EFFECTS OF RELAXING TIME WINDOWS ON VEHICLE ROUTING AND SCHEDULING

Ali Gul Qureshi **, Eiichi Taniguchi*** and Tadashi Yamada****

1. Introduction

With the increase in the economy and industrial activities, large cities are experiencing increase in transportation requirements in terms of both passengers and freight. A high proportion of total goods movement occurs within cities[1)] and most of this movement is based on road transport. Traffic congestion, noise, vibrations, generation of NOx, SPM, CO_2 and other environmental problems, accidents, loading and off-loading on streets are common problems caused by the road based freight transport in urban areas.

With the innovations in logistics, like Just-In-Time (JIT) systems and E-Commerce, urban freight related problems have further aggravated. Increase in E-Commerce, especially Business to Consumers (B2C) increases the negative effects on environment and traffic conditions, unless a major proportion of consumers use B2C, so that the reduced shopping trips may have considerable positive impact[2)].

Vehicle Routing Problem with Time Windows (VRPTW) is a common tool, used to investigate a variety of city logistics measures. In hard time windows (VRP-HTW), delivery is not possible outside the customer specified time windows. While in soft time windows variant (VRP-STW), any violation of time windows causes a penalty but the delivery is still possible.

Practical logistic problems are mostly set in soft windows environment. Mostly heuristic (approximate) solutions of VRPTW are used in city logistics related research[3)-4)], where the solution is required within reasonable time. Heuristic techniques are procedures that seek to find good (i.e. near exact) solutions for mathematical programming problems. While they are sometimes faster and easily implemented, yet they do not guarantee to identify the exact solution or state how close to the exact solution a particular feasible solution is[5)]. With the rapid increase in computer and information technology, it has now been possible to solve hard time windows vehicle routing problems of limited nature to optimality by using exact solution techniques within reasonable time.

This paper presents the initial stage results of a study focused on to identify the effects of hard time windows and soft time windows on the total running time, cost of delivery, and on generation of NOx, SPM and CO_2. At this stage, heuristic solution for soft time windows is compared with heuristic and exact solution of hard time windows, while future work is focused on incorporating soft time windows constraints in exact formulation of VRPTW.

2. Vehicle Routing and Scheduling Problem with Time Windows (VRPTW)

VRPTW is defined on a network (V, A). The node set includes the depot node 1 and the set of customers $C = \{2, 3,\ldots\ldots\ldots, n\}$. With every node of V associated a demand q_i, with $q_1 = 0$, and a time window $[a_i, b_i]$ representing the earliest and the latest possible service start times. K represents the set of identical vehicles with capacity Q. The arc set A consists of all the feasible arcs (i, j), $i, j \in V$. A cost c_{ij} and time t_{ij} is associated with each arc (i, j) $\in A$. t_{ij} include the service time at node i. The object of the VRPTW is to design a set of feasible routes of minimum cost, starting and ending at depot node. A feasible route is defined as ordered set of customer nodes, which satisfies the capacity constraint of the vehicle and time windows of customers.

Kohl et al.[6)], mathematically formulated VRPTW as

$$\min \sum_{k\in K} \sum_{(i,j)\in A} c_{ij} X_{ij}^{k} \tag{1}$$

subject to

$$\sum_{k\in K} \sum_{j\in V} X_{ij}^{k} = 1, \qquad \forall\, i \in C \tag{2}$$

* Key words: logistics planning, vehicle routing

** Student Member of JSCE, M.E, Doctoral Student, Faculty of Engineering, Kyoto University.
Yoshida honmachi, Sakyo-ku, Kyoto-shi, Kyoto-fu, Tel. 075-753-4788, Fax. 075-752-5303.
*** Fellow Member of JSCE, Dr. Eng., Faculty of Engineering, Kyoto University.
Yoshida honmachi, Sakyo-ku, Kyoto-shi, Kyoto-fu, Tel. 075-753-4789, Fax. 075-752-5303.
**** Full Member of JSCE, Dr. Eng., Faculty of Engineering, Kyoto University.
Yoshida honmachi, Sakyo-ku, Kyoto-shi, Kyoto-fu, Tel. 075-753-4787, Fax. 075-752-5303.

$$\sum_{i\in C} q_i \sum_{j\in V} X_{ij}^k \le Q, \quad \forall\, k \in K \tag{3}$$

$$\sum_{j\in V} X_{1j}^k = 1, \quad \forall\, k \in K \tag{4}$$

$$\sum_{j\in V} X_{ih}^k - \sum_{j\in V} X_{hj}^k = 0, \quad \forall\, h \in C \quad \forall\, k \in K \tag{5}$$

$$\sum_{i\in V} X_{i1}^k = 1, \quad \forall\, k \in K \tag{6}$$

$$S_i^k + t_{ij} - S_j^k \le \left(1 - X_{ij}^k\right) M_{ij}^k, \quad \forall\, (i, j) \in A \quad \forall\, k \in K \tag{7}$$

$$a_i \le S_i^k \le b_i, \quad \forall\, i \in V, \quad \forall\, k \in K \tag{8}$$

$$X_{ij}^k \in \{0,1\}, \quad \forall\, (i, j) \in A \quad \forall\, k \in K \tag{9}$$

Where, X^k_{ij} is 1 if the vehicle *k* travels from node *i* to *j* and 0 otherwise and S^k_i is the service start time at node *i* by vehicle *k*. M^k_{ij} is a big constant. Constraint set (2) enforces that every customer must only be serviced once while constraints (3) are capacity constraints. Constraints (4), (5) and (6) are flow preservation constraints. Constraints (8) and (9) are time windows constraints

(1) Genetic Algorithms

Many researchers has focused their work on VRPTW solution based of heuristics such as tabu search[7)-8)], simulated annealing and genetic algorithms[9)-13)]. This study used genetic algorithms (GA)[10)] to find the approximate solution of VRP-HTW with waiting allowed at no cost. The GA uses integer chromosome representation and ordered based crossover with swap mutation. The population size is fixed at 1000, but number of generations was taken 300 times the number of customer nodes. Cross-over rate is taken as 98% while the mutation is applied at 10%.

(2) Column Generation

After the earlier work of Kolen et al. [14)], the body of literature on VRP-HTW has been increasing with time. Most of the research has taken along two optimal approaches namely Lagrangean Relaxation[15)-16)] and Column Generation[6) and 17)-20)].

Column generation or Dantzig-Wolfe decomposition, decomposes the VRPTW problem (1 – 9) into an elementary shortest path problem with resource constraints (ESPPRC) and a set partitioning problem. ESPPRC is a NP-hard problem in strong sense[21)], it gives the feasible shortest path subjected to the constraints (3 – 9). The master problem, which now consists of selecting a set of feasible paths of minimum cost is mathematically described in Feillet et al.[19)] as:

$$\min \sum_{p\in P} c_p Y_p \tag{10}$$

subject to

$$\sum_{p\in P} a_{ip} Y_p = 1, \quad \forall i \in C \tag{11}$$

$$Y_p \in \{0,1\} \quad \forall p \in P \tag{12}$$

Where Y_p takes value 1 if the path *p* is selected and 0 otherwise. a_{ip} represents the number of times path *p* serves customer *i*. *P* is the set of all feasible paths. In actual application the set covering master problem is solved by replacing constraint (11) by (13), as linear programming relaxation of set covering type master problem is more stable than set partitioning type[17)].

$$\sum_{p\in P} a_{ip} Y_p \ge 1, \quad \forall i \in C \tag{13}$$

Desrochers et al.[17)] used 2-cycle elimination while solving shortest path sub-problem whereas Irnich and Villeneuve[18)] used k-cycle elimination with k>3. Feillet et al.[19)] and Chabrier[20)] used ESPPRC as the sub-problem. This study uses an adaptation of ESPPRC algorithms given by Feillet et al. [19)] and Irnich and Villeneuve[18)], to generate columns for a set covering master problem. At this stage, hard time window variant is considered where delivery is not possible outside the time windows. Vehicle is allowed to wait at no cost if it arrives earlier than start time window of customer. This permits to exclude all the arcs (i, j) which does not satisfy the inequality $a_i + t_{ij} \le b_i$.

3. Test Problems

At this stage the comparison between the approximate GA solution and exact solution has been done using Solomon's C-type benchmark problems. The test problems consist of 25 and 50 customers each and are taken from Solomon's C101 benchmark problem[22)]. These problems are the most widely used test problems in VRPTW related research to test the worst-case behavior of various algorithms. Two case settings were used for both VRP-HTW and VRP-STW. In Case1, waiting was allowed at no cost while in Case2, waiting was not allowed at all for VRP-HTW and for VRP-STW waiting caused an early arrival penalty with penalty being equal to the operating cost of the vehicle.

4. Results and Discussions

Travel time, travel cost, generation of NOx, SPM and CO_2 were used to compare the approximate GA solution of VRP-STW with GA and exact solution of VRP-HTW. Table 1 gives comparison of travel time, travel cost and number of vehicles for Case 1 heuristic solution and exact solution. In Solomon's benchmark problems distance and travel time between customer nodes is usually taken as their Euclidean distance. Travel time column represents only the running time excluding service as well as waiting time, similarly travel cost represents only running travel cost and fixed cost of vehicles. It can be seen that soft time windows (STW) have lesser travel time and cost for C101-50 compared with GA solution of hard time windows (HTW). As expected, there is a gap between exact and heuristic solution of hard time windows, which also describes the poor performance of soft time windows when heuristic solution of STW is compared with exact solution of HTW. This emphasizes the need of an exact approach for VRP-STW in order to examine the relative advantages or disadvantages of both soft time windows and hard time windows in exact terms.

Table 1: Comparison of Heuristic and Exact Solution of VRP-HTW with Heuristic Solution of VRP-STW.

Test Problems	Travel Time			Travel cost*			No. of Vehicles		
	VRP-HTW		VRP-STW	VRP-HTW		VRP-STW	VRP-HTW		VRP-STW
	Exact	Case 1	Case 1	Exact	Case 1	Case 1	Exact	Case 1	Case 1
C101-25	191.3	191.3	191.3	33934.5	33934.5	33934.5	3	3	3
C101-50	362.4	488.6	426	57168.4	69355.2	68477.5	5	6	6

**All costs are in Japanese yen.*

To calculate evaluation parameters an average running speed of 20 kmph is used for delivery vehicles. Also small delivery vehicle with capacity of 2000 kg is assumed. It is very clear that exact solution is far better than the GA solution for the tested problem. Table 2, gives the comparison of various environmental related evaluation parameters results. Again, soft time windows have comparatively better results when compared with hard time windows heuristically.

Table 2: Comparison of NOx, SPM and CO_2 for Soft and Hard Time Windows.

Test Problems	NOx			SPM			CO_2		
	VRP-HTW		VRP-STW	VRP-HTW		VRP-STW	VRP-HTW		VRP-STW
	Exact	Case 1	Case 1	Exact	Case 1	Case 1	Exact	Case 1	Case 1
C101-25	13.93	13.93	13.93	2.89	2.89	2.89	13.13	13.13	13.13
C101-50	26.36	35.57	31.01	5.47	7.38	6.43	24.88	33.54	29.25

Table 3, gives the waiting time for hard time windows using exact and GA for Case 1; and GA solution for soft time windows using both case settings i.e. when waiting is allowed at no cost (Case 1) and when waiting cause an early arrival penalty (Case 2). For HTW, Case 2 settings did not allow waiting at all.

Table 3: Comparison of Waiting Time

Test Problems	VRP-HTW		VRP-STW	
	Case 1	Exact	Case 1	Case 2
C101_25	440.5	440.5	440.5	83.3
C101_50	650.1	151.3	608.1	74.8

Table 4: Comparison of True STW and HTW

Travel Time		Travel cost			No. of Vehicles	
VRP-HTW	VRP-STW	VRP-HTW		VRP-STW	VRP-HTW	VRP-STW
Case 2	Case 2	Case 2	Case 1*	Case 2	Case 2	Case 2
215.2	294.8	44687.1	40110.3	35385.6	4	3
605.6	666.2	91830.5	78469.6	71845.1	8	6

Waiting time was less in STW than in HTW in GA solution of both cases. In fact while considering pure soft time windows (with early arrival penalty), soft time windows have considerable low waiting time than hard time windows. Though, compared with GA solution of hard time windows in Case 1, GA solution of soft time windows in Case 2 results in more travel distance (Table 4) but if a similar penalty is applied for waiting time of hard time windows environment in calculating its cost (Col. Case 1*, In Table 4), soft time windows have less over all cost. Reduction in waiting time may not only be translated in savings in labor and vehicle utilization costs but also it has meaningful effect on on-street parking of delivery vehicles in urban areas.

5. Conclusion and Future Work

Analysis results show that relaxing the time windows have better effects on solution of VRPTW. The analysis presented in this paper was based on benchmark problems and using exact solution of VRP-HTW, shows that there is a large gap between exact and heuristic solution. Though the benchmark problems are much harder than the practical problems, this gap could still be there and could be recovered using exact approaches for the practical problems as well. The future studies will be focused on using actual practical problems instead of benchmark problems. In addition, to incorporate soft time windows constraints in exact solution technique of VRPTW, and apply that model to some practical problems, which often set in soft time windows environment.

References:

1) Taniguchi, E., *et al.*: City Logistics; Network Modeling and Intelligent Transport Systems, Pergamon, Oxford, 2001.
2) Taniguchi, E., and Kakimoto, Y: Effects of e-commerce on urban distribution and the environment, Journal of the Eastern Asia Society for Transportation Studies, Vol.5, pp. 2355-2366, 2003.
3) Taniguchi, E., & Ando, N.: An experimental study on the performance of probabilistic vehicle routing and scheduling with ITS, in Recent Advances in City Logistics proceedings of the 4th International Conference on City Logistics, Langkawi, Malaysia, 2005, Taniguchi , E. and Thompson, R.G. eds., Elsevier, Oxford, pp. 59-73, 2005.
4) Qureshi, A. & Hanaoka, S.: Analysis of the effects of cooperative delivery system in Bangkok, in Recent Advances in City Logistics proceedings of the 4th International Conference on City Logistics, Langkawi, Malaysia, 2005, Taniguchi , E. and Thompson, R.G. eds., Elsevier, Oxford, pp. 293-306, 2005.
5) Thompson, R.G., & Duin, R.V.: Vehicle routing and scheduling, in Innovations in Freight Transportation, Taniguchi , E. and Thompson, R.G. eds., WIT Press, Southampton, pp. 47-63, 2003.
6) Kohl, N., *et al.*: 2-Path cuts for the vehicle routing problem with time windows, Transportation Science, Vol.33(1), pp. 101-116, 1999.
7) Badeau, P., *et al.*: A parallel tabu search heuristic for the vehicle routing problem with time windows, Transportation Research (C), Vol.5(2), pp. 109-122, 1997.
8) Taillard, E., *et al.*: A tabu search heuristic for the vehicle routing problem with soft time windows, Transportation Science, Vol.31(2), pp. 170-186, 1997.
9) Taniguchi, E., & Thompson, R.G.: Modeling city logistics, Transportation Research Records, Vol. 1790, pp. 45-51. 2002.
10) Qureshi, A.: Analysis of the Effects of Cooperative Delivery System in Bangkok, AIT Thesis No. TE-04-10, 2005.
11) Tavares J., *et al.*: GVR delivers it on time, 4th Asia Pacific Conference on Simulated Evolution and Learning (SEAL'02), Singapore 2002.
12) Tan, K.C., *et al.*: Artificial intelligence heuristics in solving vehicle routing problems with time window constraints, Engineering Applications of Artificial Intelligence, Vol.14, pp. 825-837, 2001.
13) Tan, K.C., *et al.*: Heuristic methods for vehicle routing problem with time windows, Engineering Applications of Artificial Intelligence, Vol.15, pp. 281-295, 2001.
14) Kolen, A. W. J., et al.: Vehicle routing with time windows, Operations Research, Vol.35(2), pp. 266-273, 1987.
15) Fisher, M. L., *et al.*: Vehicle routing with time windows – Two optimization algorithms, Operations Research, Vol.45(3), pp. 488-498, 1997.
16) Kallehauge, B., *et al.*: Lagrangian duality applied to the vehicle routing problem with time windows, Computers & Operations Research Vol.33, pp. 1464-1487, 2006.
17) Desrochers, M., *et al.*: A new optimization algorithm for the vehicle routing problem with time windows, Operations Research, Vol.40(2), pp. 342-354, 1992.
18) Irnich, S., & Villeneuve, D.: The Shortest Path Problem with K-Cycle Elimination (K3): Improving a Branch and Price Algorithm for the VRPTW, Technical Report G-2003-55, GERAD, 2003.
19) Feillet, D., *et al.*: An exact algorithm for the elementary shortest path problem with resource constraints: Application to some vehicle routing problems, Networks, pp. 216-229, 2004.
20) Chabrier, A.: Vehicle routing problem with elementary shortest path based column generation, Computers & Operations Research Vol.33, pp. 2972-2990, 2006.
21) Dror, M.: Note on the complexity of the shortest path models for column generation in VRPTW, Operations Research, Vol.42(5), pp. 977-978, 1994.
22) Solomon, M. M. Algorithms for the vehicle routing and scheduling problem with time windows constraints, Operations Research, Vol.35(2), pp. 254-265, 1987.