

DYNAMIC ESTIMATION OF TRAFFIC STATE ON SURFACE STREET NETWORK^{*}

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1. Introduction

Reliable traffic information is essential for the development of efficient traffic control and management strategies for traffic networks in advanced traffic management systems (ATMS) and advanced traveler information systems (ATIS). Real-time traffic information is utilized for various purposes such as dynamic route guidance, incident detection, and control of variable message signs. Traffic information can be obtained from several vehicle detectors installed on the road network. However, using only detectors cannot give us a complete view of current traffic state on the whole network since detector can measure traffic state only at a point where it is installed. The mathematical model of traffic flow such as the macroscopic model (e.g., the LWR model proposed by Lighthill and Whitham¹⁾ and Richards²⁾) can then be used to connect the local view of measured traffic state from all detectors to the estimated complete view for the whole network. Nevertheless, when the network is complicated and has some uncertainties as in the case of surface street, the traffic flow model alone may sometimes fail to replicate the real traffic situation. Moreover, detector data may also include some noises due to the measurement error. To compensate for these problems, several adaptive filtering techniques such as the Kalman filter (KF) and its extension (extended Kalman filter: EKF) were introduced and integrated into the macroscopic model for real-time estimation of traffic state³⁾. The objectives of this paper are twofold: 1) to model the surface street network using discretized macroscopic model and 2) to evaluate the effectiveness of integrating the Kalman filter with macroscopic model.

In chapter two, the macroscopic model and its numerical method for the case of surface street network are discussed. Third, the brief outline of how to integrate the EKF with the macroscopic model is given. In chapter four, the method is tested on the sample network and compared with the results obtained from simulation software. Finally, the conclusion is drawn and the area for future research is discussed.

2. Macroscopic Model

In the LWR model, traffic is viewed as a continuous media and characterized by traffic density (ρ), travel speed (v), and flow rate (q) as a function of space (x) and time (t). The model consists of three basic equations: 1) the conservation of vehicles, 2) the definition of travel speed v , and 3) the equilibrium speed-density relationship.

$$\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial x} = 0 \quad (1)$$

$$q = \rho v \quad (2)$$

$$v = V_e(\rho) \quad (3)$$

Daganzo⁴⁾ proposed a numerical method for this model based on the discretization of space and time. Later, Lebacque⁵⁾ generalized this method into the demand and supply concept and showed that it is equivalent to the Godunov scheme. Based on this method, the discrete version of the LWR model for segment i of link l at any time step k is as follows

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$$\rho_i^l(k+1) = \rho_i^l(k) + \frac{\Delta t}{\Delta x} (qb_{i-1}^l(k) - qb_i^l(k)) \quad (4)$$

$$v_i^l(k) = V_e(\rho_i^l(k)) \quad (5)$$

$$q_i^l(k) = Q_e(\rho_i^l(k)) = \rho_i^l(k) \cdot V_e(\rho_i^l(k)) \quad (6)$$

where Δx is the segment length, Δt is the time interval of one simulation step, and $qb_i^l(k)$ is the outflow from segment i of link l during time between $(k-1)\Delta t$ and $k\Delta t$. Using demand-supply concept, the outflow from segment i of link l is defined as

$$qb_i^l(k) = \text{Min}\{D(\rho_i^l(k)), S(\rho_{i+1}^l(k))\} \quad (7)$$

where $D(\rho_i^l(k)) = \begin{cases} Q_e(\rho_i^l(k)) & \text{if } \rho_i^l(k) \leq \rho_c^l \\ qc_i^l & \text{if } \rho_i^l(k) \geq \rho_c^l \end{cases}$, $S(\rho_{i+1}^l(k)) = \begin{cases} qc_{i+1}^l & \text{if } \rho_{i+1}^l(k) \leq \rho_c^l \\ Q_e(\rho_{i+1}^l(k)) & \text{if } \rho_{i+1}^l(k) \geq \rho_c^l \end{cases}$, qc is the saturation flow rate, and ρ_c is the density at saturation flow. The flow through signalized intersection can be modeled using the modified version of the general intersection model⁵⁾. The flow $qb_{mn}(k)$ from last segment of link m to the first segment of link n at time k is modeled as

$$qb_{mn}(k) = A_{mn}(k)G_{mn}(k) \cdot \text{Min}\{\gamma_{mn}(k) \cdot D_m(k), S_{mn}^*(k)\}. \quad (8)$$

$D_m(k)$ is the demand at the last segment of link m at time k . $S_{mn}^*(k)$ is the actual supply of link n for the flow from link m at time k . $\gamma_{mn}(k)$ is the turning ratio from m to n at time k . $A_{mn}(k) = 1$ if signal allows turning from m to n at time k , and $A_{mn}(k) = 0$ otherwise. $G_{mn}(k) = 1$ if the gap on the opposing flow at time step k is accepted by the vehicles turning from m to n , and $G_{mn}(k) = 0$ otherwise. Since traffic signal is utilized in order to reduce the conflict at intersection, we may postulate the priority rule for merging from any of upstream link m to a downstream link n as follows. For example, if link m has first priority and link j has second priority to merge to link n at time k , then $S_{mn}^*(k) = S(\rho_1^n(k))$ and $S_{jn}^*(k) = S(\rho_1^n(k)) - qb_{mn}(k)$. Based on this set of equations and assumptions, we can then estimate the dynamic of traffic state on the surface street network if the initial density on all segments in the network, the boundary inflow into the network, turning ratio at intersections, and signal phasing and timing are known.

3. Macroscopic Model with EKF

In this paper, for the implementation of EKF with macroscopic model, the state vector $\mathbf{x}$ is a set of density at all segments in the network and the observation vector $\mathbf{y}$ is a set of flow and speed at all observation locations. A set of state and observation equations is as follows

$$\text{State equation:} \quad \rho_i^l(k+1) = \rho_i^l(k) + \frac{\Delta t}{\Delta x} (qb_{i-1}^l(k) - qb_i^l(k)) + \varphi_i^l(k) \quad (9)$$

$$\text{Observation equation 1:} \quad v_i^l(k) = V_e(\rho_i^l(k)) + \psi_i^{v,l}(k) \quad (10)$$

$$\text{Observation equation 2:} \quad q_i^l(k) = Q_e(\rho_i^l(k)) = \rho_i^l(k) \cdot V_e(\rho_i^l(k)) + \psi_i^{q,l}(k). \quad (11)$$

where φ and ψ are the noise of system and observation equation, respectively. From these equations, it is possible to define the state-space model as follows

$$\text{State equation:} \quad \mathbf{x}(k+1) = \mathbf{f}(\mathbf{x}(k), \mathbf{u}(k)) + \boldsymbol{\varphi}(k)$$

$$\text{Observation equation:} \quad \mathbf{y}(k) = \mathbf{g}(\mathbf{x}(k), \mathbf{u}(k)) + \boldsymbol{\psi}(k).$$

Since equations (9) – (11) are non-linear, the EKF is more suitable than the standard KF. The detail of changing state-space model to the form suitable for EKF and the algorithm of calculating and updating can be found in the general textbook of KF.

4. Numerical Example

A schematic diagram of a hypothetical network of a four-leg intersection, two lanes and 200 m. length on each leg is shown in Figure 1. The speed-density diagram for all links is assumed to follow the linear model with free flow speed = 60 km/hr, jam density = 120 veh/km/lane, and saturation flow rate = 1,800 veh/hr/lane. The signal has 120 seconds of cycle length, 4 phases of 30 seconds each. Each leg is allowed to move through intersection in all directions at only one phase. Three different traffic scenarios are considered in this study as shown in Table 1. The first scenario represents the case of deterministic where flow and turning ratio are constant and known. The second and third scenarios include changes of flow and turning ratio. Note that the average of varying turning ratios is the same as in the case of constant. The actual values are used in generating traffic data using simulation software but the average values are used in the macroscopic model in order to represent the case of imperfect prior knowledge of turning ratio. The main purpose is to compare between the macroscopic model alone and the macroscopic with EKF on the accuracy of estimating traffic state in case of knowing only average turning ratios.

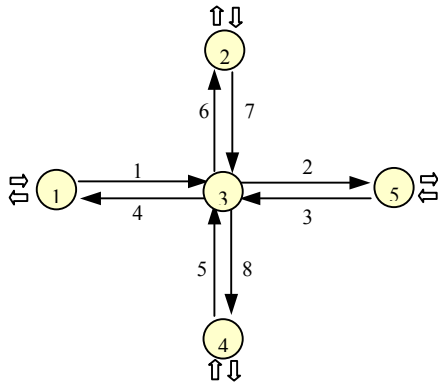


Figure 1: Schematic diagram of a sample network

Table 1: Traffic scenarios

Scenario	Inflow pattern	Turning ratio pattern
1	Constant	Constant
	(700) & (600)	(0.3, 0.6, 0.1) & (0.2, 0.7, 0.1)
2	Constant	Varying
	(700) & (600)	(0.25, 0.68, 0.07) & (0.35, 0.6, 0.05) (0.35, 0.52, 0.13) & (0.05, 0.8, 0.15)
3	Varying	Varying
	(700) & (600) (900) & (800)	(0.25, 0.68, 0.07) & (0.35, 0.6, 0.05) (0.35, 0.52, 0.13) & (0.05, 0.8, 0.15)

Remarks: In case of varying, the second line represents the value using during the first half in simulation software while the third line represents the value during the last half. The number in the first parenthesis of inflow pattern is inflow of link 1 and 3 (in veh/hr) while the second parenthesis is for link 5 and 7. The number in the first parenthesis of turning ratio pattern is for link 1 and 7 (left, through, right) while the second parenthesis is for link 3 and 5.

The road network is discretized in space and time with $\Delta x = 40$ m and $\Delta t = 2$ sec. The total simulation time is 1800 sec. Eight detectors are installed at the entry and exit point of the network and data from these detectors are used as the observation values in the model with EKF. Another set of detectors installed at the midpoint of exit link is used as the check stations for verifying the model. The aggregation interval of observed flow and speed is 120 sec. Aggregated inflow rate measured by detector at the entry point is used as the input inflow boundary for the model. The results obtained from model are averaged every 120 sec interval and are compared with the one obtained from simulation software, INTEGRATION. In order to evaluate the performance of estimations quantitatively, error indices such as root mean square error (RMSE) and mean absolute relative error (MARE) are used. They are defined as:

$$RMSE = \sqrt{\frac{1}{I \cdot K} \sum_k \sum_i (x_i^k - \hat{x}_i^k)^2} \quad \text{and} \quad MARE = \frac{1}{I \cdot K} \sum_k \sum_i \frac{|x_i^k - \hat{x}_i^k|}{x_i^k} \quad (12)$$

where I = number of observation points, K = number of calculation time steps, x_i^k = the actual value of the reference variable of point i at time k , and $\hat{x}_i^k$ = the estimated value of the reference variable of point i at time k . Table 2 shows the performance of the model for all traffic scenarios. The plots of estimation and observed flow at check points on link 4 and 8 are shown in Figure 2.

Table 2: Model performance

Scenario	Model	RMSE _q (vph)	MARE _q	RMSE _v (kph)	MARE _v
1	Macro	25.78	0.035	7.24	0.144
	Macro + EKF	31.33	0.043	7.35	0.146
2	Macro	91.47	0.124	7.58	0.150
	Macro + EKF	89.56	0.124	7.69	0.152
3	Macro	106.16	0.130	7.58	0.151
	Macro + EKF	102.08	0.126	7.72	0.154

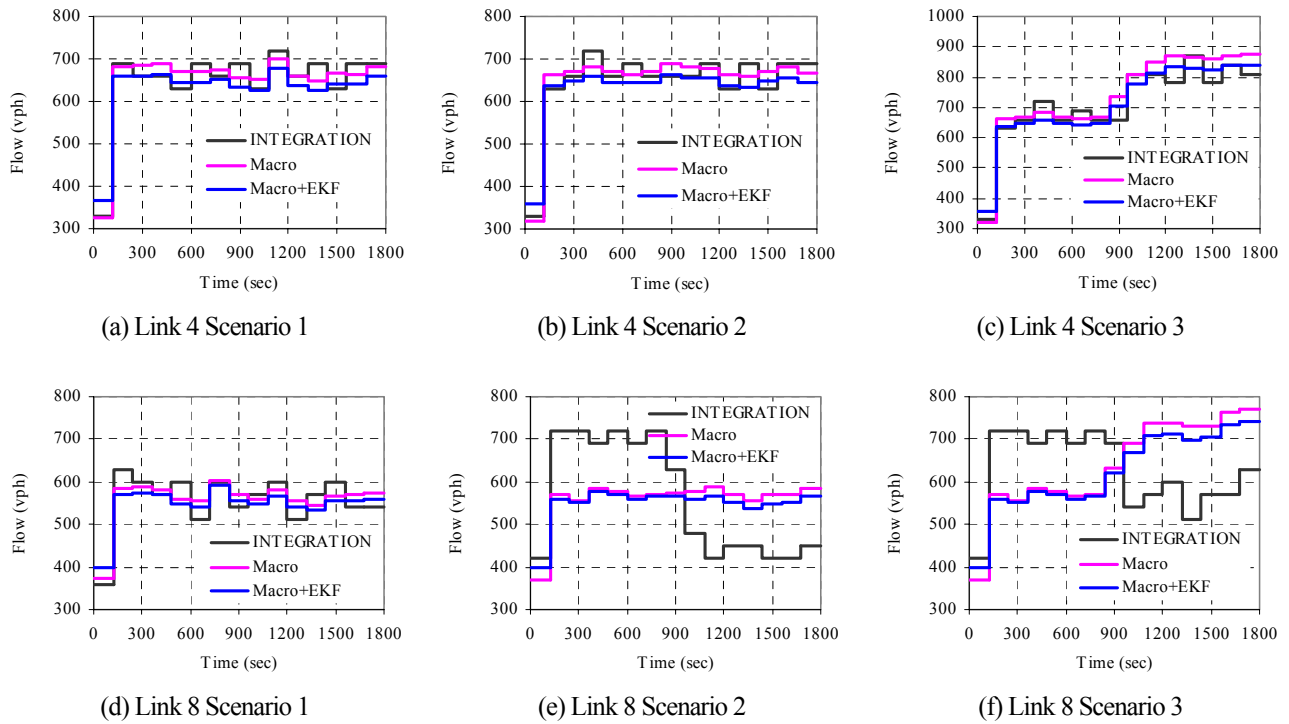


Figure 2: Comparison of observed and estimated flow on link 4 and 8

From the above results, we can see that macroscopic model alone gives slightly better speed estimation in all scenarios. In term of flow estimation, macroscopic model with EKF seems to be better particularly when only the average turning ratio is available. Both models give a very good estimate for link 4 since the combination of changes of turning ratio and inflow results in a small and gradual change of flow. In contrast, the combination of changes in turning ratio and inflow results in a sudden and large change of flow on link 8 and hence both models give a poor estimate. Therefore, the better knowledge of changes in turning ratio would give a better estimation results.

5. Conclusion

In this paper, two dynamic traffic estimation models were proposed. One is based on the macroscopic traffic model alone and another is based on both macroscopic model and the extended Kalman filter. The numerical method of macroscopic model using demand and supply concept was adopted. In order to make it more suitable for surface street network with signalized intersection, the modified version of general intersection model was implemented. The models were tested with hypothetical network under the mixed between constant/varying inflow and constant/varying turning ratio. The models give reasonable and well estimate in case of gradual changes of traffic condition. The future research may go to the dual estimation of both traffic state and turning ratio.

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