DIAGNOSIS OF VALUE OF TIME: VALUE OF TIME AS A RESOURCE, VALUE OF SAVING TIME, VALUE OF EXTENDING TIME AND VALUE OF ACTIVITY TIME

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Abstract: The objective of this study is to diagnose the value of time and its different concepts. In its traditional sense, value of time in transportation research corresponds to the value of saving time in travel. However, there are different concepts of value of time other than value of saving time, such as value of time as a resource, value of extending time and value of activity time. In this paper, we formulate a microeconomic model of utility maximization in order to derive and explain each of these concepts and their relationships. These concepts and relationships are useful for evaluating transportation projects.

1. Introduction

In its traditional sense, the value of time (commonly known as, *VOT*) in transportation engineering and planning corresponds to the value of saving time (*VOST*) in travel; benefits of travel time saving are extensively used in the evaluation of transportation projects. This is the reason why the existing researches are limited particularly to the estimation of value of saving time (*VOST*) in travel. However, it should be clearly understood that the term '*VOT*" is rather confusing and should better be refined with particular circumstances. It should be noted that there should be some restrictions (constraints) on time use in order to observe some value that an individual is willing to pay when the associated restriction is released (relaxed), if he or she prefers such relaxation.

While allocating individual's total available time to different activities, we generally face two types of time use constraints: time resource constraint and consumption time constraint (Bain, 1976). Time resource is the fixed time endowment for doing different activities, and hence, a constraint. This time resource constraint requires that the amount of time allocated to a specific activity add up to the total time available. This relationship follows directly from the assumption that activities are performed one at a time and all available time is allocated to the activities. It is not always the case that only one activity can be done at one time. Several activities can be done at the same time, such as, reading newspaper while traveling by transit, listening music while working and so on. However, we can classify the activities in order to satisfy the above-mentioned criteria.

Each activity can be performed only at the expense of time. The amount of time allocated to a consumption activity is partly a matter of choice, and partly a matter of necessity. The consumption time constraint applies only when it is binding. The binding constraint is either upper-bounded or lower-bounded. For example, we have to spend a minimum necessary travel time in traveling, but we generally prefer to spend as little travel time as possible. Hence, travel is considered as a lower-bounded consumption activity. However, we can spend only fixed duration at seeing movie in a theater and, prefer the extended duration if the movie is really interesting. Hence viewing a movie can be classified into upper-bounded consumption activity. It should be noted that most daily consumption activities are not bounded by consumption time constraint. Example includes; eating, sleeping, participating to recreational activities, shopping etc. This paper is heavily drawn from Nepal (2005).

2. Microeconomic Model

In addition to income constraint, both resource time constraint and consumption time constraints, as discussed in the previous section, play a vital role in time valuation. To make it clearer, we clarify the different concepts of value of time (*VOT*) using a mathematical optimization problem. For this, let us classify all daily activities into three mutually exclusive and collectively exhaustive groups: (i) activities whose consumption time is unbounded (e.g., time required for eating), (ii) activities whose consumption time is lower-bounded (e.g., traveling time) and, (iii) activities whose consumption time is upper-bounded (e.g., time spent seeing a movie in a theater). Let us formulate an activity time allocation model in which an individual is assumed to maximize his or her daily utility by participating in different activities and consuming various goods (expressed in terms of composite goods) under above-mentioned constraints. The subscript for an individual is omitted for the convenience of mathematical notation.

Keywords: value of time, microeconomic model, different concepts of value of time

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For mathematical formulation, let us denote activities whose consumption times are not bounded by $(1,2,...,j,...,J)$, activities whose consumption times are lower-bounded by $(1,2,...,k,...,K)$ and, activities whose consumption times are upper-bounded by $(1,2,...,l,...,L)$. Then, under both time (both resource time and consumption time) constraint and income constraint, an extended microeconomic activity time allocation model over DeSerpa (1971) and Becker (1965) can be formulated as follows:

Maximize:

$$U = U(t_1,...,t_j,...,t_J;t_1,...,t_k,...,t_K,;t_1,...,t_l,...,t_L;Z) \tag{1}$$

Subjected to:

$$\sum_{j=1}^{J} t_j + \sum_{k=1}^{K} t_k + \sum_{l=1}^{L} t_l = T \qquad \Rightarrow \mu \tag{2}$$

$$t_k \geq t_k^{MIN} \qquad \Rightarrow \gamma_k, \forall k \in K \tag{3}$$

$$t_l \leq t_l^{MAX} \qquad \Rightarrow \gamma_l, \forall l \in L \tag{4}$$

$$F - \sum_j (r_j \times t_j) - \sum_k (r_k \times t_k) - \sum_l (r_l \times t_l) - PZ = 0 \Rightarrow \lambda \tag{5}$$

where:

U = Total utility of an individual over a period *T*;

t_j = Time allocated to non-bounded consumption activity $j \in J$;

r_j = Unit cost of participation to a non-bounded consumption activity $j \in J$;

t_k = Time allocated to lower-bounded consumption activity $k \in K$;

r_k = Unit cost of participation to a lower-bounded consumption activity $k \in K$;

t_l = Time allocated to upper-bounded consumption activity $l \in L$ during a study period;

r_l = Unit cost of participation to an upper-bounded consumption activity $l \in L$;

Z = Quantity of composite goods consumed;

P = Price of composite goods Z;

t_k^{MIN} = Minimum required consumption time to lower-bounded consumption activity $k \in K$ during a study period;

t_l^{MAX} = Maximum possible consumption time to upper-bounded consumption activity $l \in L$ during a study period;

F = Unearned income of an individual; and

$\mu, \lambda, \gamma_k, \gamma_k$ = Lagrange multipliers.

This constrained utility maximization problem clearly shows the time resource constraint, Equation (2), consumption time constraint for lower-bounded consumption activities, Equation (3), consumption time constraint for upper-bounded consumption activities, Equation (4) and income constraint, Equation (5). We define different terminologies related to value of time (*VOT*) from this constrained utility maximization problem.

3. DIFFERENT CONCEPTS OF VALUE OF TIME

1) Time Resource Constraint and Value of Time as a Resource (VOTR)

When an individual is interested to extend the total available fixed time, there is a value he or she is willing to pay for this extension. Saying in another words, if an individual is not interested to extend total fixed period of time, he or she will not be interested to pay for the extended total time period and hence, the value of such extension for him or her will be zero. However, since individuals generally prefer to have an extended longer total period of time, individuals are willing to pay some amount of money for the extended period of time, no matter how much which depends and varies over individuals. Lagrange multiplier μ, marginal utility of total available time, indicates the increase in the utility when the total time available is relaxed by a unit and can be converted into monetary term dividing by marginal utility of money, λ.

Hence the ratio of these two constraints is the value of extension of time resource or total amount of time, called value of time as a resource (*VOTR*), and is given by the following mathematical equation:

$$VOTR = \frac{\mu}{\lambda} \tag{6}$$

Note that the value of time as a resource (*VOTR*) is independent of the type of activities. So, value of time as a resource (*VOTR*) is constant and equal for all activities, irrespective of whether the consumption time of an activity is constrained or not.

2) Consumption Time Constraint and Value of Saving Time (VOST)

When the consumption time is lower-bounded i.e. consumption of an activity requires certain minimum amount of time to be allocated and in the same time, an individual prefers to allocate as minimum time as possible such as in travel, there is a value that an individual is willing to pay in order to further reduce the minimum required amount of consumption time. Lagrange multiplier γ_k, marginal utility of consumption time, indicates the increase in the utility when the minimum necessary consumption time of activity k is relaxed by a unit and can be converted into monetary term dividing by marginal utility of money, λ. Hence the ratio of these two constraints is seen as the value of saving time in lower-bounded consumption-time-constrained activity and is called value of saving time (*VOST*) in that activity. This value is fully depend on the types of activity which is constrained by the lower-bounded consumption time, and is given by the following mathematical equation:

$$VOST_k = \frac{\gamma_k}{\lambda} \tag{7}$$

In this case, the value of saving time (*VOST*) in consumption-time-constrained activity depends upon the type of activity under consideration. In transportation, value of saving time (*VOST*) in travel has important implications. The terminologies, value of time (*VOT*), value of travel time savings (*VTTS*) or subjective value of travel time savings (*SVTTS*) are commonly used to represent this quantity.

Since the majority of transportation analysis is directly related to this quantity, the value of saving time (*VOST*) in travel and its role in the transportation projects is the major theme of the researches in transportation economics. Not only the travel demand modeling and forecasting for transport service is influenced by the economic value of time but also it plays a central role for evaluating transportation projects because of being a single largest contributor to the benefits in deciding about investment appraisal and policy, and has important welfare implications. The value of saving time (*VOST*) in travel are being adopted by transport planning agencies as only one of the major determinant of road project evaluation, urban transit appraisal and transport infrastructure investment analysis. It can shed important light as to whether any traffic control measures (*TCM*) will increase social welfare or not and help us understand how commuters make their travel decisions. Quantifying the value of travel time and associated benefit is a very important but challenging issue for the transportation planner and policy makers. Travel time saving benefit comprises the main component among other benefits that transportation infrastructures provide, not only to the transport service users and but also for the non-users.

3) Consumption Time Constraint and Value of Extending Time (VOET)

When the consumption time is upper-bounded, i.e., consumption of an activity requires certain maximum possible amount of time to be allocated and in the same time, an individual prefers to allocate as maximum time as possible such as movie show duration, there is a value that an individual is willing to pay in order to further increase this maximum possible amount of consumption time. Lagrange multiplier γ_l, marginal utility of consumption time, indicates the increase in the utility when the maximum possible consumption time of an activity, l, is relaxed by a unit. It can be converted into monetary term dividing by marginal utility of money, λ. Hence the ratio of these two constraints is seen as the value of extending time in upper- bounded- consumption- time- constrained activity and is called value of extending time (*VOET*) in that particular activity, given by:

$$VOET_l = \frac{\gamma_l}{\lambda} \tag{8}$$

Similar with value of saving time (*VOST*), the value of extending time (*VOET*) in upper- bounded- consumption- time-constrained activity also depends on the type of activity under consideration. Since transportation professionals, in general, do not deal with such activities; the value of extending (*VOET*) is not analyzed in sufficient detail till date. However, we should take into account such activities, if we maximize the daily activity time allocation model using total utility of the day.

4) Variation in Consumption Time and Value of Activity Time (VOAT)

Whether or not there is a consumption time constraint (lower-bounded or upper-bounded), an individual's utility changes with the changes in the duration of consumption activity time. In this case, the valuation of time allocated to an activity due to direct variation in utility can be seen just the value as a commodity and is known as value of activity time

(*VOAT*). It is sometimes useful to evaluate directly how important is the activity for an individual. The value of activity time (*VOAT*) may be defined as *"Value of activity time (VOAT) is the marginal rate of substitution of time allocated to an activity for money and usually estimated by taking the ratio of marginal utility of time allocated to a particular activity and marginal utility of total money budget constraint"*. The terms value of activity time, value of time as a commodity and activity time value are used interchangeably.

In some activities, the extension of consumption time duration increases the utility such as for recreational activities (positive *VOAT*). But in other activities, the extension of consumption time duration decreases the utility such as for travel time (negative *VOAT*). In our model, the value of time allocated to activity $q \in j,k,l$ can be estimated by taking the ratio of the marginal utility of time allocated to activity $q \in j,k,l$ and the marginal utility of money budget constraint as shown below:

$$VOAT_q = \frac{\partial U / \partial t_q}{\lambda}, \qquad q \in j,k,l \tag{9}$$

3. RELATIONSHIPS BETWEEN DIFFERENT CONCEPTS OF VALUE OF TIME

We can derive the relationships among these different concepts of value of time (*VOT)* using a general activity time allocation model. Lagrange function of the optimization problem in Equation (1), Equation (2), Equation (3), Equation (4) and Equation (5) is:

$$\begin{aligned} L = U(.) + \mu\left(T - \sum_{j=1}^{J} t_j - \sum_{k=1}^{K} t_k - \sum_{l=1}^{L} t_l\right) + \sum_k \gamma_k \left(t_k - t_k^{MIN}\right) + \\ \sum_l \gamma_l \left(t_l^{MAX} - t_l\right) + \lambda\left(F - \sum_j (r_j \times t_j) - \sum_k (r_k \times t_k) - \sum_l (r_l \times t_l) - PZ\right) \end{aligned} \tag{10}$$

The first order conditions are:

$$\frac{\partial L}{\partial t_j} = \frac{\partial U}{\partial t_j} + \mu(-1) + \lambda(-r_j) = 0 \Rightarrow \frac{\partial U}{\partial t_j} = \mu + \lambda \times r_j \tag{11}$$

$$\frac{\partial L}{\partial t_k} = \frac{\partial U}{\partial t_k} + \mu(-1) + \gamma_k(1) + \lambda(-r_j) = 0 \Rightarrow \frac{\partial U}{\partial t_j} = \mu + \lambda \times r_j - \gamma_k \tag{12}$$

$$\frac{\partial L}{\partial t_l} = \frac{\partial U}{\partial t_l} + \mu(-1) + \gamma_l(-1) + \lambda(-r_l) = 0 \Rightarrow \frac{\partial U}{\partial t_l} = \mu + \lambda \times r_l + \gamma_l \tag{13}$$

From these first order conditions and the definitions of different concepts of value of time (*VOT*) in the above mathematical equations, the following relationships among these different values can be established:

$$VOAT_j = VOTR + r_j \tag{14}$$

$$VOAT_k = VOTR + r_k - VOST_k \tag{15}$$

$$VOAT_l = VOTR + r_l + VOET_l \tag{16}$$

These relationships shows the different concepts of time value (*VOT*) are interrelated and estimating all of these components are very difficult unless we develop an integrated modeling framework.

4. SYNTHESIS AND CONCLUSIONS

The paper discussed in-depth the value of time and its different concepts. In addition to value of saving time in travel, there are different concepts of value of time, such as value of time as a resource, value of extending time and value of activity time. We formulated a microeconomic model of utility maximization in order to derive and explain each of these concepts and their relationships. Empirical estimation of these values are left for further research.

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