A DYNAMIC GROSS INVESTMENT MODEL USING OVERLAPPING GENERATION AND GENERAL EQUILIBRIUM CONCEPT*

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Abstract

This paper develops a dynamic gross investment model considering overlapping generation (OLG) and dynamic computable general equilibrium (CGE) concept. Most of the previous works used only two period OLG model where investment policy is analyzed. But in this work, seven period OLG model is used where investment as well as reinvestment policy is analyzed for consumers and firms. This model is based on intertemporal dynamic optimization behaviors of consumers and firms. A final demand is classified into consumption and investment goods for analyzing dynamic CGE. Not only gross investment but also the domestic and foreign investment are taken into consideration in this investment model. The effect of present gross investment, next period gross domestic product and changes in interest rate are considered to analyze the dynamic gross investment of the economy. The international input-output (IO) transaction table is used for analyzing Japan-U.S economy in details.

1   Introduction

Investment has a major impact on domestic output and incomes of a country for analyzing the economic condition of that country. In international trade, a substantial share of domestic output of a country is exported to foreign countries and a large part of total expenditure is imported from foreign countries. So, it can be said that a dynamic gross investment also has an important role for the trade prediction that is most important for the port plan. More generally, investment depends upon the revenues that will be generated by the state of overall economic activity. The aim of this study is to construct a dynamic gross investment model for analyzing the Japan-US economy considering OLG concepts.

There are several previous IO and CGE models which analyze supply and demand balance of goods. Leontief et. al. (1977)[1] extended IO model to the trade model among many countries based on general equilibrium theory. Multiregional Input-Output (MRIO) models developed by Isard (1951)[2], Moses (1955)[3], Leontief and Shrout (1963)[4] are capable of describing the interregional trade flows and the interindustry transactions in regional and industrial details. C. K. Liew and C. J. Liew (1985)[5] introduced a model to measure the development impact of a transportation system considering regional input-output coefficients, trade coefficients and trade flows. Whalley (1985)[6] developed two models (four-region and seven region model) for analyzing the impact of changes in trade policies among developed countries. A. feltenstein and A. Shah (1995)[7] found that a corporate tax reduction has the most stimulate impact on investment. Leon Bettendorf (1998)[8] analyzed a investment-promoting polices in a small open economy. Simulations of a decrease of the corporate income tax rate and an increase of the investment tax credit rate are discussed and compared in his work. Dirk Willenbockel (1999)[9] analyzed a dynamic two-country model with international capital mobility and intertemporal optimizing agents. Jin Shui Zhang (2001)[10] extended the linear dynamic input-output model to the computable non-linear dynamic input-output model.

In all of the above works, neither the consumption-goods demand nor the investment-goods demand are distinguished in the final demand. But in this paper, investment and consumption goods are distinguished in the final demand. The investment goods demand can be expected to increase independently in the future economic growth but the consumption goods demand is increased when the national income is piled up. So, it might be thought that the investment and consumption goods have different influence on economy. In this paper, seven period OLG technique is used for developing a dynamic gross investment model. In OLG model, it is assumed that consumers receive disposable income for consumption and savings in lifetime budget condition. Income expenditure and savings of firms are also considered. Furthermore, it is assumed that consumers invest all savings for buying domestic/foreign bonds, and firms invest all savings into the next period capital stock. Several other papers have considered two period OLG model where only investment policy is analyzed. However, this paper considers seven period OLG model for analyzing the investment as well as reinvestment policy for consumers and firms. The same technique like OLG model is applied for developing a dynamic gross investment model using dynamic CGE concept. Not only gross investment but also the domestic and foreign investment are taken into consideration in this investment model. The effect of present gross investment, next period gross domestic product and changes in interest rate are considered to analyze the dynamic gross investment of the economy. The paper is organized as follows. Section 2 presents the framework of the model. In section 3, the model is formulized for analyzing the production activities, income distribution for household, firm and government, final demand for consumption and investment goods demand and dynamic gross investment. Finally section 4 summarizes the conclusion.
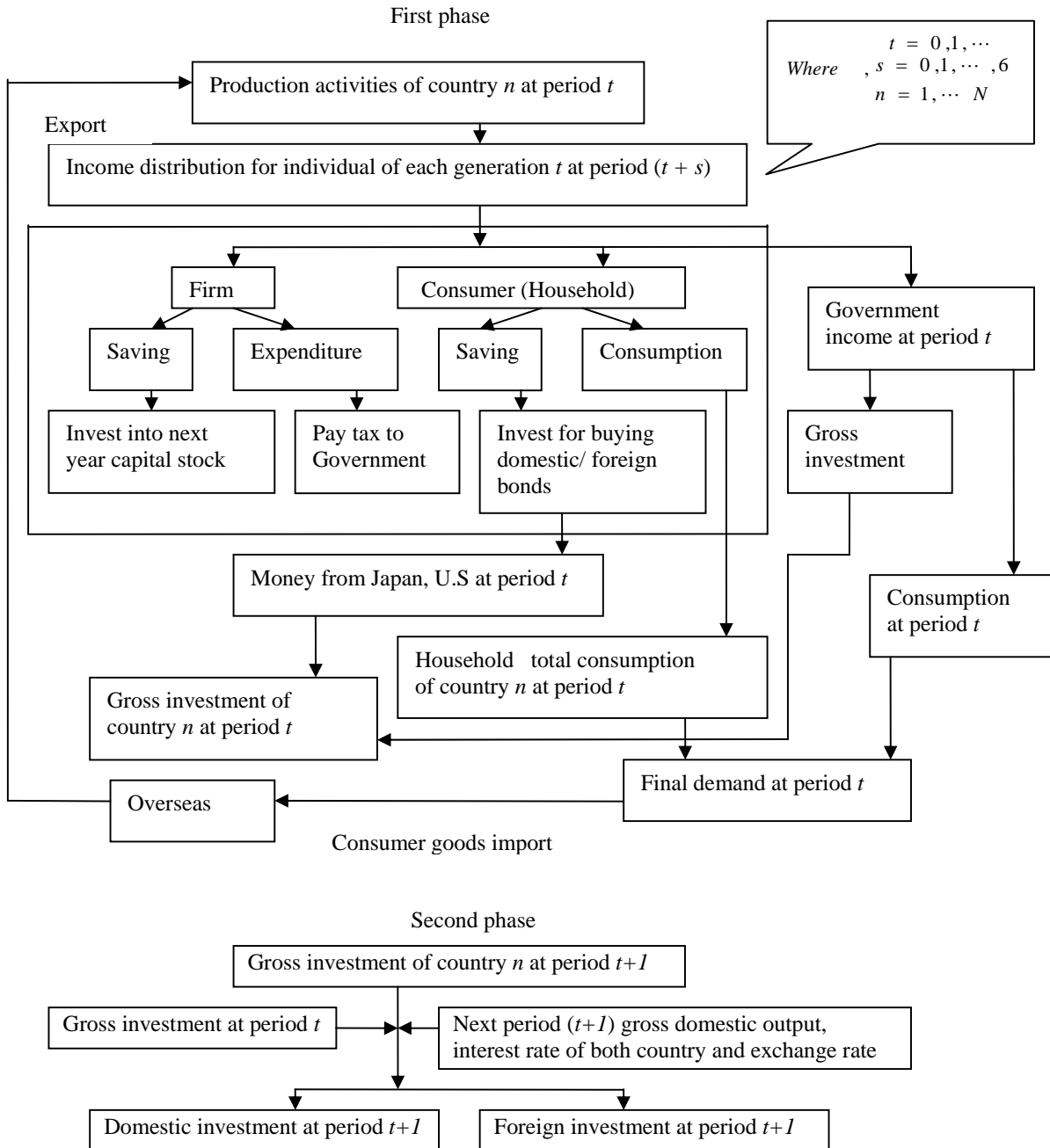
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2   Framework of Model

The dynamic gross investment model is described into two phases. In the first phase, present gross investment is calculated for describing the economic activities of a country using four calculation steps: analysis of production activities, income distribution, final demand, and accumulation of capital. In the second phase, the future gross investment is analyzed considering the effect of present gross investment, next period gross domestic product and changes in interest rate. The model is built for two or more countries considering the export and import of goods and investment among the countries. The framework of the model is given bellow.

First phase

Production activities of country n at period t

Export

Income distribution for individual of each generation t at period $(t + s)$

$$Where \quad t = 0,1,\cdots \\ , s = 0,1,\cdots,6 \\ n = 1,\cdots\ N$$

Firm

Consumer (Household)

Saving   Expenditure   Saving   Consumption

Government income at period t

Invest into next year capital stock

Pay tax to Government

Invest for buying domestic/ foreign bonds

Gross investment

Money from Japan, U.S at period t

Household   total consumption of country n at period t

Consumption at period t

Gross investment of country n at period t

Final demand at period t

Overseas

Consumer goods import

Second phase

Gross investment of country n at period $t+1$

Gross investment at period t

Next period ($t+1$) gross domestic output, interest rate of both country and exchange rate

Domestic investment at period $t+1$

Foreign investment at period $t+1$

3   Formulization of Model

3.1   *Analysis of production activities and equilibrium price*

It is assumed that the following assumptions (a)-(c) from Armington are also hold for this model.
(a) Market equilibrium is considered for all goods and excess demand is eliminated.

(b) Production factors, labor and capital movement between countries are impossible but movement between domestic industries are possible.
(c) Commodities are distinguished by place of production.

All countries except the candidate countries (m, $n=1$ to N from here after) are treated as Rest of the World (ROW). Although the activities of ROW are not taken into consideration but it has been taken only for balancing of the market. Furthermore, the classification is not considered about the commodities from ROW because all commodities from ROW are regarded as a single commodity.

Production function: In this paper, the production function of country n industry j is assumed as the Leontief type production function of concerning intermediate inputs and production factors which is expressed by the following relation.

$$X_{jt}^{n} = \min\left(\cdots , \frac{x_{ijt}^{mn}}{a_{ijt}^{mn}} , \cdots , \frac{x_{jt}^{ROWn}}{a_{jt}^{ROWn}} , \cdots \frac{Y_{jt}^{n}}{a_{Y_jt}^{n}} \right) \tag{1}$$

where

$X_{jt}^{n} = \qquad$ quantity of production of country n industry j at period t,

$x_{ijt}^{mn} = \qquad$ the intermediate input to country n industry j imported commodity i from country m at period t,

$x_{jt}^{ROWn} = \qquad$ the intermediate input to country n industry j imported commodity i from ROW at period t,

$Y_{jt}^{n} = \qquad$ production capacity of country n industry at period t,

$a_{ijt}^{mn} , a_{jt}^{ROWn} = $ input coefficient at period t, and

$a_{Y_jt}^{n} = \qquad$ a ratio of production capacity per unit quantity of production in country n at period t.

Production capacity: In above equation (1), a Cobb-Douglas type production capacity function Y_{jt}^{n} is assumed for production factors. The function can be expressed as follows.

$$Y_{jt}^{n} = \theta_{jt}^{n}\left(L_{jt}^{n}\right)^{\alpha_{L_jt}^{n}}\left(K_{jt}^{n}\right)^{\alpha_{K_jt}^{n}} \tag{2}$$

where

$$\left(\alpha_{L_jt}^{n} + \alpha_{K_jt}^{n}\right) = 1,$$

L_{jt}^{n}, $K_{jt}^{n} = $ labor and capital of country n which are supplied in industry j at period t, and

$\theta_{jt}^{n} = $ manufacturing technique parameter in country n at period t.

Equilibrium price: In the following equation (3), it is assumed that the producer price of commodity is the sum of payments to primary factors and intermediate inputs. For calculating the payments to primary factors (labor and capital), it is necessary to consider the labor and capital demand which can be found by solving the cost minimization problem. At the market equilibrium condition, an equilibrium price can be obtained from the following equation (3) which is considered as the prices of commodity of a country. A price equilibrium equation is formulized as the following equation.

$$P_{jt}^{n} = \sum_{m}\sum_{i}\left(q_{it}^{mn} a_{ijt}^{mn} \right) + q_{t}^{ROWn} a_{jt}^{ROWn} + a_{Y_jt}^{n}\left(\omega_{t}^{n} DL_{jt}^{n} + r_{t}^{n} DK_{jt}^{n}\right) \tag{3}$$

where

$P_{jt}^{n} = \qquad$ the producer price of country n industry j at period t,

$q_{it}^{mn} = \qquad$ the consumer price in country n of imported commodity i from country m at period t,

$q_{t}^{ROWn} = \qquad$ the price of commodities in ROW at period t,

$\omega_{t}^{n} = \qquad$ the rate of wages in country n at period t,

$r_t^n =$ capital service cost in country n at period t,

$DL_{jt}^n =$ labor demand per unit production capacity in country n at period t, and

$DK_{jt}^n =$ capital demand per unit production capacity in country n at period t.

Cost minimization problem: Labor demand and capital demand per unit production capacity are obtained by solving the following cost minimization problem.

$$\min_{DL_{jt}^n, DK_{jt}^n} \omega_t^n DL_{jt}^n + r_{jt}^n DK_{jt}^n$$

$$s.t \; \theta_{jt}^n \left(DL_{jt}^n \right)^{\alpha_{L_{jt}}^n} \left(DK_{jt}^n \right)^{\alpha_{K_{jt}}^n} = 1 \tag{4}$$

Therefore solving the above equation (4), labor demand and capital demand per unit production capacity is found as follows.

$$DL_{jt}^n = \frac{1}{\theta_{jt}^n} \left(\frac{\alpha_{L_j}^n r_t^n}{\alpha_{K_{jt}}^n \omega_t^n} \right)^{\alpha_{K_{jt}}^n} \; and \; DK_{jt}^n = \frac{1}{\theta_{jt}^n} \left(\frac{\alpha_{K_{jt}}^n \omega_t^n}{\alpha_{L_{jt}}^n r_t^n} \right)^{\alpha_{L_{jt}}^n} \tag{5}$$

Producer price and consumer price: The relation between consumer price and producer price of a production country with respect to domestic goods demand and overseas goods demand can be expressed as follows.

$$q_{it}^{nn} = p_{it}^n, q_{it}^{mn} = \frac{p_{it}^m}{\varepsilon^{mn}} \tag{6}$$

where

$\varepsilon^{mn} =$ exchange rate.

3.2   *Income distribution for consumer and firm*

In this section two types of income distribution, primary income distribution and secondary income distribution are considered for consumer and firm. A labor and capital income is assumed as the primary income for a consumer and a firm. For secondary income distribution, it is assumed that a household receives disposable income after paying the corporation tax to the government from labor income, and a firm receives the capital income after paying corporation tax to the government from operating surplus.

3.2.1   **Primary income distribution**

A labor income is distributed as employee compensation for a consumer (household). In this model, at first, a labor income is formulized at period t for considering OLG technique as the following equation.

$$ce_t^n = \sum_j \left(\omega_t^n dl_{jt}^n a_{Y_{jt}}^n X_{jt}^n \right) \tag{7}$$

where

$$dl_{jt}^n = \frac{DL_{jt}^n}{N_t^n}, \qquad N_t^n = L_t^n,$$

$ce_t^n =$ employee compensation for individual of country n at period t,

$N_t^n =$ total working population of country n at period t,

$L_t^n =$ total labor supply of country n at period t, and

$dl_{jt}^n =$ individual labor demand of country n for industry j at period t.

Capital income per worker at period t is served as fixed capital depreciation and operating surplus which is shown bellow.

$$fk_t^n = \sum_j \left(\delta_{jt}^n P_{K_{jt}}^n dk_{jt}^n a_{Y_{jt}}^n X_{jt}^n \right) \; and \tag{8}$$

$$os_t^n = \sum_j \left[\left(r_t^n - \delta_{jt}^n P_{K_{jt}}^n\right) dk_{jt}^n a_{Y_{jt}}^n X_{jt}^n\right]$$
(9)

where
$$dk_{jt}^n = \frac{DK_{jt}^n}{N_t^n},$$

$fk_t^n =$ fixed capital depreciation amount per worker of country n at period t,

$\delta_{jt}^n =$ the rate of depreciation of capital stock at period t,

$os_t^n =$ operating surplus per worker of country n at period t,

$P_{K_{jt}}^n =$ the price of capital stock at period t, and

$dk_{jt}^n =$ individual capital demand of country n for industry j at period t.

       Next, the primary income distribution is formulized for each generation t at period $t+s$. Because, in this paper, seven period OLG model is considered for income distribution. For simplicity, it is assumed that consumers live seven periods in infinite horizon, so that seven generations coexist in every period t. Furthermore, it is assumed that consumers in each generation start to work for saving and consumption after reaching age 30 and retire at age 60. All activities for the remaining lifetime until death are considered similar and included with the retirement period. So, all consumers belonging to the same generation have the same income for every period as described by the following equations. In all equations from (10) to equation (20),the superscript denotes the year of birth of each generation t of country n and subscript denotes the date of consumption.

       Employee compensation per worker (household) ce_{t+s}^{nt} for each generation t at period ($t+s$) is expressed as the following equations.

$$ce_{t+s}^{nt} = \sum_j \left(\omega_{t+s}^{nt} dl_{j,t+s}^{nt} a_{Y_{jt}}^n X_{jt}^n\right)$$
(10)

where,
$$dl_{j,t+s}^{nt} = \frac{1}{\theta_{j,t+s}^n}\left(\frac{\alpha_{L_{jt}}^n r_{t+s}^{nt}}{\alpha_{K_{jt}}^n \omega_{t+s}^{nt}}\right)^{\alpha_{K_{jt}}^n},$$

$dl_{j,t+s}^{nt} =$ individual labor demand of industry j for each generation t at period $t+s$,

$t = 0,1,\cdots, \quad and \quad s = 0,1,2,\cdots,6$ from here after.

       Capital income distribution per worker for each generation t at period ($t+s$) is formulized by the following equations.

$$fk_{t+s}^{nt} = \sum_j \left(\delta_{j,t}^n P_{K_{jt}}^n dk_{j,t+s}^{nt} a_{Y_{jt}}^n X_{jt}^n\right) \quad and$$
(11)

$$os_{t+s}^{nt} = \sum_j \left[\left(r_{t+s}^{nt} - \delta_{j,t}^n P_{K_{jt}}^n\right) dk_{j,t+s}^{nt} a_{Y_{jt}}^n X_{jt}^n\right]$$
(12)

where
$$dk_{j,t+s}^{nt} = \frac{1}{\theta_{j,t+s}^{nt}}\left(\frac{\alpha_{K_{jt}}^n \omega_{t+s}^{nt}}{\alpha_{L_{jt}}^n r_{t+s}^{nt}}\right)^{\alpha_{L_{jt}}^n},$$

$fk_{t+s}^{nt} =$ fixed capital depreciation per worker of country n for each generation t at every period $t+s$,

$os_{t+s}^{nt} =$ operating surplus per worker of country n for each *generation t* at every period $t+s$, and

$dk_{t+s}^{nt} =$ individual capital demand of country n for industry j for each generation t at every period $t+s$.

From first order zero profit maximization problem, the labor wage rate and capital service rate can be calculated for each generation at his lifetime.

3.2.2 Secondary income distribution

A household receives disposable income after paying income tax to the government from labor income. He also receives transfer income from government for pension, social security, health, transportation etc. which is included with disposable income. The relation can be expressed as

$$di_{t+s}^{nt} = ce_{t+s}^{nt} - dt_{h,t+s}^{nt} + trg_{t+s}^{nt} \tag{13}$$

where

$$dt_{h,t+s}^{nt} = \tau_{h,t}^{nt} ce_{t+s}^{nt},$$

$di_{t+s}^{nt} =$ household disposable income for each generation t at period $t+s$ of country n,

$dt_{h,t+s}^{nt} =$ income tax for each generation t at period $t+s$ of country n,

$trg_{t+s}^{nt} =$ transfer income for each generation t of country n, and

$\tau_{h,t}^{nt} =$ income tax rate which is same for all over the model.

In this paper, it is assumed that a household spends his disposable income for saving and consumption. All savings are invested/reinvested for buying domestic/foreign bonds in each generation t at his lifetime. The household lifetime budget constraint is formulized by the following equation.

$$s_{h,t+s}^{nt} = di_{t+s}^{nt} + s_{h,t-1+s}^{nt}(1 + R_s) - c_{h,t+s}^{nt} \tag{14}$$

where

$s_{h,t+s}^{nt} =$ household saving of country n for each generation t at period $t+s$, and

$c_{h,t+s}^{nt} =$ household consumption of country n for each generation t at period $t+s$,

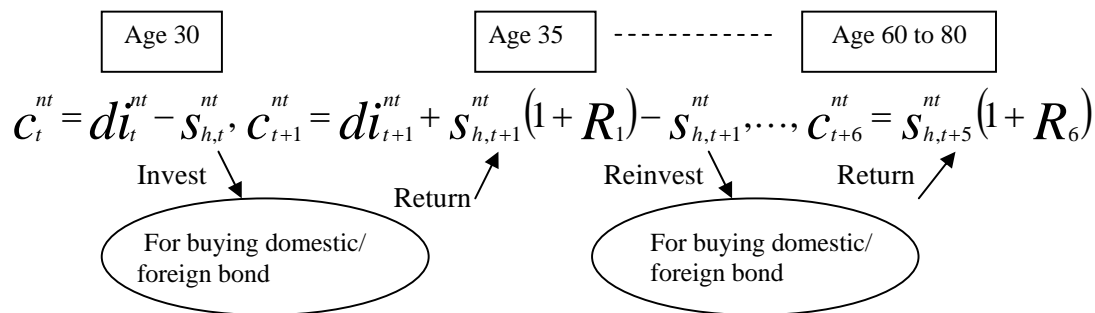Since bequests are not considered, initial return is zero that is,

$s = 0, s_{h,t-1+s}^{nt}(1 + R_s) = 0$

Household saving and disposable income are also zero at the retirement period that is,

$s = 6, s_{h,t+s}^{nt} = 0, di_{t+s}^{nt} = 0$

The household budget constraint is split for understanding the model clearly which is illustrated below.

Household consumption demand is obtained by solving the following Utility maximization problem subject to household lifetime budget constraint for each generation t of country n which is formulized as follows.

$$\max u^{nt}(c) = \sum_{s=0}^{6} \beta^s u\left(c_{t+s}^{nt}\right)$$

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$$s.t. \cdot c_t^{nt} + \frac{c_{t+1}^{nt}}{(1+R_1)} + \cdots + \frac{c_{t+6}^{nt} = \sum_{s=6}^{10} c_{t+s}^{nt}}{(1+R_1) \cdots (1+R_6)} = DI^{nt} \tag{15}$$

where

$$DI^{nt} = di_t^{nt} + \frac{di_{t+1}^{nt}}{(1+R_1)} + \cdots + \frac{di_{t+5}^{nt}}{(1+R_1) \cdots (1+R_5)},$$

DI^{nt} = lifetime disposable income for each generation t of country n, and

R = interest rate.

      Firm receives the money after paying corporation tax to the government from operating surplus that is considered as savings of firm. All savings of firm are invested/ reinvested into the next year capital stock of country n. Firm's saving and capital stock for each generation t at period $(t+s)$ are formulized by the following equations.

$$s_{f,t+s}^{nt} = os_{t+s}^{nt} - dt_{f,t+s}^{nt}, \tag{16}$$

$$dt_{f,t+s}^{nt} = \tau_{f,t}^{nt} os_{t+s}^{nt}, \tag{17}$$

$$s_{f,t+s}^{nt} = I_{f,t+s}^{nt} \quad and \tag{18}$$

$$k_{t+s}^{nt} = k_{t+s-1}^{nt}(1-\delta_t^n) + I_{f,t+s-1}^{nt}, \quad when, s=0, k_t^n = k_t^{nt} \tag{19}$$

where

$s_{f,t+s}^{nt}$ = firm savings for each generation t at period $t+s$ of country n,

$dt_{f,t+s}^{nt}$ = corporation tax for each generation t at period $t+s$ of country n,

$\tau_{f,t}^{nt}$ = corporation tax rate which is same for all over the model.

k_{t+s}^{nt} = per worker capital stock for each generation t at period $t+s$ of country n

$I_{f,t+s}^{nt}$ = investment amount of firm for each generation t at period $t+s$ of country n.

      In this model, seven generations coexist in every period t. For calculating the economic activities of country n in period t, it is necessary to calculate the economic activities of country n for each generation t at period $t+s$. For this reason, all above equations from (10) to (21) are formulized for each generation t at period $t+s$ of country n. Now, considering these equations, the following activities for consumers and firms of country n can be easily formulized in every period t.

      Total employee compensation CE_t^n, total disposable income DI_t^n, total income tax $DT_{h,t}^n$, total consumption C_t^n and total savings $S_{h,t}^n$ of households of country n in period t are formulized as follows:

$$CE_t^n = \sum_{s=0}^{5} N^{t-s} ce_t^{t-s}, \quad DI_t^n = \sum_{s=0}^{5} N^{t-s} di_t^{t-s}, \quad DT_{h,t}^n = \sum_{s=0}^{5} N^{t-s} dt_{h,t}^{t-s},$$

$$C_t^n = \sum_{s=0}^{6} N^{t-s} c_t^{n,t-s} and \quad S_{h,t}^n = \sum_{s=0}^{5} N^{t-s} s_{h,t}^{n,t-s} \tag{20}$$

where

$$N^t = (1+\eta) N^{t-1}, \quad N_t^n = \sum_{s=0}^{6} N^{t-s}$$

η = working population growth rate for generation wise.

Total fixed capital depreciation FK_t^n, total operating surplus OS_t^n, total corporation tax (expenditure of firm) $DT_{f,t}^n$ and total savings $S_{f,t}^n$ of firms of country n in period t are formulized as follows:

$$FK_t^n = \sum_{s=0}^{6} N^{t-s} fk_t^{n-s}, \quad OS_t^n = \sum_{s=0}^{5} N^{t-s} os_t^{t-s}, \quad DT_{f,t}^n = \sum_{s=0}^{5} N^{t-s} dt_{f,t}^{t-s} \text{ and } \quad S_{f,t}^n = \sum_{s=0}^{5} N^{t-s} s_{f,t}^{t-s} \qquad (21)$$

3.3 Income expenditure for government

Although the government behavior is not considered for each generation t at period $t+s$ like other agents but it is analyzed in every period t. Government receives income after paying transfer income and returning on bonds issued to households from his revenues. Government revenues are obtained by tax collection from household and firm and bonds issuing for household. Government spends his income for consumption and gross investment as described bellow.

$$G_t^n = DT_{h,t}^n + DT_{f,t}^n + \sum_{m=Japan,U.S} Bn_t^{nm} - \sum_{m=Japan,U.S} Bn_{t-1}^{nm} - TRG_t^n \qquad (22)$$

$$GI_{g,t}^n = sav_{g,t}^n G_t^n, \quad GC_t^n = \left(1 - sav_{g,t}^n\right) G_t^n \qquad (23)$$

where

$G_t^n =$ government income of country n in period t,

$GI_{g,t}^n =$ government gross investment of country n in period t,

$GC_t^n =$ government consumption of country n in period t,

$\sum_m Bn_t^{nm} =$ amount of bonds issued by government of country n in period ,

$\sum_m Bn_{t-1}^{nm} =$ amount of bond issued by government at last period of country n, and

$sav_{g,t}^n =$ gross investment rate of government of country n in period t.

3.4 Final demand

In this paper, final demand is classified into consumption goods demand and investment goods demand. Consumption goods demand is performed by households and the government consumption demand. If utility maximization problem subject to consumption restrictions is assumed Cobb-Douglas type, then the household consumption demand for commodity can be formulized as follows.

$$\max, U_t^n = \left[\prod_m \prod_i \left(c_{it}^{mn} \right)^{\beta_{it}^{mn}} \right] \left(c_t^{ROWn} \right)^{\beta_t^{ROWn}}$$

$$s.t \left(\sum_m \sum_i q_{it}^{mn} c_{it}^{mn} \right) + q_t^{ROWn} c_t^{ROWn} = \sum_i C_{it}^n \qquad (24)$$

$$\text{where} \quad C_t^n = \sum_i C_{it}^n$$

The household consumption demand for commodity is given by

$$c_{it}^{mn} = \frac{\beta_{it}^{mn} C_t^n}{q_{it}^{mn}}, \qquad c_t^{ROWn} = \frac{\beta_t^{ROWn} C_t^n}{q_t^{ROWn}} \qquad (25)$$

where

$$\left(\sum_m \sum_i \beta_{it}^{mn} \right) + \beta_t^{ROWn} = 1,$$

$c_{it}^{mn} =$ the household consumption demand of country n in period t for commodity i imported from country m,

$\beta_{it}^{mn}, \beta_t^{ROWn} =$ preference parameter in period t, and

$c_t^{ROWn} =$ the household consumption demand of country n in period t for commodities imported from *ROW*.

Similarly, the government consumption demand function also can be formulized as follows.

$$g_{it}^{mn} = \frac{\zeta_{it}^{mn} GC_t^n}{q_{it}^{mn}}, \qquad g_t^{ROWn} = \frac{\zeta_t^{ROWn} GC_t^n}{q_t^{ROWn}} \tag{26}$$

where

$g_{it}^{mn} =$ the government consumption demand of country n for commodity i imported from country m, and

$\zeta_{it}^{mn}, \zeta_t^{ROWn} =$ government preference parameter in period t.

The investment good demand depends on the real investment of a country that is settled from the dynamic gross investment model. The investment goods influence supply and demand balance of goods of a country. The real investment function is considered as the Leontief type technology that is expressed as follows.

$$GCF_t^n = \min\left(\ldots, \frac{q_{it}^{mn} cp_{it}^{mn}}{\phi_{it}^{mn}}, \ldots, \frac{q_{it}^{ROWN} cp_{it}^{ROWn}}{\phi_{it}^{ROWn}} \right) \tag{27}$$

Therefore, the investment goods demand for each commodity can be derived from the above equation no. (27) is expressed as follows.

$$cp_{it}^{mn} = \frac{\phi_{it}^{mn} GCF_t^n}{q_{it}^{mn}}, \qquad cp_{it}^{ROWn} = \frac{\phi_{it}^{ROWn} GCF_t^n}{q_{it}^{ROWn}} \tag{28}$$

where

$$\sum_m \sum_m \left(\phi_{it}^{mn} \right) + \phi_{it}^{ROWn} = 1,$$

GCF_{it}^n : the real investment frame of country n at period t,

ϕ_{it}^{mn} : the parameter of real investment demand technology at period ,

cp_{it}^{mn} : real investment demand of the goods in country n at period t.

3.5 Accumulation of capital

In this paper, it is assumed that households invest their savings for buying domestic/foreign bonds. The investment to ROW is not considered. So, the gross investment of a country is estimated by the amount of money from domestic country (household investment amount and government gross investment) and foreign countries (household investment amount) as formulized by the following equations.

$$S_{h,t}^n = I_{h,t}^n = \sum_{m=Japan,U.S} I_t^{nm} = I_{Db,t}^{nn} + I_{Fb,t}^{nm} \tag{29}$$

$$GI_t^n = I_{Db,t}^{nn} + I_{Fb,t}^{mn} + GI_{g,t}^n \tag{30}$$

$$\sum_m Bn_t^{nm} = I_{Db,t}^{nn} + I_{Fb,t}^{mn} \tag{31}$$

where

$I_{Db,t}^{nn} =$ the amount of money of country n that is invested for buying domestic bonds in period t,

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$I^{mn}_{Fb,t} =$ the amount of money of country n that is invested for buying foreign bonds in period t,

$GI^{n}_{t} =$ gross investment of country n in period t, and

$I^{mn}_{Fb,t} =$ the amount of money of foreign country that is invested for buying bonds of country n in period t.

3.6 Dynamic gross domestic product and gross investment model

Investment has a major impact on domestic output and incomes of a country for analyzing the economic condition of that country. So, it can be said that the analysis of dynamic gross domestic product is important to analyze the dynamic gross investment of a country.

Dynamic gross domestic product: The gross domestic product GDP^{n}_{t} for each country at each period is the sum of operating surplus, employee compensation, and fixed capital depreciation of that country. In this paper, future GDP^{n}_{t+1} is analyzed for each country using the OLG technique as formulized by the following equations.

$$GDP^{n}_{t} = OS^{n}_{t} + CE^{n}_{t} + FK^{n}_{t} \quad and \tag{32}$$

$$GDP^{n}_{t+1} = OS^{n}_{t+1} + CE^{n}_{t+1} + FK^{n}_{t+1} \tag{33}$$

where

$$OS^{n}_{t+1} = \left(\sum_{j} \left[\left(r^{n}_{t+1} - \delta^{n}_{j,t+1} P^{n}_{K_{j}t} \right) dk^{n}_{j,t+1} a^{n}_{Y_{jt}} X^{n}_{j,t+1} \right] \right) N^{n}_{t+1},$$

$$CE^{n}_{t+1} = \left(\sum_{j} \left(\omega^{n}_{t+1} dl^{n}_{j,t+1} a^{n}_{Y_{jt}} X^{n}_{j,t+1} \right) \right) N^{n}_{t+1},$$

$$FK^{n}_{t+1} = \left(\sum_{j} \left(\delta^{n}_{j,t} P^{n}_{K_{j}t} dk^{n}_{j,t+1} a^{n}_{Y_{jt}} X^{n}_{j,t+1} \right) \right) N^{n}_{t+1}.$$

Dynamic gross investment model: In this model, the future gross investment of a country is considered as a function of present gross investment, next period gross domestic product, interest rate of both country and exchange rate based on Keynesian marginal efficiency theory. Not only gross investment but also the domestic and foreign investments are taken into consideration in this investment model. The amount of foreign investment is estimated by determining the rate of the foreign investment to each country into the investment model, and the remainder is considered as the amount of domestic investment. However, this dynamic gross investment model does not consider the investment to ROW. If the above is summarized, an investment model will be formulized as follows.

$$GI^{n}_{t+1} = f^{n}_{GI} \left(GI^{n}_{t}, R^{n}_{t+1}, R^{m}_{t+1}, GDP^{n}_{t+1}, \varepsilon_{t+1} \right) \tag{34}$$

$$FA^{mn}_{t+1} = rfa^{mn}_{FA,t+1} \cdot GI^{n}_{t+1} \tag{35}$$

$$GCF^{n}_{t+1} = \left[1 - \sum_{m} \left(raf^{mn}_{FA,t+1} \right) \right] \cdot GI^{n}_{t+1} \tag{36}$$

where

$raf^{mn}_{FA,t+1}$: the rate of foreign investment,

R^{n}_{t+1} : the interest rate of country n at period $t+1$,

ε_{t+1} : exchange rate at period $t+1$,

FA^{mn}_{t+t} : net increase of the asset in country m (investment to country m by country n).

3.7 Equilibrium conditions

The intertemporal general equilibrium requires that all markets clear in each country and in each period. Clearing is required on (1) the commodity market, (2) the labor market, and (3) the capital market as expressed by the following equations.

(1) Supply and demand balance of the commodity for each country at period t and $t+1$ is shown bellow.

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$$X_{it}^{m} = \sum_{n}\left[\sum_{j}\left(a_{ijt}^{mn} X_{jt}^{n}\right) + c_{it}^{mn} + g_{it}^{mn} + k_{it}^{mn}\right] + e_{it}^{mROW} \tag{37}$$

$$X_{i,t+1}^{m} = \sum_{n}\left[\sum_{j}\left(a_{ijt}^{mn} X_{j,t+1}^{n}\right) + c_{i,t+1}^{mn} + g_{i,t+1}^{mn} + k_{i,t+1}^{mn}\right] + e_{it}^{mROW} \tag{38}$$

where
$$c_{i,t+1}^{mn} = \frac{\beta_{it}^{mn} C_{t+1}^{n}}{q_{i,t+1}^{mn}}, \quad , \quad g_{i,t+1}^{mn} = \frac{\zeta_{it}^{mn} GC_{t+1}^{n}}{q_{i,t+1}^{mn}}, \quad cp_{i,t+1}^{mn} = \frac{\phi_{it}^{mn} GCF_{t+1}^{n}}{q_{i,t+1}^{mn}}$$

$$C_{h,t+1}^{n} = C_{t+1}^{n} = \sum_{i} C_{i,t+1}^{n} = \left(1 - sav_{h,t+1}^{n}\right) DI_{t+1}^{n},$$

$$DI_{t+1}^{n} = CE_{t+1}^{n} - DT_{h,t+1}^{n} + TRG_{t+1}^{n},$$

$C_{t+1}^{n} =$ total consumption of each country n at period $t+1$,

$DI_{t+1}^{n} =$ total household disposable income of each country n at period $t+1$,

$sav_{h,t+1}^{n} =$ households saving rate at period $t+1$, and

$e_{it}^{mROW} =$ Export to ROW from country m commodity i at period t.

Similarly, the government consumption of each country for future can be calculated.

(2) Supply and demand balance of labor for each country at period t and $t+1$ is shown bellow.

$$L_{t}^{n} = \sum_{j}\left(DL_{jt}^{n} a_{y_j t}^{n} X_{jt}^{n}\right) \tag{39}$$

$$L_{t+1}^{n} = \sum_{j}\left(DL_{j,t+1}^{n} a_{y_j t}^{n} X_{j,t+1}^{n}\right) \tag{40}$$

where
$$L_{t+1}^{n} = \left(1 + \eta\right) L_{t}^{n},$$

$\eta =$ periodic labor growth rate.

(3) Supply and demand balance of capital for each country at period t and $t+1$ is shown bellow.

$$K_{t}^{n} = \sum_{j}\left(DK_{jt}^{n} a_{y_j t}^{n} X_{jt}^{n}\right) \tag{41}$$

$$K_{t+1}^{n} = \sum_{j}\left(DK_{j,t+1}^{n} a_{y_j t}^{n} X_{j,t+1}^{n}\right) \tag{42}$$

where $K_{t+1}^{n} = \left(1 - \delta_{t}^{n}\right) K_{t}^{n} + I_{f,t}^{n}$

4   Conclusion

Labor and capital demand are considered for per worker to formulize the primary income distribution for consumers and firms. Next, the primary income distribution is formulized for each generation at seven periods for considering seven period OLG techniques. Several other papers have considered two period OLG model where only investment policy is analyzed. However, this paper considers seven period OLG model for analyzing the investment as well as reinvestment policy for consumers and firms. In two period model, last period capital accumulation is not considered because first period capital will fully depreciate in the next period. The main difficulty of seven period model is that a lot of capital is accumulated at last period for each generation. In the secondary income distribution, it is assumed that consumers receive disposable income for consumption and savings in seven period budget conditions. Income expenditure and savings of firms are also considered. Furthermore, it is assumed that consumers invest all savings for buying domestic/foreign bonds, and firms invest all savings into the next period capital stock. Households and firms investment policy are distinguished. A firm will invest all savings into the next period capital stock because of the dynamic

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nature and for calculating dynamic wage and capital rental price. In cost minimization problem, it is observed that household wage, capital rental price and production technology have an influence on labor and capital demand.

The OLG model also allows the utility maximization problem subject to a household lifetime budget constraint for obtaining the household consumption. In all previous works, neither the dynamic consumption-goods demand nor the dynamic investment-goods demand are distinguished in the final demand. But in this work, a final demand is classified into consumption and investment goods demand at every period for analyzing dynamic CGE. The main contribution is that the same technique like seven period OLG model is applied for developing a dynamic gross investment model using dynamic CGE concept. In this model, the future gross investment of a country is considered as a function of present gross investment, next period gross domestic product, interest rate of both country and exchange rate based on Keynesian marginal efficiency theory. This problem will be solved by multiple regression model. Not only gross investment but also the domestic and foreign investments are taken into consideration in this investment model. The amount of foreign investment is estimated by determining the rate of the foreign investment to each country into the investment model, and the remainder is considered as the amount of domestic investment. In the real world, foreign investment is distributed for foreign securities, for buying bonds, for corporate stock etc. Type of foreign investment is not distinguished in this model. It is considered as owned assets to abroad from a country. The domestic investment is considered as a real investment of a country that has an influence on investment goods demand.

Currently the work is in data analysis phase where tables seven periods (1965, 1970, 1975, 1980, 1985, 1990 and 1995) international input-output transaction table, national accounts statistics and time series data are being used by the model for analyzing Japan-U.S economy in details. In this paper, seven periods value-added part for different sectors are used for analyzing the model. For deficiency of data, the value-added part for different sectors for four periods (1965,1970,1975 and 1980) are manipulated based on actual total yearly GDP data and three periods (1985, 1990 and 1995) international I-O transaction tables. The remaining three periods (1985, 1990 and 1995) value-added data are used from actual I-O table. The complete international I-O table is used for only year 1995.

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