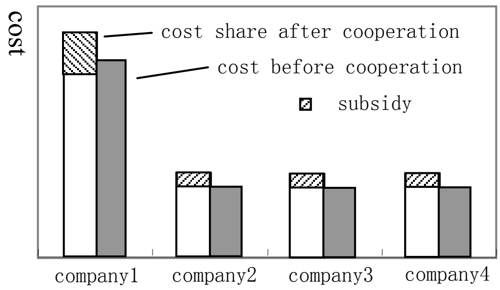

Figure 7: Companies' cost situation of case 2

Because of the high OC, the companies in both cases should pay a higher cost after cooperation than before. By using ε-core, we obtained the bigger cost share of each company in two cases after cooperation as the left rectangles shown in Figure 6 and 7. In terms of core, the coalition can not be built in this condition. However, with financial support by local public sector, as the top left rectangles show, situation can be changed and alliance can be formed. How large the top left rectangles should be depends on the ε we get. In case 1, it is special that the subsidy for each company is just the difference between its cost of before and after cooperation. However, the problem of subsidy giving is absolutely not a simple problem of filling the vacancy. This ε should satisfy that each company's cost share in the grand coalition should be less than its possible cost share in any other subcoalitions as well in individual. So in case 2, a more general situation, the subsidy which the large company obtained is more than the difference between its cost of before and after cooperation. That is mainly because if a large company, in any subcoalitions, has the chance to get more payoffs, than it can get in the grand coalition, it naturally prefers to join the subcoalition instead of the grand colaition. So, it is necessary to offer the larger company more subsidy, so as to let its actual obtaining in the grand coalition, more than any expected obtaining in any other subcoalitions, to keep them in the grand coalition. This is also one of most important meanings in Cooperative Game theory. Anyway, under the conclusions that figure 6 and 7 shows, the grand coalition can be constructed with such optimal subsidy.

5. Future research

In the future, we want to improve our study in two aspects. Firstly, as the return to the subsidy given by local public sector, not only size of companies but also the environmental and social factors need to be considered into this model. Secondly, a realistic case should be investigated and tested by the model.

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