OPTIMAL CONGESTION PRICING DESIGN METHODS IN INTEGRATED LOCATION/TRANSPORT MODELS

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1. Introduction

Congestion pricing has been widely recognized as an effective means for reducing traffic congestion in urban areas. Theoretical researches have been extensively conducted on how to design optimal congestion pricing policies in transport networks. In particular, marginal cost pricing is considered as the best pricing scheme that optimizes certain welfare functions defined in transport network models. Various second best pricing schemes, including Cordon pricing, area pricing, have been designed when the marginal cost pricing can not be implemented due to practical and technical restrictions. However, most theoretical results are based on pure transport network models, and may fail to be optimal in a framework that takes into account the interaction between transport and other related activities. Most recently, there is a strong concern regarding what may be the best pricing principles from social economic point of view, see Rothengatter (2003) and Nash (2003), because theoretically sound and practically realizable pricing principles are indispensable for making national and international transport pricing policies. To address the problem of optimality of pricing principles, the ideal is to work on a mathematical model that considers most of the important factors related to transport. We believe that working on integrated location/transport models may serve as a first step toward this ideal.

The researches of urban economists on congestion pricing problems do consider location problems in their models, where the transport network structure is usually simplified in order to obtain analytical conclusions (see, e.g., Fujita, 1989). On the other hand, researches on computational methods for solving optimal design problems in integrated location/transport models have been conducted, but without rigorous mathematical algorithms for dealing with congestible transport networks (see Coelho and Williams, 1978; Boyce and Mattsson, 1999).

In this paper we propose a sensitivity analysis based algorithm for solving the optimal pricing problems in an integrated location/transport model. Although this algorithm is developed for a particular model, it illustrates a general principle of optimization in FOR general integrated location/transport models.

2. An integrated location/transport model

A transport network is represented by a directed graph which consists of a set of links $A=\{a,b,...\}$, and a set of nodes $N=\{r,s,...\}$. Let W denote the set of OD (origin destination) pairs. Let q_{rs} be the volume of travel demand for OD pair $rs\in W$. Let R_{rs} be the set of paths connecting r and s, $rs\in W$. Let $\boldsymbol{x}=(x_a, a\in A)$ denote the vector of link flows. Link travel time on link a is assumed to be a function of x_a: $t_a=t_a(x_a),\ a\in A$. The travel time of path k from r to s is written as $C_k^{rs}=\sum_{a\in A} t_a\delta_{k,a}^{rs}$ .The equilibrium link flows are given by

$$x_a=\sum_{rs\in W} q_{rs}\frac{\exp\left(-\theta C_k^{rs}(\boldsymbol{x})\right)\delta_{k,a}^{rs}}{\sum_{k\in R_{rs}}\left(-\theta C_k^{rs}(\boldsymbol{x})\right)},\ a\in A. \tag{1}$$

The expected minimum cost (disutility) S_{rs} for a trip from r to s is given by

*Keywords: congestion pricing, integrated location / transportation model, optimization
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$$S_{rs}=-\frac{1}{\theta}\ln\left(\sum\nolimits_{k\in R_{rs}}\exp(-\theta C_k^{rs}(\boldsymbol{x}))\right),\ rs\in W\ . \tag{2}$$

In the following a model consisting of employment and residential zones is considered. Let Z_E be the set of employment zones, Z_R the set of residential zones. It is assumed that each household has one worker commuting from a residential zone r to an employment zone s. Let D_s be the fixed number of employment in zone $s\in Z_E$. Travel demand q_{rs} from zone r to zone s is equal to the number of people choosing residence in zone $r\in Z_R$ and working in zone s. Assume $W=Z_R\times Z_E$, where W is the set of OD pairs.

Let $y_r=\sum\nolimits_{s\in Z_R}\left(nq_{rs}\right)$ denote the population in zone $r\in Z_R$, where n is a constant coefficient indicating the average number of residents per commuting worker; in the rest we assume $n=1$. Assume that the cost for locating in zone r is a function $c_r=c_r(y_r)$. The disutility of locating in r and working in s is given by $S_{rs}+c_r$. Assume that this disutility is perceived with an i.i.d. Gumbel random error of scale $1/\alpha$. Then the travel demands are functions given as follows

$$q_{rs}=D_s\frac{\exp(-\alpha(S_{rs}+c_r(\boldsymbol{y})))}{\sum\nolimits_{r\in Z_R}\exp(-\alpha(S_{rs}+c_r(\boldsymbol{y})))},\ rs\in W\ . \tag{3}$$

The aggregate utility of workers in employment zone s is (see Williams, 1977).

$$U_s=\frac{1}{\alpha}\ln\left(\sum\nolimits_{r\in Z_R}\exp(-\alpha(S_{rs}+c_r))\right) \tag{4}$$

The traffic flows and location population are in an equilibrium state if the following equations are satisfied

$$y_r-\sum\nolimits_{s\in Z_E}D_s\frac{\exp\left(-\alpha(S_{rs}+c_r(y_r))\right)}{\sum\nolimits_{r\in Z_R}\exp(-\alpha(S_{rs}+c_r(y_r)))}=0,\ r\in Z_R\ , \tag{5}$$

$$x_a-\sum\nolimits_{rs\in W}q_{rs}\frac{\exp\left(-\theta C_k^{rs}(\boldsymbol{x})\right)\delta_{k,ar}^{rs}}{\sum\nolimits_{k\in R_{rs}}\left(-\theta C_k^{rs}(\boldsymbol{x})\right)}=0,\ a\in A\ . \tag{6}$$

For later use, rewrite the equations in the following symbolic form

$$\begin{cases}G(\boldsymbol{y};\ S)=\ 0,\\ F(\boldsymbol{x};\ Q)=\ 0,\end{cases} \tag{7}$$

where $S=S(\boldsymbol{x}),\ Q=Q(\boldsymbol{y},\ S)=\ Q(\boldsymbol{y},S(\boldsymbol{x}))$.

3. Optimal pricing problem

Suppose that a toll T_a is imposed on traffic link $a\in A$, and a tax (or subsidy) L_r is imposed on location $r\in Z_R$. Then the total social welfare can be defined as the sum of the residents' total utility and the total taxes collected

$$SW=\sum\nolimits_{s\in Z_E}D_sU_s+\sum\nolimits_{a\in A}x_aT_a+\sum\nolimits_{r\in Z_R}y_rL_r\ , \tag{8}$$

Denote $f(\boldsymbol{x},\boldsymbol{y},\ T,L)=SW=U+\sum\nolimits_{a\in A}x_aT_a+\sum\nolimits_{r\in Z_R}y_rL_r$ as the objective function to be optimized. The optimal pricing scheme design problem can be formulated as follows.

$$\max\nolimits_{\{T,L\}}\ f(\boldsymbol{x},\boldsymbol{y},\ T,L) \tag{9}$$

subject to

$$\begin{cases}G(\boldsymbol{y};\ L,\ S)=\ 0,\\ F(\boldsymbol{x};\ T,\ Q)=\ 0,\end{cases} \tag{10}$$

$$\begin{cases} S = S(\boldsymbol{x},\ T), \\ Q = Q(\boldsymbol{y},\ L,\ S) = \ Q(\boldsymbol{y},\ L,\ S(\boldsymbol{x},\ T)), \end{cases} \tag{11}$$

$$T = \{T_a, a \in A\} \in \Omega_T,\ L = \{L_r, r \in Z_R\} \in \Omega_L,$$

where Ω_T and Ω_L are feasible ranges for traffic link tolls and location taxes, respectively. If the gradient $\left(\frac{\partial f}{\partial T}, \frac{\partial f}{\partial L}\right) = \left(f_x \frac{\partial \boldsymbol{x}}{\partial T} + f_y \frac{\partial \boldsymbol{y}}{\partial T} + f_T, f_x \frac{\partial \boldsymbol{x}}{\partial L} + f_y \frac{\partial \boldsymbol{y}}{\partial L} + f_L \right)$ of f with respect to the price variables is known, then existing mathematical programming algorithms can be applied to solve the optimization problem. This amounts to computing the following vector of partial derivatives $\left(\frac{\partial \boldsymbol{x}}{\partial T}, \frac{\partial \boldsymbol{y}}{\partial T}, \frac{\partial \boldsymbol{x}}{\partial L}, \frac{\partial \boldsymbol{y}}{\partial L}\right)$. This vector can be solved for by the following equations.

$$\begin{cases} \frac{\partial G}{\partial \boldsymbol{y}} \frac{\partial \boldsymbol{y}}{\partial L} + \frac{\partial G}{\partial S} \frac{\partial S}{\partial \boldsymbol{x}} \frac{\partial \boldsymbol{x}}{\partial L} + \frac{\partial G}{\partial L} = 0, \\ \frac{\partial F}{\partial \boldsymbol{x}} \frac{\partial \boldsymbol{x}}{\partial L} + \frac{\partial F}{\partial Q} \left[\frac{\partial Q}{\partial L} + \frac{\partial Q}{\partial \boldsymbol{y}} \frac{\partial \boldsymbol{y}}{\partial L} + \frac{\partial Q}{\partial S} \frac{\partial S}{\partial \boldsymbol{x}} \frac{\partial \boldsymbol{x}}{\partial L} \right] = 0 \end{cases} \tag{12}$$

$$\begin{cases} \frac{\partial G}{\partial \boldsymbol{y}} \frac{\partial \boldsymbol{y}}{\partial T} + \frac{\partial G}{\partial S} \frac{\partial S}{\partial \boldsymbol{x}} \frac{\partial \boldsymbol{x}}{\partial T} + \frac{\partial G}{\partial S} \frac{\partial S}{\partial T} = 0, \\ \frac{\partial F}{\partial \boldsymbol{x}} \frac{\partial \boldsymbol{x}}{\partial T} + \frac{\partial F}{\partial Q} \left[\frac{\partial Q}{\partial \boldsymbol{y}} \frac{\partial \boldsymbol{y}}{\partial T} + \frac{\partial Q}{\partial S} \frac{\partial S}{\partial T} + \frac{\partial Q}{\partial S} \frac{\partial S}{\partial \boldsymbol{x}} \frac{\partial \boldsymbol{x}}{\partial T} \right] + \frac{\partial F}{\partial T} = 0 \end{cases} \tag{13}$$

The cofficients of the above equations can be divided into two parts,

$$\left(\frac{\partial G}{\partial \boldsymbol{y}}, \frac{\partial Q}{\partial \boldsymbol{y}}, \frac{\partial G}{\partial S}, \frac{\partial G}{\partial L}, \frac{\partial Q}{\partial S}, \frac{\partial Q}{\partial L}\right) \quad \text{and} \quad \left(\frac{\partial F}{\partial \boldsymbol{x}}, \frac{\partial S}{\partial \boldsymbol{x}}, \frac{\partial F}{\partial Q}, \frac{\partial F}{\partial T}, \frac{\partial S}{\partial T}\right).$$

The first part can be calculated solely from the location equilibrium equations, for fixed transport costs; and the second part can be calculated solely from the traffic network equilibrium equations, for fixed transport demands (Ying and Miyagi, 2001).

4. A numerical example.

As shown in Figure 1, the network in question consists of seven residential zones (1,…,7) and two employment zones (8,9). The employment are given as $D_8 = D_9 = 500$. The location cost functions are assumed to have the form $c_r = c_{r0} + b_r(y_r - M_r)^2, r \in Z_R$, where M_r is intended to be the optimal population for location r. The parameters for the cost functions are given in Table 1.

Figure 1. Location Sites and Transport network.

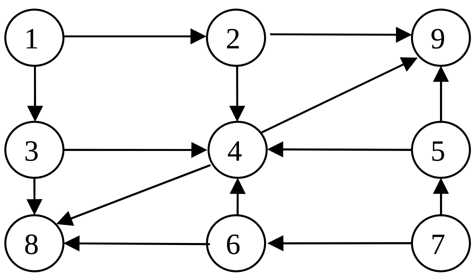

Table 1. Coefficients for Location Cost Functions.

zone	1	2	3	4	5	6	7
c_{r0}	100	100	100	200	100	100	100
b_r	0.002	0.002	0.002	0.001	0.002	0.002	0.002
M_r	20	20	20	20	40	40	40

Link travel time function is assumed to be of the BPR form $t_a = c_{a0}\left(1 + b_a\left(x_a / Cap_a\right)^4\right)$. These coefficients are assumed to be $c_{a0} = 10,\ Cap_a = 100,\ b_a = 0.15$, for all $a \in A = \{1,2,...,14\}$. The random error scale parameters in route and location choices are assumed to be $\theta = 0.5$ and $\alpha = 0.1$, respectively. Computations were carried out for the following cases.

- NoPolicy: No location taxes (subsidies), no tolls (residents incur average costs)
- MCPLT: MCP on both location and transport (residents incur marginal costs)
- MCPT: MCP on transport
- MCPL: MCP on location
- OptLT: Both location taxes (or subsidies) and tolls (negative tolls are forbidden) are designed based on optimization method
- OptT: Tolls are designed based on optimization method
- OptL: Location taxes (or subsidies) are designed based on optimization method

Table 2. Social Welfare, Travel Time, and Travel Disutility, for Various Pricing Schemes.

	NoPolicy	MCPLT	OptLT	MCPT	OptT	MCPL	OptL
-SW	39415.1	35842.7 (-9.1%)	35842.7 (-9.1%)	36388.5 (-7.7%)	36197.1 (-8.2%)	36029.5 (-8.6%)	35957.1 (-8.8%)
Travel Time	26928.1	25080.2 (-6.9%)	25086 (-6.8%	24691.7 (-8.3%)	24604.3 (-8.6%)	25497.1 (-5.3%)	25390.2 (-5.7%)
Travel Disutility	26899.3	71714 (+166.6%)	28922.7 (+7.5%)	71643.4 (+166.3%)	36103.2 (+34.2%)	25473.7 (-5.3%)	25368.2 (-5.7%)

From these results of the hypothetical example, it can be observed that:

(1) Social welfare optimum can be achieved by MCP rules or by optimization methods applied to both transport and location.

(2) Among pricing schemes restricted to transport network, MCP is not the best one, the best can be obtained by optimization methods.

(3) Location taxes or subsidies may be used to reduce travel time and to improve total social welfare.

(4) Travel disutility may be worsened by imposing tolls. Optimization method may provide more flexibility for deciding the levels of tolls.

5. Concluding remarks

An important step toward practical application of the proposed method is the implementation of the proposed method for optimal pricing scheme design in established integrated location/ transport models. And, public transit has not been considered in the paper, for practical application we need to include public transit system as alternative transport mode. All these topics remain to be studied in the future.

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