EXACT MEASUREMENT OF VALUE OF TIME*

by Hisa MORISUGI*, Kouta KAZAMA ** and Jane ROMERO ***

1. Introduction

The role of measurement in the efficient allocation of resources is especially important in cases of public goods as in the case of transportation projects. An improvement in resource allocation requires that the benefits of a decision exceed its costs, which in turn requires the measurement of benefits and costs. Value of time is used to measure the time savings in cost benefit analysis of transportation projects or to calculate the generalized cost in the traffic demand forecasting model. It is defined as

$$VOT \equiv -\left.\frac{dp}{dt}\right|_{u\ or\ \pi=const.} \tag{1}$$

where p is transport cost, t is necessary transport time, u is utility and π is profit. The subjective value of time is the marginal rate of substitution between travel time and travel cost under constant utility or profit level. It is commonly referred to as the monetary appraisal of value of time or the willingness to pay for savings in travel time.

However, it is difficult to grasp the function form of the utility or profit given by (1). In practice, value of time is obtained by making use of the formulation in (2).

$$VOT \equiv -\left.\frac{dp}{dt}\right|_{X\ or\ c=const.} \tag{2}$$

where p is transport cost, t is necessary transport time, X is demand quantity and c is generalized cost. The value of time obtained from (2) is under the assumption of constant demand or generalized cost.

The value for the marginal substitution of traffic cost p relative to travel time t differs depending on the method applied to derive the value of time. Using (1) is not equal to using (2), as in the latter assumes the demand quantity or the generalized cost as fixed. Needless to say, it is dubious to obtain an accurate value of time from (2). Thus, the formulation in (1) is used in measurement of value of time.

*Keywords: value of time, observability of VOT, demand functions

*Fellow of JSCE, Ph.D., Professor
Graduate School of Information Sciences, Tohoku University
(Aoba 06, Aoba, Sendai, Japan,TEL:22-217-7501,FAX:22-217-7500)
** Member of JSCE, Graduate Student
Graduate School of Engineering, University of Tokyo
(Hongo 7-3-1, Bunkyo-ku, Tokyo 113-8656, Japan)
*** Graduate Student, Graduate School of Information Sciences, Tohoku University
(Aoba 06, Aoba, Sendai, Japan,TEL:22-217-7501,FAX:22-217-7500)

The current practice derives the value of time from specified functional forms of demand or utility. Previous papers on the valuation of passenger travel time have assumed a specified utility and demand functions. The limitation of this method is that the results are purely indicative, coming from specified functional form.

We intend to measure the value of time defined by (1) in terms of the observable demand. As utility is usually not easily observed, we need to find a way to observe the manifestations how the changes in utility are valued. Our approach is to begin with the observed market demand and then derive the unobserved indirect utility function and expenditure function. This approach is applicable to all types of trip purposes. In this paper it will be applied to the following person and freight trips as shown in Table 1. However, the case of person business trips will no longer be discussed, as it is already well known that its VOT is equal to wage rate.

Table 1. Trip and freight purposes

	Non-business	Business
Person trips	• Commuting trips • Shopping trips	• Business trips (VOT = wage rate)
Freight	• Door-to-door parcel delivery	• Freight delivery

In Section 2 is the specification of several types of demand functions. Interestingly, the relationship between the true value of time (under constant utility or profit) and the practical value of time (under constant demand or generalized cost) could be analyzed with the varying of the demand functions. Thus, exact definitions of various VOTs are shown here. Section 3 is devoted to expressing the VOT in terms of observable demand. The individual non-business trips and non-business freight will be evaluated jointly while the analysis of business freight will be in another sub-section. Section 5 summarizes and concludes the paper.

2. Exact definition of VOTs

As the only observable data are market data, it is important to find a function that fits the data well. Here we consider three cases of demand functions to define the corresponding VOTs under the assumptions of constant demand and constant utility or profit.

Linear: $$x = \alpha p + \delta y + \eta t + \beta \tag{3}$$

Log-linear: $$x = Ae^{\eta t} p^{\alpha} y^{\delta} \tag{4}$$

Semi-Log Linear: $$x = Ke^{\alpha p + \delta y + \eta t} \tag{5}$$

where x is transport service demand, p is transport cost per trip, t is trip duration, y is income and the parameters satisfy these conditions as $\alpha < 0, \delta \geq 0, \eta < 0$.

This section uses the Roy's identity given by $x = -\frac{\partial v/\partial p}{\partial v/\partial y}$ or $x_i = \frac{V_{q_i}}{\sum q_j V_{q_i}}$, $q_i = \frac{p_i}{y}$ $V_{q_i} = \frac{\partial v}{\partial q_i}$, and Hotelling Lemma, $\pi_p = -x$. Two cases are considered, when $\delta \neq 0$ and $\delta = 0$ where δ is the income parameter. The method applied is similar to the various demand functions so as an example, only the case of linear demand $\delta \neq 0$ will be discussed below.

The exact definition of VOTs under constant demand, constant utility (for both for the individual non-business trips, non-business freight) and constant profit (for business freight) will be tabulated at the end of this subsection.

2.1 Linear demand function for non-business trips and freight

The linear demand function given in (3) is a simple case often used in empirical analysis, especially when a separability assumption between the good whose price changes and the other goods is appropriate. For more discussions, refer to Hausman (1981).

(i). when $\delta \neq 0$

VOT under constant demand is as follows:

$$VOT_x \equiv -\left.\frac{dp}{dt}\right|_{x=const.} = \frac{\partial x/\partial t}{\partial x/\partial p} = \frac{\eta}{\alpha} \tag{6}$$

Next is the derivation of VOT under constant utility. Given an indifference curve, to stay along the path of price change within the curve,

$$0 = dv = \frac{\partial v(p(t), y(t))}{\partial p(t)}\frac{\partial p(t)}{\partial t} + \frac{\partial v(p(t), y(t))}{\partial y(t)}\frac{\partial y(t)}{\partial t} \tag{7}$$

Then, using the implicit function theorem and Roy's identity ,

$$\frac{dy(p)}{dp} = \alpha p + \delta y + \eta t + \beta \tag{8}$$

Now y is expressed in terms of p and solving the ordinary differential equation (8) to find

$$y(p) = ce^{\delta p} - \frac{1}{\delta}\left(\alpha p + \frac{\alpha}{\delta} + \eta t + \beta\right) \tag{9}$$

where c, the constant of integration depends on the initial utility level. Here we assume c as our cardinal utility index, which is equal to the indirect utility function v.

$$v(p, t, y) = c = e^{-\delta p}\left\{y + \frac{1}{\delta}\left(\alpha p + \frac{\alpha}{\delta} + \eta t + \beta\right)\right\} \tag{10}$$

Accordingly, VOT_v is solved as

$$VOT_v \equiv -\left.\frac{dp}{dt}\right|_{v=const.} = \frac{\partial v/\partial t}{\partial v/\partial p}$$

$$= -\frac{\alpha}{\delta(\alpha p + \delta y + \eta t + \beta)}\frac{\eta}{\alpha} = -\frac{\eta}{\delta}\frac{1}{x} \tag{11}$$

The relationship between VOT_x and VOT_u when $\delta \neq 0$ is

$$VOT_v = -\frac{\alpha}{\delta}\frac{1}{x}VOT_x \tag{12}$$

Thus in this case, exact VOT is not equal to the practical VOT under constant demand.

2.2 Types of demand functions for both business and non-business trips

Table 2 summarizes the various exact forms of the VOTs under constant demand and constant utility for the individual non-business trips and non business freight trips derived from the various demand functions.

Table 2. Demand function and the corresponding exact definitions of VOT_x and VOT_v

Demand Function	δ	VOT_v	VOT_x	Relationship
Linear	$\delta \neq 0$	$-\frac{\eta}{\delta}\frac{1}{x}$	$\frac{\eta}{\alpha}$	$VOT_v = -\frac{\alpha}{\delta}\frac{1}{x}VOT_x$
	$\delta = 0$	$\eta\frac{p^{**}}{x}$	$\frac{\eta}{\alpha}$	*****
Log-linear	$\delta \neq 0$	$\frac{\eta}{\alpha+1}p$	$\frac{\eta}{\alpha}p$	$VOT_v = \frac{\alpha}{\alpha+1}VOT_x$
	$\delta = 0$	$\frac{\eta}{\alpha+1}p$	$\frac{\eta}{\alpha}p$	$VOT_v = \frac{\alpha}{\alpha+1}VOT_x$
Semi – log linear	$\delta \neq 0$	$\frac{\eta}{\alpha}$	$\frac{\eta}{\alpha}$	$VOT_v = VOT_x$
	$\delta = 0$	$\frac{\eta}{\alpha}$	$\frac{\eta}{\alpha}$	$VOT_v = VOT_x$

**improper formulation

The VOT_v derived from the linear demand model when $\delta = 0$ yields a negative value for VOT, which contradicts conventional consumer behavior. Therefore, it may imply that the linear form when $\delta = 0$ is inappropriate form for demand .

The above results show that the semi-log linear demand model is consistent under both conditions and is the best model to represent the travel demand function.

Table 3 shows the various exact forms of the VOTs for the business freight trips derived from the various demand functions by applying Hotelling Lemma $\pi_p = -x$.

Table 3. Demand function and the corresponding exact definitions of VOT_x and VOT_π

Demand Function	VOT_v	VOT_x	Relationship
Linear	$\eta \frac{p}{x}$	$\frac{\eta}{\alpha}$	$VOT_v = \frac{p\alpha}{x} VOT_x$
Log-linear	$\frac{\eta}{\alpha+1} p$	$\frac{\eta}{\alpha} p$	$VOT_v = \frac{\alpha}{\alpha+1} VOT_x$
Semi-log linear	$\frac{\eta}{\alpha}$	$\frac{\eta}{\alpha}$	$VOT_v = VOT_x$

As in the case of the individual non-business trips and non-business freight, the semi-log linear demand function is the best-suited function for analyzing business freight trips.

To estimate the VOT from the semi-log linear demand model, the ratio of two normal variables could be applied as well as the t-test method or Fieller's theorem. However, for the VOTs from the linear demand model and log-linear demand model, as it is difficult to predict the corresponding distribution of the variables, non-parametric methods like Bootstrap could be used.

2.3 Logit model

Using this formulation of another type of Roy's identity, let

$$x_i = \frac{V_{q_i}}{\sum q_j V_{q_i}}, \text{ where } q_i = \frac{p_i}{y},\ V_{q_i} = \frac{\partial v}{\partial q_i} \tag{13}$$

Then, V is equal to the following indirect utility functions

$$V = V(p, y) = V(p_i / y_i, 1) \equiv v(q_i) \tag{14}$$

In order to justify the logit model, let

$$V_{q_i} = e^{a - bt_i - cq_i} \tag{15}$$

be consistent with classical consumers theory. Then, demand share is solved by substituting (15) to (13) and is calculated as

$$x_i = \frac{e^{(a-bt_i-cq_i)}}{\sum q_j e^{(a-bt_i-cq_i)}} \tag{16}$$

Assuming a logit model with 3 alternative modes, (16) is equal to

$$x_i = \frac{e^{(a-bt_i-cq_i)}}{\sum_{j=2}^{3} e^{(a-bt_j-cq_j)}} \frac{\sum_{j=2}^{3} e^{(a-bt_j-cq_j)}}{\sum_{k=1}^{n} q_k e^{(a-bt_k-cq_k)}} \tag{17}$$

where the first term of the right-hand side is the modal split of modes 2 and 3 while the second term is the total trip demand.

One of the specific solutions could be the indirect utility function that is suitable for (15) is

$$V = -\sum \frac{1}{c_i} e^{(a_i - b_i t_i - c_i q_i)} \tag{18}$$

The specification of (18) could yield to a case where $VOT_{v=const.} = VOT_{share=const.} = VOT_{demand=const.}$, which would be more applicable in actual practice as even modal split share could be derived. However, this equality of VOTs only satisfies the logit case and may not hold for other cases.

3. Derivation of VOT in terms of *observable* demand

Using the definition of VOT in (1), which is under constant utility, now we intend to express the value of time in terms of observable demand function. The individual non-business trips and non-business freight will be evaluated separately from business freight.

3.1 Individual non-business trip and non-business freight

First, we need to formulate an individual behavior on a non-business trip, that is, either commuting or shopping. Let $u(z,x,l,t)$ be the individual preference function. The utility of an individual is assumed to be a function of the demand for commodity goods z whose price is normalized to 1, transport service demand x, leisure time l and duration of trip t. The individual maximizes utility subject to income y and total available time T. The indirect utility function, $v(1,p,t,T,y)$, is given by

$$v(1, p, t, T, y) = \max_{z,x,l} u(z, x, l, t) \tag{19}$$

$$\text{s.t.} \quad z + px = y, \qquad tx + l = T \tag{20}$$

We assume that it is competitive equilibria that are observable. At the most disaggregated level, one can observe the demand of individuals as prices and the allocation of endowments or revenue vary. The subscript A denotes the situation of observation. The value of time in terms of this indirect utility function is given as

$$VOT = -\left.\frac{dp}{dt}\right|_{v=const.at\ A} = \left.\frac{v_t}{v_p}\right|_A = \left.\frac{v_t}{-v_y x}\right|_A \tag{21}$$

where subscript is partial derivative and by using $-x = v_p / v_y$, given from Roy's identity. Similar functional form for VOT is also derived for the non-business case.

Two methods could be used to express the VOT in terms of demand function x, by implying weak complementarity or citing neutrality as described in Larson (1992). The method of finding neutral goods to represent the VOT will be discussed here.

By envelope theorem, VOT can be expressed in terms of the expenditure function at A,

$$VOT = -\left.\frac{dp}{dt}\right|_{v=const.} = -\left.\frac{dp}{dt}\right|_{e=const} = \frac{e_t}{e_p} = \frac{e_t}{x} \tag{22}$$

Now the problem is on how to express e_t in terms of x. By Slutsky-Hicks equation, e_t can be expressed in terms x.

$$\frac{\partial x_i^C}{\partial t} = \frac{\partial x_i}{\partial t} + e_t \frac{\partial x_i}{\partial y} \quad (23)$$

where x_i is the Marshallian demand and x_i^C is the compensating demand. Now assuming that there is at least a neutral good with respect to t such that

$$\frac{\partial x_i}{\partial t} = \frac{\partial x_i}{\partial y} = 0 \Rightarrow \frac{\partial x_i^C}{\partial t} = 0 \quad (24)$$

Then expressing e_t in terms of expenditure on substitutes and complements E_1 and E_2,

$$e_t = \frac{\partial E_1 / \partial t + \partial E_2 / \partial t}{(\partial E_1 / \partial y + \partial E_2 / dy) - 1} \quad (25)$$

$$\partial E_1 / \partial y + \partial E_2 / \partial y \neq 1, \text{ where } E_k = \sum p_{k_i} x_{k_i} \quad (26)$$

Finally, the VOT is expressed as

$$VOT = \frac{(e_t)_A}{x_A} = \frac{1}{x_A} \frac{\partial E_1 / \partial t + \partial E_2 / \partial t}{(\partial E_1 / \partial y + \partial E_2 / \partial y) - 1} \quad (27)$$

Here VOT is observable by looking at the change in expenditure of the non-neutral goods with respect to transport time. Accordingly, the time saving benefit B can be measured as

$$B \int_{t_A}^{t_B} e_t \, dt = \int_{t_A}^{t_B} \frac{\partial E / \partial t}{\partial E / \partial y - 1} dt \text{ , where } E = E_1 + E_2 \quad (28)$$

3.2 Business freight

The profit maximization function of a company using freight delivery service is

$$\pi(1, w, p) = \max[y - (wL + pX)] \quad (29)$$

$$\text{s.t.} \quad Y = f(L, X, t) \quad (30)$$

where L is labor force, X is demand of the delivery service, w iswage rate, p is price of the delivery per one unit freight, t: is necessary time for the delivery per one unit freight, Y is production (the price of goods is assumed to be equal to 1) and f is production function

By substituting (30) to (29), the value of time can be computed as follows

$$\pi(1, w, p, t) = Y(1, p, w,) - wL(1, w, p, t) - px(1, w, p, t) \quad (31)$$

$$VOT = \frac{\pi_t}{\pi_p} = \frac{Y_t - (wL_t + pX_t)}{X} \quad (32)$$

In (32), Y_t is the variation of the production function due to the improvement of the necessary time while the second term of the right-hand side of the equation is the variation of the total cost due to the improvement of the necessary travel time. X is the quantity of demand of delivery service. So the time saving benefit B can be measured by

$$B = \int_{t_A}^{t_B} \pi_t = \int_{t_A}^{t_B} Y_t - C_t \text{, where } C_t = wL_t + pX_t \quad (33)$$

4. Conclusions

First, we showed the difference between the value of time under constant demand and value of time under constant utility. In Section 2, exact definitions of VOTs are evaluated under constant demand, constant utility and constant profit corresponding to the different specifications of the demand function. The semi-log linear demand function is the best so far as its result is consistent under the various conditions.

Using Roy's identity $x_i = V_{q_i} \Big/ \sum q_j V_{q_i}$ and assuming a logit demand function could lead to an indirect utility function that could at the same time calculate modal split share. This would be of practical use as the current methods are only capable to yield aggregate results.

Lastly, it is shown that the exact measurement of VOT by expressing it in terms of observable demand is possible. In the case of individual non-business trip and non-business freight, it is by examining the change in expenditure of the non-neutral goods with respect to travel time. In the case of the business freight, it is observable based on the change in output level and total cost due to travel time.

References

1) Haab, T.C., McConell, K.E.: *Valuing Environmental and Natural Resources,* New Horizons in Environmental Economics, Edward Elgar Pub. Ltd., Massachusettes, USA, 2003.
2) Hausman, J.A.: Exact consumer's surplus and deadweight loss, *The American Economic Review*, Vol.71, No.4, pp.662-676, 1981.
3) Jara-Diaz, S.R.: Allocation and valuation of travel-time savings, in D.A. Hensher and K.J. Button eds., *Handbook of Transport Modelling,* Handbooks in transport Vol.1, Pergamon, Amsterdam, 2000.
4) Jiang M., Morikawa, T.: Theoretical amalysis on the variation of value of travel time savings, *Transportation Research Part A,* Vol.38, pp.551-571, 2004.
5) Larson, D.M.: Further results on willingness to pay for nonmarket goods, *Journal of Environmental Economics and Management,* Vol.23, pp. 101-122, 1992.
6) Neill,J.R.: Another theorem on using market demands to determine willingness to pay for nontraded goods, *Journal of Environmental Economics and Management,* Vol.15, pp.224-232, 1988.
7) Varian, H.R.: *Microeconomic Analysis,* W.W. Norton & Company Inc., New York, 1992.
8) Vartia, Y.O.: Efficient methods of measuring welfare change and compensated income in terms of ordinary demand functions, *Econometrica,* Vol. 51, pp. 79-98, 1983.