# 私的人流における時間価値の観測可能性とその時間節約便益計測への応用BENEFIT MEASUREMENT OF VALUE OF TIME USING OBSERVABLE DEMAND*

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## 1. Introduction

The role of measurement in the efficient allocation of resources is especially important in cases of public goods as in the case of transportation projects. An improvement in resource allocation requires that the benefits of a decision exceed its costs, which in turn requires the measurement of benefits and costs. Value of time is used to measure the time savings in cost benefit analysis of transportation projects or to calculate the generalized cost in the traffic demand forecasting model. It is defined as

$$(1) \qquad VOT \equiv -\frac{dp}{dt}\bigg|_{u \, or \, \pi = const.}$$

where $p$ is transport cost, $t$ is necessary transport time, $u$ is utility and $\pi$ is profit. The subjective value of time is the marginal rate of substitution between travel time and travel cost under constant utility or profit level. It is commonly referred to as the monetary appraisal of value of time or the willingness to pay for savings in travel time.

The current practice derives the value of time from specified functional forms of demand or utility. Previous papers on the valuation of passenger travel time have assumed specified demand and utility functions. Different specifications of the demand curve instead, not the utility function, would be fit with the best-fitting demand equation chosen to base the applied welfare analysis on.

Our approach is to measure the benefits of value of time in terms of *observable demand* that is consistent with utility-theoretic model. The use of utility-theoretic model allows for exact welfare measurement of the effects of changes in trip costs, travel duration and environmental quality. As utility is usually not easily observed, we need to find a way to observe the manifestations how the changes in utility are valued.

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We begin with the observed market demand and then derive the unobserved indirect utility function and expenditure function. This approach is analogous to the approach of Hausman (1981) and Larson (2001) who showed that the basic idea used in deriving the exact measure of consumer's surplus is to use the observed market demand curve to derive the unobserved compensated demand curve as it is the latter demand curve that leads to the compensating variation and equivalent variation.

The paper is organized as follows. First is the derivation of the value of time in terms of the observable demand $x$. The indirect utility function and expenditure function lead to the exact benefit measurement of the value of time. The equivalent variation approach is used to measure the welfare change. Finally, a summary and conclusions of the study is given.

## 2. Derivation of VOT for non-business person trips with observable demand

### 2.1 Individual behavior

To formulate the VOT, think of an individual on a non-business trip, that is, either commuting or shopping. Assuming neutrality in time, let $u(z,x,l)$ be the individual's preference function. The utility of an individual is assumed to be a function of the demand for commodity goods $z$ whose price is normalized to 1, transport service demand $x$, leisure time $l$ and duration of trip $t$. The individual maximizes utility subject to income $y$ and total available time $T$. The indirect utility function, $v(1,p,t,T,y)$, is given by

$$(2) \qquad v(1,p,t,T,y) = \max_{z,x,l} u(z,x,l)$$

$$(3) \quad \text{s.t.} \qquad z + px = y, \qquad tx + l = T$$

The utility function is assumed to be differentiable and quasi-concave in $x$, and the constraints are differentiable and linear in $x$ and in both money and time prices and money and time budgets. The Lagrangian function of (2) and (3) is represented by

(4)
$$V(1,p,t,T,y) = \max_{x} u(z,x,l) + \lambda(y - z - px) + \mu(T - l - tx)$$

The Lagrange multipliers $\lambda$ and $\mu$ represent the shadow values of money and time, respectively. Intuitively, $\lambda$ is the marginal utility of income and $\mu$ is the marginal utility of time.

The Marshallian demand $x(z, p, t, T, y)$ can be recovered through two separate versions of Roy's identity:

(5) $V_p = -V_y x$ , $V_t = -V_T x$

## 2.2 VOT expressed by observable demand

Recalling (1) and applying Roy's identities above, the value of time can be expressed as

(6) $$VOT \equiv -\left.\frac{dp}{dt}\right|_{v=const.} = \frac{V_t}{V_p} = \frac{-V_T x}{-V_y x} = \frac{V_T}{V_y} = \frac{\mu}{\lambda}$$

where subscript is partial derivative

Taking the derivative of (5) with respect to $t$ and $p$ to express the VOT in terms of $x$,

(7) $$\begin{aligned} V_{pt} &= (-V_y x)_t = -V_{ty} x - V_y x_t = -(-V_T x)_y x - V_y x_t \\ &= V_{Ty} x^2 + V_T x_y x - V_y x_t \end{aligned}$$

(8) $$\begin{aligned} V_{tp} &= (-V_T x)_p = -V_{pT} x - V_T x_p = -(-V_y x)_T x - V_T x_p \\ &= V_{yT} x^2 + V_y x_T x - V_T x_p \end{aligned}$$

Simplifying (7) and (8),

(9) $V_T(x_p + x x_y) = V_y(x_t + x_T x)$

Finally, VOT can be expressed in terms of observable demand $x$ as

(10) $$\left.VOT\right|_{v=const.} = \frac{V_T}{V_y} = \frac{x_t + x x_T}{x_p + x x_y}$$

From (10), value of time under the assumption of constant utility can be measured from observable changes in the Marshallian demand $x$ where $x_t$ is the change in demand with respect to change in time, $x_p$ is the change in demand with respect to change in transport cost, $x_T$ is the change in demand with respect to change in total available time and $x_y$ is the change in demand with respect to change in income.

## 2.3 Compensated VOT and extended Slutsky equations

The dual approach to the problem is to consider the associated minimization problem, which defines the expenditure function

(11) $$e(1, p, t, T, \bar{u}) \equiv \min_{h_0, h} h_0 + ph$$

(12) s.t. $th + h_2 = T,\ u(h_0, h, h_2) = \bar{u}$

where $h_i\ (i = 0,2)$ represents the demand for commodity goods $z$ and leisure $l$ respectively, $h$ is the compensated transport demand, $t$ is transport time, $T$ is total available time and $\bar{u}$ is given level of utility level.

The expenditure function may be expressed as the minimization of the Lagrange function,

(13) $$e = L = h_0 + ph + \xi(T - th - h_2) + \varphi(\bar{u} - u(h_0, h, h_2))$$

Then the following are derived from the envelope theorem applied to (15):

(14) $e_p = h$ , $e_t = -\xi h = -e_T h$ and $e_T = \xi$

Now going back to the definition of the value of time, it can also be expressed in terms of the expenditure function by substituting the envelope results in (14),

(15) $$VOT = -\left.\frac{dp}{dt}\right|_{\substack{e=const. \\ \bar{u}=const.}} = \frac{e_t}{e_p} = \frac{-e_T h}{h} = -e_T \equiv \rho$$

where $(-e_T)$ can be interpreted as the compensated value of time.

Now, to express the VOT in terms of the compensated demand $h$, take the derivative of the $e_p = h$ with respect tof $t$ and $e_t = -\xi h = -e_T h$ with respect to $p$, which results to $e_{pt} = h_t$ and $e_{tp} = (-e_T h)_p = -e_{pT} h - e_T h_p = -h_T h - e_T h_p$ .

Accordingly, it can be shown that

(16) $\rho = (h_t + h h_T)/h_p$

This compensated VOT can be expressed by the compensated demand function. The compensating demand function is always equal to the usual demand function at income equal to $e(1, p, t, T, u)$, which can be expressed as,

(17) $h(1, p, t, T, u) \equiv x(1, p, t, T, e(1, p, t, T, u))$

Taking the derivative of (17) with respect to $p$, $t$ and $T$, and recall that when $u$ is at $v(1, p, t, T, y)$ then it follows that $e(1, p, t, T, v(1, p, t, T, y, v)) \equiv y$ and accordingly, it can be derived that $x_e = x_y$, $h = x$ and $VOT = \rho = -e_T$, it can be shown that

(18a) $h_p = x_p + x x_y$

(18b) $h_t = x_t + (VOT) x x_y$

(18c) $h_T = x_T - (VOT) x_y$

Notice that (18a)-(18c) show the relationship between the observable uncompensated Marshallian demand and the unobservable Hicksian demand. These expressions lead to evaluating the compensated demand in terms of observable demand.

Note that (18a)-(18c) can be written taking the form similar to a Slutsky equation as follows:

(19a) $\rho_p = (VOT)_p + x(VOT)_y$

(19b) $\rho_t = (VOT)_t + (VOT)x(VOT)_y$

(19c) $\rho_T = (VOT)_T + (VOT)(VOT)_y$

## 3. Exact welfare measurement

The indirect utility function and the expenditure function provide the theoretical structure for welfare estimation. The theoretical method is to use information available about the expenditure function or its accompanying Hicksian demand curves for market goods to obtain compensating or equivalent variation measures of a change in a nonmarket good.

As the definition of value of time implies, it is the willingness to pay for savings in travel time so the changes that we will consider to measure the exact welfare changes between the before ($A$) and after ($B$) scenarios are time change $(t_A \to t_B)$, price change $(p_A \to p_B)$, income change $(y_A \to y_B)$ and change in utility $\left(v^A = v(1, p^A, t^A, T, y^A) \to v^B = v(1, p^B, t^B, T, y^B)\right)$. It is possible that these changes may occur individually or simultaneously depending on the given circumstances or the need of evaluation but to measure the exact welfare changes, we will consider a whollistic approach including all the changes.

### 3.1 Two expressions of equivalent variation (EV)

There are two ways to expressing EV in terms of observable demand. First, looking at point A, EV can be expressed as

$$\begin{aligned} EV &= e(A, v^B) - e(A, v^A) \\ &= \oint_{A\to B} e_y(A, v(1, p, t, T, y))(-xdp - xVOTdt + dy) \end{aligned} \quad (20)$$

where $\oint$ is line integral, subscript is partial derivative

Evaluating $e_y(A, v)$, it can be shown that,

$$e_y(A, v) = \frac{e_v(A, v)}{v_y(1, p, t, T, y)} = \frac{v_y(1, p, t, T, y)}{v_y(1, p^A, t^A, T, y)} \quad (21)$$

So $e_y$ may be called as the marginal utility ratio of income. Note that $e_y(A, v(A, y)) \equiv y$ at any $y$, therefore $e_{yy} = 0$.

Another way of expressing EV is as,

$$\begin{aligned} EV &= e(A, v^B) - e(A, v^A) \\ &= e(A, v^B) - e(B, v^B) + y^B - y^A \\ &= \oint(-hdp - h\rho dt) + \Delta y \end{aligned} \quad (22)$$

where $\Delta y = y^B - y^A$, $h = h(i, p, t, T, v^B)$

In order to calculate and express (20) and (22) in terms of consumer's surplus, we take linear approximation with respect to the demand function $x, h$ and marginal utility ratio of income $e_y$ for both cases. This means we will take a second order approximation of EV.

### 3.2 Approximation by marginal utility ratio of income

By linear approximation, (20) can be re-written as

$$\begin{aligned} EV &= \frac{1}{2}(x^A + e_y^B x^B)(p^A - p^B) \\ &+ \frac{1}{2}(x^A VOT^A + e_y^B x^B VOT^B)(t^A - t^B) \\ &+ \frac{1}{2}(1 + e_y^B)(y^B - y^A) \end{aligned} \quad (24)$$

since $e_y^A = 1$, it needs to express only $e_y^B = e_y(A, v^B)$ in terms of observable demand. So $e_y^B$ can be linearly approximated and expressed in terms of observable demand as,

$$e_y^B = e_y^A + e_{yp}^A(p_B - p_A) + e_{yt}^A(t_B - t_A) + e_{yy}^A(y_B - y_A)$$

where

$$e_{yy}^A = 0, e_{yp}^A = e_{py}^A = \left[e_v^A(A, v)v_p\right]_y^A = \left[-e_y(A, v)x\right]_y^A = -x_y^A,$$

$$e_{yt}^A = e_{ty}^A = \left[e_v v_t\right]_y^A = \left[-e_v v_T x\right]_y^A = -\left[xVOT\right]_y^A$$

Simplifying, it becomes,

$$e_y^B = 1 - x_y^A(p_B - p_A) - (xVOT)_y^A(t_B - t_A) \quad (23)$$

The above formula, (24), is the well-known trapezoidal formula for area with modification of $x^B$ by $e_y^B x^B$ where $e_y^B$ is an addition by income effect.

From (22), it can be shown that by linear approximation,

$$\begin{aligned} EV &= -\frac{1}{2}(h(A, v^B) + h^B)(p^B - p^A) \\ &- \frac{1}{2}(h(A, v^B)\rho(A, v^B))(t^B - t^A) + \Delta y \end{aligned} \quad (25)$$

where $h^B = h(B, v^B) = x^B$, $\rho^B = \rho(B, v^B) = (VOT)^B$

The term $h(A, v^B)$ in (25) can be linearly approximated from point B as,

$$h(A, v^B) = h(B, v^B) + h_p^B(p_A - p_B) + h_t^B(t_A - t_B)$$

where
$$h_p^B = x_p^B + x^B x_y^B$$
$$h_t^B = x_t^B + (VOT)^B x^B x_y^B$$

Simplifying,

$$
(26) \quad h(A, v^B) = x^B + \left(x_p^B + x^B x_y^B\right)\left(p^A - p^B\right) + \left(x_t^B + (VOT)^B x^B x_y^B\right)\left(t^A - t^B\right)
$$

Thus $h(A, v^B)$ can be expressed as an addition to the Marshallian demand due to price and time substitution effect.

### 3.3 Comparison of the two methods

The difference between the two methods become clear by looking only at the price change as illustrated in Figure 1.

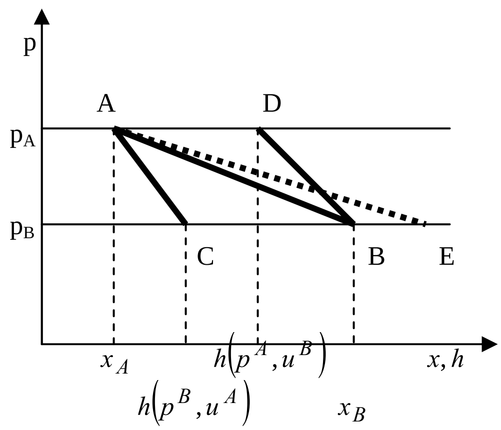


Figure 1

AB: Marshallian demand function

AC,DB: Hicksian demand at $v^A$ and $v^B$

AE= $e_y^A(A, v)x$ : modified demand function

Equation (24) is the expression of

EV= trapezoid $p_A AEp^B$

while (25) is the expression of

EV= trapezoid $p_A DBp^B$

Time change can be expressed graphically the same except by expressing demand in terms of $(xVOT)$ or $(h\rho)$.

## 4. Summary and Conclusions

This paper develops an approach for welfare measurement of the value of time based on observable demand. While the current practice derives the value of time assuming specified utility and demand functions, analysis thru observable demand is possible and is consistent with utility-theoretic model. The method is applicable to the general case evaluating business and non-business trips both for persons and freight.

It has been known that the use of compensated demand curves lead to appropriate welfare measures. However, as applied to the benefit measurement of the value of time it has not been shown how can this be derived from observable demand. The indirect utility function and expenditure function provide the appropriate compensated demand curve and thus the appropriate welfare function.

Exact welfare measurement for the value of time by equivalent variation considering time change, price change, income change and utility change is demonstrated. Two methods are shown on how to express the welfare change and the comparison of the methods is shown graphically. From the methods shown, exact welfare measurement is calculated consistently from observable demand.

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