TRAVEL TIME RELIABILITY ANALYSIS IN MIXED TRAFFIC NETWORK UNDER PROVISION OF ADVANCED TRAVELER INFORMATION SYSTEM *

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1. Introduction

Recently, transportation network reliability has attracted more attention from many researchers and planners along with the increasing need of the society for a more reliable transportation system which is very important for people's daily commute. In urban network, unreliable travel time is one of the major disturbances for commuters, since it is the main source of uncertainty in arriving at the destination within an acceptable travel time so that it is difficult for travelers to schedule their trip. This problem gives rise to the study of travel time reliability. It has been argued that travel time variation can be caused by daily fluctuation of supply side or non-recurrent congestion[1)]; and day-to-day variation of demand[2)]. Therefore, some drivers may try to seek any possible mean that might help them to improve their travel time reliability by accepting to pay some money.

In the age of advanced technology and development of ITS, the Advanced Traveler Information System (ATIS) which provides predictive information or the information on current network condition at pre-trip stage can be an attractive mean for them. However, whether ATIS can help to improve the reliability of equipped drivers is very questionable; and not so many researches have devoted in this field. To our knowledge, Lam *et al.*[3)] are the first authors who analyze the reliability in terms of generalized travel cost under provision of predictive information while traffic flow is subject to random day-to-day fluctuation.

It should be noted that while realized network always consists of various kinds of vehicle types (e.g. large and small, or slow and fast vehicles) which have different impacts on each other, none of the above models has incorporated mixed traffic into the study of reliability. For this reason, previous studies may provide a misleading interpretation in real world application, especially when network exhibits many modes having strongly different performance (e.g. car and motorbike).

In this paper, the mixed traffic condition is explicitly handled, by assuming the performance function of each mode as endogenously given in the analysis of travel time reliability under day-to-day variation, when the information on current network condition is provided to equipped drivers.

2. Notations and Assumptions

For the sake of simplicity, the notations that will be used throughout the paper are defined as follows: q_{od} is the mean OD demand (given by OD matrix) corresponding to random variable Q_{od} (person trips/hr). σ_{od}^2 is the variance of OD demand. q_{od}^i is the mean OD demand of mode i corresponding to random variable Q_{od}^i (i = 1, 2, …, n) measured in person trips per hour. σ_{od}^{ij} is the covariance of OD demand by mode i and j. $p_{i|od}$ is the probability of choosing mode i for a given OD pair. R_{od} represents the index set of paths connecting the pair OD. $p_{r|i}^{od}$ is probability or proportion that the driver of mode i chooses path r ($r \in R_{od}$) to travel from O to D. λ_i^{od} is the vehicle occupancy of mode i for a given OD pair (passengers/vehicle). f_{ri}^{od} is the stationary mean flow of mode i on path r ($r \in R_{od}$). v_{ai} is the realized flow of mode i on link a (a = 1, 2, …, A). μ_{ai} is the mean flow of mode i on link a. σ_a^{ij} is the covariance of flow by mode i and j on link a. α_{ij} is the flow converting factor of mode i to mode j which is equal to 1.0 if $i = j$. C_{ai} is the capacity of link a for mode i (vehicle/hr). t_{0ai} is the free-flow travel time of link a for mode i. t_{ai} is realized travel time on link a for mode i. $\mu_{T_{ai}}$, $\sigma_{T_{ai}}^2$ are mean and variance respectively of travel time on link a for mode i. $c_{r|i}^{od}$ is the realized travel time on path r for mode i traveling from O to D. $\mu_{C_{r|i}^{od}}$, $\sigma_{C_{r|i}^{od}}^2$ are the mean and variance respectively of travel time on path r for mode i. $\mu_{i,od}$, $\sigma_{i,od}^2$ are the mean and variance of OD travel time for mode i corresponding to random variable $T_{i,od}$. γ_i, β_i are non-negative constant and integer respectively. δ_{ar}^{od} is link-path incidence index for a pair OD being equal to 1 if link a is part of path r and 0 otherwise. V_{ai}, $C_{r|i}^{od}$, T_{ai} are random variables of v_{ai}, $c_{r|i}^{od}$, t_{ai}.

In the study of day-to-day dynamics, the period that is considered to analyze the system variation can be part of the day (e.g. morning rush hour) or the whole day. Hereafter in this paper the term *reference period* and *day* will be used interchangeably to refer to the period under study. The reference period is further divided into *subintervals* since it is more relevant in the framework of providing pre-trip information to drivers. Before proceeding, the following assumptions are adopted throughout: 1) The actual demand for each OD pair is randomly distributed in the day-to-day context; and all OD demand on each day is statistically independent across OD pairs. The OD demand is inelastic with network level of service. 2) For each OD demand of any one day, travelers choose independently between alternative modes i (i = 1, 2,…, n) with constant probabilities $p_{i|od}$. Over time, these probabilities remain unchanged regardless of network travel time. 3) Conditional on OD demand realized on any one day, driver on a given mode i is assumed to choose independently between alternatives paths $r \in R_{od}$ with constant probability (route choice fraction) $p_{r|i}^{od}$. 4) The departure rate is uniformly distributed within the reference period, i.e. the same number of travelers departs at each subinterval; and the duration of each subinterval is large enough so that usual definition of

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link-path incidence maintains its significance. Moreover, travelers are assumed not to have incentive to change their departure time from one day to another so that route choice is the only decision open to them. 5) All drivers have the knowledge of travel time history for the time they depart through their own experience for the routes they used and via the information provided by media for the unused routes. Additionally, the drivers equipped with ATIS can have the knowledge about network travel time in the previous intervals of the same day through information supplied by ATIS. 6) The pre-trip information about the current network condition is provided at every subinterval of the reference period.

3. Model Formulation

(1) Determination of link flow moments

Given the relationship $Q_{od}^{i} = p_{i|od}\, Q_{od}$, the mean, variance and covariance of modal OD demand can be determined by $q_{od}^{i} = p_{i|od}\, q_{od}$, $\sigma_{od}^{ii} = p_{i|od}^{2}\, \sigma_{od}^{2}$, and $\sigma_{od}^{ij} = p_{i|od}\, p_{j|od}\, \sigma_{od}^{2}$ respectively. Therefore, link flow which is related to OD demand via relation $V_{ai} = \sum_{od}\sum_{r\in R_{od}} (1/\lambda_i^{od})\delta_{ar}^{od}\, p_{r|i}^{od}\, Q_{od}^{i}$, can be deduced as follows:

Mean $$\mu_{ai} = \sum_{od}\sum_{r\in R_{od}} (1/\lambda_i^{od})\,\delta_{ar}^{od}\, p_{r|i}^{od}\, q_{od}^{i} = \sum_{od}\sum_{r\in R_{od}} (1/\lambda_i^{od})\,\delta_{ar}^{od}\, p_{r|i}^{od}\, p_{i|od}\, q_{od} \tag{1}$$

Variance $$\sigma_a^{ii} = \sum_{od} (1/\lambda_i^{od\,2})\left(\sum_{r\in R_{od}} \delta_{ar}^{od}\, p_{r|i}^{od}\right)^2 \sigma_{od}^{ii} = \sum_{od} (1/\lambda_i^{od\,2})\left(\sum_{r\in R_{od}} \delta_{ar}^{od}\, p_{r|i}^{od}\right)^2 p_{i|od}^{2}\, \sigma_{od}^{2} \tag{2}$$

Covariance $$\sigma_a^{ij} = \sum_{od}\sum_{r\in R_{od}}\sum_{r'\in R_{od}} (1/\lambda_i^{od}\lambda_j^{od})\,\delta_{ar}^{od}\delta_{ar'}^{od}\, p_{r|i}^{od}\, p_{r'|j}^{od}\, p_{i|od}\, p_{j|od}\, \sigma_{od}^{2} \tag{3}$$

Equation (3) can be demonstrated by the following arguments:

$$\sigma_a^{ij} = Cov(V_{ai}, V_{aj}) = Cov\left(\sum_{od}\sum_{r\in R_{od}} (1/\lambda_i^{od})\,\delta_{ar}^{od}\, p_{r|i}^{od}\, Q_{od}^{i}, \sum_{o'd'}\sum_{r'\in R_{o'd'}} (1/\lambda_j^{o'd'})\,\delta_{ar'}^{o'd'}\, p_{r'|j}^{o'd'}\, Q_{o'd'}^{j}\right)$$

$$= \sum_{od}\sum_{o'd'}\sum_{r\in R_{od}}\sum_{r'\in R_{o'd'}} (1/\lambda_i^{od}\lambda_j^{o'd'})\,\delta_{ar}^{od}\delta_{ar'}^{o'd'}\, p_{r|i}^{od}\, p_{r'|j}^{o'd'}\, Cov(Q_{od}^{i}, Q_{o'd'}^{j})$$, where $Cov(Q_{od}^{i}, Q_{o'd'}^{j}) = p_{i|od}\, p_{j|o'd'}\, Cov(Q_{od}, Q_{o'd'})$

According to the assumptions above, the random OD demand of different OD pairs are mutually independent, implying that $Cov(Q_{od}, Q_{o'd'}) = 0$. Therefore, $Cov(Q_{od}^{i}, Q_{o'd'}^{j}) = 0$ and $Cov(Q_{od}^{i}, Q_{od}^{j}) = \sigma_{od}^{ij}$ from which the above equation can be reduced to $\sigma_a^{ij} = \sum_{od}\sum_{r\in R_{od}}\sum_{r'\in R_{od}} (1/\lambda_i^{od}\lambda_j^{od})\,\delta_{ar}^{od}\delta_{ar'}^{od}\, p_{r|i}^{od}\, p_{r'|j}^{od}\, \sigma_{od}^{ij}$, which is equivalent to (3).

It is worth to note that by Central Limit Theorem, (1), (2) and (3) are approximately the moments of multivariate Normal V_{ai}. This approximation becomes more realistic the larger the number of OD pairs and paths passing through link *a*. Moreover, it is also important to note that in the above formulation route choice fraction is assumed to be constant. Therefore, the specification of $p_{r|i}^{od}$ is apparently required. In this model, we adopt route choice fraction that is derived from the stationary probability distribution of the stochastic process (SP) proposed by Cascetta[4)] based on mean OD demand. The reason that SP model is adopted in our multi-modal assignment is due to its powerful results in route choices and flows when the system state evolves over time. It has been proved that the temporal evolution of the transportation system is dynamically stable and ergodic under some mild conditions regardless of link performance function, i.e. it admits steady-state probabilities from which mean and moments of link flows can be deduced analytically. Moreover, the stationarity and ergodicity of the system still hold when drivers update network travel time up to their departure interval based on travel time that occurred in the previous subintervals of the same reference period, which can be accomplished only by provision of pre-trip information[5)]. From the computational point of view, although the stationary probability distribution can be calculated analytically, Monte Carlo simulation approach exhibits more advantages in large network problem[4),5)] so that it will be adopted in this paper (cf. chapter 4).

(2) Calculation of link travel time moments

For the sake of simplicity, the index *a* will be dropped from the formulation below. Then, since there are *n* modes on a link, the travel time of any one mode is affected by flows of the others. So, the performance function of mode *j* can be expressed by:

$$T_j = t_{0j}\left(1 + \gamma_j\left(\sum_{i=1}^{n}\alpha_{ij}V_i / C_j\right)^{\beta_j}\right) \tag{4}$$

Then, moments: $\mu_{T_j} = t_{0j} + \left(\gamma_j t_{0j} / C_j^{\beta_j}\right) E\left[\left(\sum_{i=1}^{n}\alpha_{ij}V_i\right)^{\beta_j}\right]$, and $\sigma_{T_j}^{2} = \left(\gamma_j^{2} t_{0j}^{2} / C_j^{2\beta_j}\right)\left\{E\left[\left(\sum_{i=1}^{n}\alpha_{ij}V_i\right)^{2\beta_j}\right] - E^{2}\left[\left(\sum_{i=1}^{n}\alpha_{ij}V_i\right)^{\beta_j}\right]\right\}$

So, we need to calculate the terms $S_j = E\left[\left(\sum_{i=1}^{n}\alpha_{ij}V_i\right)^{\beta_j}\right]$ and $S_j^{'} = E\left[\left(\sum_{i=1}^{n}\alpha_{ij}V_i\right)^{2\beta_j}\right]$ by using Multinomial series.

Firstly, define the set $A(n,\beta_j)$ of *n*-dimensional non-negative integers a_i, where $A(n,\beta_j) = \left\{a_i : i = 1,2,...,n \text{ and } \sum_{i=1}^{n} a_i = \beta_j\right\}$

By Multinomial expansion, S_j can be written as: $$S_j = \sum_{a_i\in A(n,\beta_j)} \frac{\beta_j!}{\prod_{i=1}^{n} a_i!} E\left[\prod_{i=1}^{n}\alpha_{ij}^{a_i}\, V_i^{a_i}\right] = \sum_{a_i\in A(n,\beta_j)} \frac{\beta_j!}{\prod_{i=1}^{n} a_i!}\prod_{i=1}^{n}\alpha_{ij}^{a_i}\cdot E\left[\prod_{i=1}^{n} V_i^{a_i}\right] \tag{5}$$

Then, we can decompose the right side of (5) by: $$E\left[\prod_{i=1}^{n} V_i^{a_i}\right] = E\left[\prod_{i=1}^{n}\left[(V_i - \mu_i) + \mu_i\right]^{a_i}\right] \tag{6}$$

By using Binomial expansion, we can have: $\left[(V_i - \mu_i) + \mu_i\right]^{a_i} = \sum_{k_i=0}^{a_i}\binom{a_i}{k_i}\mu_i^{a_i-k_i}\,(V_i - \mu_i)^{k_i}$, where $\binom{a_i}{k_i} = \dfrac{a_i!}{k_i!\,(a_i - k_i)!}$

Then, we can write (6) into a new form:

$$E\left[\prod_{i=1}^{n}\left[(V_i-\mu_i)+\mu_i\right]^{a_i}\right]=E\left[\prod_{i=1}^{n}\left(\sum_{k_i=0}^{a_i}\binom{a_i}{k_i}\mu_i^{a_i-k_i}\,(V_i-\mu_i)^{k_i}\right)\right]=E\left[\sum_{k_1=0}^{a_1}\cdots\sum_{k_n=0}^{a_n}\left\{\prod_{i=1}^{n}\binom{a_i}{k_i}\mu_i^{a_i-k_i}\cdot\prod_{i=1}^{n}(V_i-\mu_i)^{k_i}\right\}\right]$$

Finally, we get: $$E\left[\prod_{i=1}^{n}\left[(V_i-\mu_i)+\mu_i\right]^{a_i}\right]=\sum_{k_1=0}^{a_1}\cdots\sum_{k_n=0}^{a_n}\left\{\prod_{i=1}^{n}\left(\binom{a_i}{k_i}\mu_i^{a_i-k_i}\right)\cdot E\left[\prod_{i=1}^{n}(V_i-\mu_i)^{k_i}\right]\right\} \tag{7}$$

Substituting (7) into (6), then (5) takes the form: $$S_j=\beta_j!\sum_{\substack{a_i\in A(n,\beta_j)\\(i=1,2,\cdots,n)}}\sum_{k_1=0}^{a_1}\cdots\sum_{k_n=0}^{a_n}\frac{\prod_{i=1}^{n}\left(\alpha_{ij}^{a_i}\,\mu_i^{a_i-k_i}\,a_i!\right)}{\prod_{i=1}^{n}\left(a_i!\,k_i!\,(a_i-k_i)!\right)}E\left[\prod_{i=1}^{n}(V_i-\mu_i)^{k_i}\right] \tag{8}$$

Similarly, $S_j^{'}$ can be evaluated in the same way as S_j. The calculation of $E\left[\prod_{i=1}^{n}(V_i-\mu_i)^{k_i}\right]$ can be referred to Clark and Watling[2)]. Therefore, the first and second order moments of link travel time for mode j can be deduced analytically once S_j is estimated by equation (8).

(3) Evaluation of travel time reliability

It is widely known that there are two types of travel time reliability: path travel time reliability and OD travel time reliability. While the former is defined as the probability that the travel time of a given path is within an acceptable threshold, the latter is the probability that the weighted average travel time of a given OD pair is within a threshold. In this study, the OD travel time reliability which is an aggregate measure for the level of service between O and D is focused since this measure is an important proxy to evaluate the performance of an OD pair. It is reasonable to postulate that travel time on a path consisting of many links follows Normal distribution regardless of link travel time distribution[1)], whose mean and standard deviation (SD) can be written as $\mu_{C_{r|i}^{od}}=\sum_a \delta_{ar}^{od}\mu_{T_{ai}}$ and $\sigma_{C_{r|i}^{od}}=\sqrt{\sum_a \delta_{ar}^{od}\sigma_{T_{ai}}^2}$. Thence, OD travel time which is the weighted average of path travel time also follows the Normal distribution. Therefore, the OD travel time reliability can be expressed mathematically as $\Pr(T_{i,od}\le t)=\Phi\left((t-\mu_{i,od})/\sigma_{i,od}\right)$, where t is a given threshold and $\Phi(.)$ is the cumulative standard Normal distribution.

4. Simulation Framework under Information Provision

As stated above, the estimation of base route choice fraction is the most demanding step. In this chapter, the simulation procedure is provided from which stationary route choice fraction is obtained based on mean OD demand. The procedure is:

Step 1: Initialize day t = 0, and initialize updated past travel time

Step 2: Set next day $t = t + 1$, initialize subinterval h = 0, and initialize travel time supplied by ATIS

Step 3: Set next interval $h = h + 1$

Step 4: Based on the departure rate of the current interval, perform traffic assignment of *non-equipped drivers* using updated past travel time, and *equipped drivers* using combination of updated past travel time and travel time supplied by ATIS

Step 5: Calculate travel time and flow for current interval. If the reference period ends, go to step 6. Otherwise, update travel time provided by ATIS and go to step 3

Step 6: Perform stationarity test. If the test is satisfied, calculate the stationary mean flow for each interval (by discarding some initial days) and stop. Otherwise, update past travel time up to day t and go to step 2

Once the stationary mean flow is obtained, stationary route choice fraction can be calculated accordingly by dividing route flow by the mean OD demand.

5. Numerical Example

In this section the travel time reliability in mixed traffic network is investigated numerically by adopting a five-link network with one OD pair given in Figure 1 and Table 1. It is assumed that the network is operated by only two modes: car (c) and motorbike (m). The updating mechanism of equipped drivers is assumed to follow the function known as exponentially weighted moving average having the form: *Updated TT* = (1-φ) * *Updated past TT* + φ * *TT supplied by ATIS*; ($0\le\varphi\le1$). Mode c is assumed to be the only mode that is equipped with ATIS with φ=0.8. Moreover, all drivers are assumed to remember travel time of the past 10 days with equal weights imposed on each day. The reference period is split into 10 subintervals, and that only the last subinterval is subject to study. The flow converting factor is set to α_{cm}=2.0 and α_{mc}=0.3. Distribution of OD demand is $Q_{od}\sim N(q_{od}, \omega.q_{od})$ with ω=0.2. Mode choice probabilities are 0.60 and 0.40 for mode c and m respectively; and the vehicle occupancy of both modes are 1. The Probit-based traffic assignment is used with link perception error $\xi_a\sim N(0, \tau.t_a)$, where τ is equal to 0.2 and 0.3 for mode c and m. Firstly, the efficiency of the proposed analytic method of estimating moments

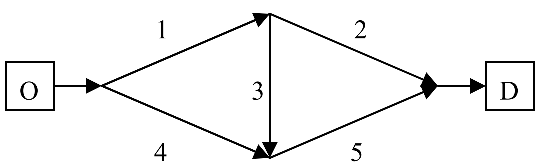

Figure 1: Test network, q_{od} = 100 person trips/hr (Route 1: 1→2, Route 2: 1→3→5, and Route 3: 4→5)

Table 1: Network characteristics

Link No.	t_{0a} (min)		C_a (veh/hr)		β_a		γ_a	
	c	m	c	m	c	m	c	m
1	4.00	6.00	40.00	60.00	4	2	0.40	0.50
2	5.00	5.00	40.00	60.00	4	2	0.40	0.50
3	2.00	1.00	60.00	80.00	4	2	0.40	0.50
4	6.00	8.00	40.00	60.00	4	2	0.40	0.50
5	3.00	3.00	40.00	60.00	4	2	0.40	0.50

of travel time is tested by comparing with the results of Monte Carlo simulation with each pseudo-random draw of Normally distributed Q_{od} is assigned probabilistically using base route choice fraction. The resulting route choice fractions, for 50% of level of market penetration of ATIS, for route 1, 2 and 3 are 0.3638, 0.1000, 0.5362 for mode *c*; and 0.5818, 0.3000, 0.1183 for mode *m* respectively. It is noted that the results from simulation can be obtained from two directions: first is based on the collection of travel time statistics; and second is based on the collection of flow statistics. The discrepancy of the results from analytic and simulation approaches is approximately 2% for both average travel time and travel time SD. This small difference emphasizes the significance of the proposed analytic model so that it can be used in the subsequent analysis.

In the analysis below, the effect of ATIS on travel time reliability is investigated. So, to compare this effect across all drivers, the initial travel time (travel time without information) is used as the threshold value for calculating the reliability. This consideration is very useful in judging whether the provision of ATIS is beneficial or not compared with the initial condition.

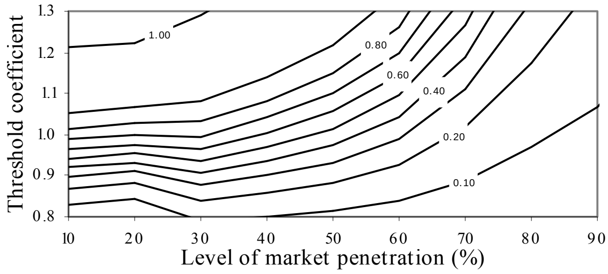

Figure 2: Reliability contour for equipped drivers

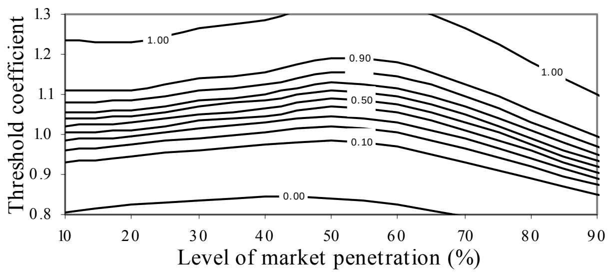

Figure 3: Reliability contour for all non-equipped drivers

In Figure 2 and 3, the coefficient of initial travel time is used instead of absolute travel time, i.e. threshold coefficient of 1.2 is equivalent to the threshold travel time which is 20% higher than initial travel time. From these figures, it is remarkable that when the number of ATIS users exceeds some levels, the reliability of equipped drivers tends to decrease sharply, and the reliability of non-equipped drivers of all modes seems to increase. The deterioration of reliability of informed drivers becomes faster after 50% of level of market penetration. This result might disappoint ATIS users who expect to achieve a better reliability by purchasing the devices. The reason is because the phenomenon of overreaction and concentration occurs among drivers equipped with ATIS. The comparison of reliability across all drivers is also depicted in Figure 4. It is observable that at the level of market penetration of less than 30% the equipped drivers are better off while non-equipped drivers have a rather stable reliability. However, when the fraction of ATIS users reaches 40%, the equipped drivers start to be worse off while non-equipped drivers of mode *c* tend to be better off; and at the same time the reliability of drivers of mode *m* remains unaffected. The reliability of mode *m*'s drivers increases remarkably after the proportion of informed drivers reaches 50%, and they even outperform the equipped drivers. In addition to the separate observation on equipped and non-equipped drivers, it is also useful to monitor the total reliability of all drivers. It is clear that total reliability remains stable up to 60% of level of market penetration, and it starts to decrease after this level. This tendency is a very important indicator for planners in carefully deciding the number of ATIS to be distributed, in order to maintain network performance at an acceptable level. It is mindful that the above results depend strongly on simulation setting.

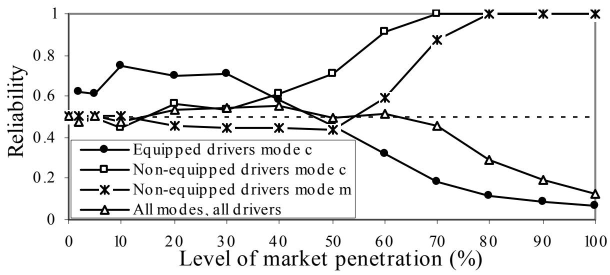

Figure 4: Level of market penetration of ATIS v/s reliability

6. Conclusion

In this paper, the analysis of travel time reliability considering mixed traffic condition is explicitly given with the main elements, i.e. mean and variance of travel time readily available based on an analytical approach. The results of the simulation can offer another view about the effect of ATIS on travel time reliability. It has been proved that with low market penetration, the informed drivers can well improve their reliability, whereas at higher market penetration their reliability tends to decrease; and the benefits seem to be transferred to non-equipped drivers who pay nothing for the system. In addition, the policy issue is also raised up concerning market penetration of ATIS to assure the reliability of transportation system users as a whole.

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