ESTIMATION OF MULTICLASS TIME-VARYING ORIGIN-DESTINATION MATRICES FROM TRAFFIC COUNTS*

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1. Introduction

An origin-destination (O-D) is an essential of transportation planning and traffic operation. Traditionally, O-D matrices are derived from interview, roadside surveys or forecasting from transportation planning model. However, data collection is difficult, costly and labor intensive. Estimation of O-D matrices from traffic counts have been paid attention for more than two decades since the estimation method is relatively inexpensive to collect the data. Traffic counts, therefore, are attractive and took a growing interest. Several works have been proposed to use link counts for O-D matrix estimation e.g. Van Zuylen and Willumsen[1)], Cascetta[2)] and Bell[3)].

O-D matrices estimation approaches can be broadly divided into two groups as static O-D matrices and time-varying O-D matrices estimation. The former considers traffic counts started and completed in the same period while the latter considers time series of traffic counts. The latter is more interesting to use in traffic management or network performance assessment.

Recently, many works has been paid attention to the traffic counts and historical O-D matrices to get the unique result. Cascetta, Inaudi and Marquis[4)] presented the estimators using both mentioned data in static and time-varying O-D matrices estimation. Ashok and Ben-Akiva[5)-6)] also developed the models for estimation and prediction of time-varying O-D matrices based on the assignment matrix. The path choice fraction and the link-path fraction are the main component of the assignment matrix. For general networks, stochastic user equilibrium (SUE) by Sheffi[7)] and Yang, Meng and Bell[8)] and markov process by Akamatsu[9)], can be applied to calculate path choice fraction in congestion effect and without path enumeration. However, these works considered only the single vehicle type. Wong *et al*.[10)] estimated the static multiclass O-D matrices for a real large-scale network to obtain the advantage from the internal inconsistency elimination of mix traffic flows.

As the amount of truck is significant, it affects to other vehicle types and is necessary for transportation planning and appraising the policy. Additionally, there are a few works provided the truck O-D together with the passenger car O-D. Our study, therefore, focuses on time-varying O-D matrices estimation of multi-class including light truck, heavy truck and passenger car on general network. By using the SUE, the model can perform on the congestion area and the real large scale network.

This study aims to estimate the time-varying O-D matrices of three vehicle types using SUE on general network. Furthermore, we propose the stochastic link-path fraction into the formulation. The model is expected to provide more accurate and useful results for planning in practical work. We organize the study as follows. Chapter 2 describes the concept of the model more details. Chapter 3 discusses the conclusion.

2. Time-varying O-D estimation Formulation

Generally, many researchers presented the relationship between observed link flow and historical O-D using an assignment matrix. Since all vehicles in the static O-D matrices were assumed to be complete trips within concentrate period. The assignment matrix can be represented by path choice fraction calculated from user equilibrium (UE), SUE and other concepts. In time-varying O-D matrices, some vehicles stayed on the network more than one time interval. A link-path fraction additionally represents the fraction of vehicles stayed on the network one or more than one time interval. The assignment matrix, therefore, depends on the path choice fraction and link-path fraction. The main concept of our model is illustrated in Figure 1.

(1) Relationship between link flows and O-D matrices

Consider a network with n_{OD} O-D pairs and n_{LK} links. Let a period of length λ divided into equal intervals of length T. This study also assumes that n_l links are equipped with detectors. Available useful information, therefore, consists of link flow obtained from traffic counts and historical O-D matrices. To use the existing information, the fundamental relationship between link flow from traffic counts and O-D flows illustrated by Cascetta, Inaudi and Marquis[4)] can be stated as follow:

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$$y^{m}_{l,h} = \sum_{g=h-g'}^{h} \sum_{od=1}^{n_{OD}} \left(a^{g,m}_{l,od,h} x^{m}_{l,od,g} \right) + \nu_{l,h} \tag{1}$$

where

$y^{m}_{l,h}$ is the flow of vehicle type *m* on link *l* during interval *h*.

$a^{g,m}_{l,od,h}$ is fraction of the trips type *m* with O-D pair *od* that departed the origin during interval *g* and crossed the link *l* during interval *h*, defined as the assignment matrix.

$x^{m}_{l,od,g}$ is the flow of type *m* with O-D pair *od* during interval *h* that departed the origin during interval.

$\nu_{l,h}$ is the measurement error.

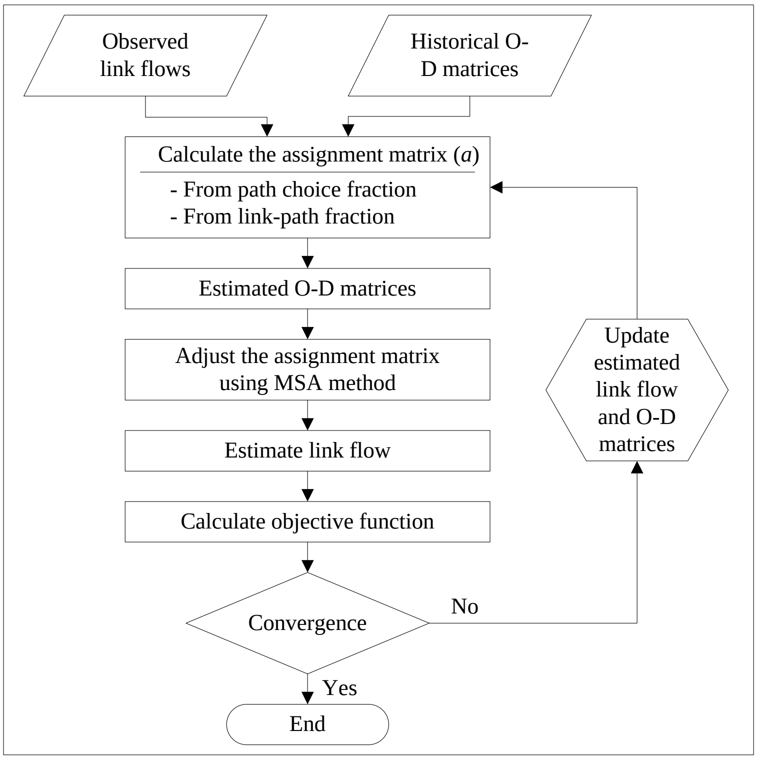

Figure 1: O-D estimation concept

(2) The assignment matrix

Let R_r represent the set of all paths between O-D pair $od \in n_{OD}$. The assignment matrix represents the relationship of each O-D flows and link flows. It can be obtained directly from the link-path fractions and the path choice fraction as shown by Ashok and Ben-Akiva[6] as follow:

$$a^{g,m}_{l,od,h} = \sum_{r \in R_r} \left(\alpha^{r,g,m}_{l,od,h} P^{r,m}_{od,g} \right) \tag{2}$$

where

$\alpha^{r,g,m}_{l,od,h}$ is the link-path fraction of flows type *m* with OD pair *od* of the r^{th} path that departed the during interval *g* and crossed the link *l* during interval *h*.

$p^{r,m}_{od,g}$ is the path choice fraction of flows type *m* that selected the r^{th} path with OD pair *od* and departed interval *g*.

The expression of these parameters can be obtained from stochastic link travel times and SUE concept. As mentioned previously, the link-path fraction in the static O-D matrix estimation would be one or zero that different concept from time-varying O-D matrices estimation. In this study, the ranges of these three parameters are placed from zero to one. If these parameters are less than zero, we truncate them to zero automatically. Additionally, if these values are greater than one, we truncate them to one automatically.

Since some vehicles may stay on the network more than one time interval, the assignment matrices have to obey the relationship as follow:

$$\sum_{g=h-g'}^{h} a^{g,m}_{l,od} = 1 \tag{3}$$

a) The path choice fraction using the logit-based SUE

The logit-based SUE is used to represent the distribution of flows over the network particularly on the urban network and congested network. The logit model based approach is recognized to be one of the most popular. However, the well-known property of the independence from irrelevant alternatives (IIA) of the logit model may sometimes provide unrealistic results.

The path choice fraction derived by Yang, Meng and Bell[8] is expressed as follows:

$$P_{od,h}^{r,m}(\theta,\mathbf{y}_h)=\frac{\exp\left[-\theta\sum_{l\in R_l}\left(t_{l,h}^{m}(y_{l,h})\delta_{l,od,h}^{r}\right)\right]}{\sum_{r\in R_{od}}\exp\left[-\theta\sum_{l\in R_l}^{\infty}\left(t_{l,h}^{m}\delta_{l,od,h}^{r}\right)\right]} \tag{4}$$

where

$P_{od,h}^{r,m}(\theta,\mathbf{y}_h)$ is the probability of flows of vehicle type *m* take the r^{th} path between nodes *o* and *d*.

θ is the path choice dispersion coefficient and greater than zero.

$\mathbf{y}_h=(\ldots,y_{l,h},\ldots)$ is a vector of all link flows.

$\delta_{l,od,h}^{r}$ is 1 if the r^{th} path between nodes *o* and *d* passes through link *l*, 0 otherwise.

$t_{l,h}(y_{l,h})$ is the travel time of vehicle type *m* on link *l* during interval *h*, which can be obtained from many concepts such as by BPR formulation.

b) Link-path fraction

Both previous works from Cascetta *et al.*[4] and Ashok and Ben-Akiva[6] derived the link-path fraction assumed that vehicle within a group (*r, g*) are uniformly distribute by a constant time headway during time *T*. In this study, the length *T* is also divided into equal intervals as *n* subintervals for data collection. We also assume that all vehicle groups follow the first in first out concept. The proposed link-path fraction can be expressed as follows:

$$\alpha_{l,od,h}^{m}=\begin{cases}=1 & if\quad (h-1)<\eta_{l,h}^{m,s},\forall s<hT\\ \dfrac{\sum_{\exists s}y_{l,h}^{m,s}}{\sum_{s=1}^{n}y_{l,h}^{m,s}} & if\quad (h-1)<\eta_{l,h}^{m,s},\exists s<hT\\ =0 & if\quad \eta_{l,h}^{m,s},\forall s<(h-1)\quad or\quad hT<\eta_{l,h}^{m,s},\forall s\end{cases} \tag{5}$$

where

$$\eta_{l,h}^{m,s}(y_h)=t(g)+\frac{(2s-1)T}{2n}+\tau_{o->l,h}^{m}(y_h) \tag{6}$$

$\sum_{\exists s}y_{l,h}^{m,s}$ is the summation of the vehicle during subinterval *s* crossed link *l* during interval *h* that satisfied the constraint.

$\eta_{l,h}^{m,s}(y_h)$ is the crossing time of each subinterval *s* of vehicle type *m* that departed the origin during interval g with OD pair *od* and crossed detector *l* during interval *h*.

t(g) is the lower boundary time of vehicle during subinterval *s* that crossed link l during interval *h* and departed origin *o* during interval *g*.

$\tau_{1,o->l,h}^{m}(y_h)$ is the cumulative of stochastic travel time of vehicle type *m* departed origin *o* and crossed link *l* during interval *h*.

(3) Time-varying O-D model formulation

By using all available information, observed link flows and best historical O-D matrices of multiclass vehicle are performed in the model to obtain the unique O-D matrices. The sequential model is useful to estimate the O-D matrix at each time interval. Moreover, it reduces the complexity of calculation comparing with the simultaneous model. The objective function, therefore, is to minimize the error of all parameters for each time interval using sequential model. The estimation problem can be rewritten as follow:

$$Min.\quad Z_h^m(\boldsymbol{\theta}_h,\mathbf{x}_h^m)=F_1(\mathbf{y}_h^m(\theta_h),\overline{\mathbf{y}}_h^m)+F_2(\mathbf{x}_h^m,\overline{\mathbf{x}}_h^m) \tag{7}$$

where F_1 and F_2 are assumed to be the Generalized Least Squares (GLS) functions of the O-D matrix and link flow vector as follows:

$$F_1(\mathbf{y}_h^m(\theta_h),\overline{\mathbf{y}}_h^m)=(\mathbf{y}_h^m(\theta_h)-\overline{\mathbf{y}}_h^m)^t\mathbf{V}_{h,m}^{-1}(\mathbf{y}_h^m(\theta_h)-\overline{\mathbf{y}}_h^m) \tag{8}$$

$$F_1\left(\mathbf{x}_h^m, \overline{\mathbf{x}}_h^m\right) = \left(\mathbf{x}_h^m - \overline{\mathbf{x}}_h^m\right)^t \mathbf{W}_{h,m}^{-1}\left(\mathbf{x}_h^m - \overline{\mathbf{x}}_h^m\right) \quad (9)$$

$\mathbf{V}_{h,m}^{-1}$ and $\mathbf{W}_{h,m}^{-1}$ represent the covariance matrices for the error terms.

(4) Solution Algorithm

We apply the method of successive average (MSA) to determine the new assignment matrix (a) for the next iteration as follow.

$$\mathbf{a}_h^{m,(i+1)} = \frac{1}{i+1}\overline{\mathbf{a}}_h^{m,(i+1)} + \frac{i}{i+1}\mathbf{a}_h^{m,(i)} \quad (10)$$

where

$\overline{\mathbf{a}}_h^{m,(i+1)}$ is the auxiliary assignment matrix obtained from the assignment result.

There are two conditions to stop the iteration. Firstly, the solution has converged and stopped if the deviation of link flows is less than the acceptable error range. Secondly, the iteration stops if the iteration number exceeds a certain preset threshold. Otherwise, we increase the value *i* by one then use the estimated link flows and O-D matrices to calculate again.

The relative root mean square error (RRMSE) is also determined to measure the goodness-of-fit and to evaluate the estimated O-D matrices of each time slice with other methods.

3. Conclusion

There are three concepts proposed in this study. Firstly, the time-varying O-D matrices of multiclass have been formulated to estimate light truck, heavy truck and passenger car O-D matrices separately at the same time using sequential estimators. These results are very useful for transportation planning and network assessment. Secondly, the stochastic link-path fraction has been developed to assign the vehicles stayed on the network one or more than one interval time. Finally, the correlation between the O-D matrices during considered time interval and departed time interval from origin have been examined.

This study, therefore, expects to obtain the time-varying O-D matrices of three vehicle types, particularly truck O-D matrices. We, firstly, attempt to test the concept on simulated data. There are not so many works applied on the real large network since calculation is highly complicated and time consuming. We, then, extend work on the real large scale network to evaluate the proposed model. The accuracy and the computation time are also important issues to consider for practical work. Finally, the SUE and the stochastic assignment matrix are performed to permit the congestion and an incident on the network.

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