STABILITY ANALYSIS OF CAR FOLLOWING BEHAVIOR*

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1. Introduction

Its common phenomenon in traffic flow that a smoothly moving traffic breaks down spontaneously becoming unstable after some time and ultimately leads to stop and go traffic situation. These catastrophic events occur as initially small disturbance gets amplified as it passes down a column of vehicles. This paper deals with the factors that influence the stability of traffic flow and the criterion that determines whether such events will occur or not. The literatures reviewed indicate that reaction time and sensitivity factor are the major factors that influence the stability of traffic flow.

Car following theories explain the interaction between individual vehicles driving in a single lane road. It can be explained using a simple stimulus-response equation,

$$\text{Response} = \lambda * \text{Stimulus} \quad (1)$$

where, λ is a proportional factor also termed as sensitivity factor, the response function can be acceleration or deceleration while the stimulus function is composed of many factors: speed, relative speed, headway, accelerations, vehicle performance, driver's threshold etc. All of these factors may not have equal significance. Chandler et al. (1958) were first to introduce stability concept in car following behavior using a stimulus-response based linear car following model,

$$\ddot{x}_i(t+T) = \lambda\ [\dot{x}_i(t) - \dot{x}_{i-1}(t)] \quad (2)$$

where, $\dot{x}_i(t)$ and $\dot{x}_{i-1}(t)$ represents speed of i^{th} and $i\text{-}1^{th}$ vehicles at time t while λ and T are termed as sensitivity factor and reaction time respectively. The stability concept was explained from two different points of views: *Local stability* is concerned with the response of a following vehicle to a fluctuation in the motion of the vehicle directly in front of it, i.e. it is concerned with the localized behavior between pair of vehicles. *Asymptotic stability* is concerned with the manner in which a fluctuation in the motion of any vehicle, say the lead vehicle of a platoon, propagates through a column of vehicles. The non-oscillating and damped responses represent stable traffic condition while oscillating response, either with or without damping, represents unstable traffic condition.

Most of literatures on stability analysis are either based on theoretical or numerical approach. It is important to check the validity of these concepts using experimental data. Real Time Kinematic (RTK) GPS is featured by its outstanding accuracy in measuring position and speed dynamically and being used for data acquisition in various fields. The use of RTK GPS technology for car following experiments is ideal in a sense that the position and speed of vehicles at any time can be determined dynamically using receivers mounted on them. A car-following experiment was conducted using ten passenger cars each equipped with RTK GPS receivers in a test track of Hokkaido Developing Bureau, Japan. This experiment has tested various speed patterns for lead vehicle including four sinusoidal curve patterns, four constant speed patterns and a random speed pattern. The effect of change in lead vehicles' speed on the driving characteristics following drivers were measured in terms of position and speed data taken by RTK GPS receiver equipped in each vehicles. This paper attempts to analysis the effect of change in lead vehicle's speed on following vehicles in terms of variation in reaction time, sensitivity factor and stability factor along the platoon for different speed patterns. Although for some of the driving patterns, the variation of these characteristics of drivers seems to have some trends but it cannot be verified statistically as in most of cases no such trends are observed.

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The different concepts of car following models developed so far are summarized in next section 2. While in section 3, the fundamental driving characteristics, which are important factors that influence the stability of traffic flow, are discussed. The stability criterion developed by researchers in past are discussed in section 4. The results of preliminary analysis on driving characteristics are presented in section 5.

2. Car Following Models

This stimulus-based car following concept was further developed by Gazis, Herman, and Rothery (1961) as a generalized car following model,

$$\ddot{x}_i(t+T) = \frac{\lambda\, \dot{x}_{i-1}^{m}(t)}{(x_i(t)-x_{i-1}(t))^{l}}\,[\dot{x}_i(t)-\dot{x}_{i-1}(t)] \qquad (3)$$

where, l and m are constants, $x_i(t)$ and $x_{i-1}(t)$ represents position of i[th] and i-1[th] vehicles at time t respectively. Many researchers conducted best fit analysis to assign appropriate values for these constants, are summarized in table 1.

Table 1: The suggested values for m and l of GHR model

Source	m	l
Chandler et al. (1958)	0	0
Herman et al. (1959)	0	1
Hoefs (1972) (dcn./ dcnbr./ acn.)	1.5/0.2/0.6	0.9/0.9/3.2
Treiterer and Myers (1974) (dcn./ acn.)	0.7/0.2	2.5/1.6
Ozaki (1993) (dcn./ acn.)	0.9/-0.2	1/0.2

Besides stimulus-based concept there are collision avoidance concept developed by Gipps (1981) and psychological concept by Michaels (1963). The CA concept is that the driver of following vehicle selects his speed to ensure that he can bring his vehicle to safe stop if the vehicle ahead comes to a sudden stop. While the psychological concept is that the driver of following vehicle perceives the change in relative velocity from the change in visual angle subtended by the vehicle ahead so the response of driver is based on the change in visual angle subtended and the threshold of driver to perceive the change.

3. Driving Characteristics

(1) Reaction Time

The delay time for human reaction is common in all of car following models. Most of investigations have derived this time conducting a best-fit analysis proposed by Gazis et al. (1961) using relative speed and acceleration data, and selecting the delay value that produces highest correlation coefficient (r^2 value),

$$Rt_i = \underset{\tau}{\mathrm{Max}}\ \ \rho_i(\tau) = \underset{\tau}{\mathrm{Max}}\ \ E\left[\frac{\Delta v_i(t)\, a_i(t+T)}{S_{\Delta v_i} S_{a_i}}\right] \qquad (4)$$

where, $\Delta v_i(t) = \dot{x}_i(t) - \dot{x}_{i-1}(t)$ is relative speed, $a_i(t) = \ddot{x}_i(t)$ is acceleration, $S_{\Delta v_i}$ and S_{a_i} are standard deviations for relative speed and acceleration respectively. The reaction time proposed from such investigation in past are summarized in Table 2.

Ozaki (1993) related change in reaction time with the importance of the task required (acceleration vs. deceleration) and find that there may be a slight correlation between reaction time T, and Δx in acceleration and deceleration conditions and proposed reaction time varying between 1.7-1.9 seconds in acceleration while 1.7-2.1 seconds in deceleration.

Table 2: The values of reaction time recommended in past

Source	Reaction Time
Chandler et al. (1958)	1.6
Herman et al. (1959)	1.2
Kometani et al. (1959)	0.5
Helly (1959)	1.4
Michaels (1963)	1.4
Lee et al. (1967) (acn./dcn.)	0.4/0.6
Aron (1988) (acn./ss./dcn.)	1.8/0.5/3.9
Ozaki (1993) (acn./dcn.)	1.9/1.9
Xing (1995) (Δv/Δx)	0.8/3.4

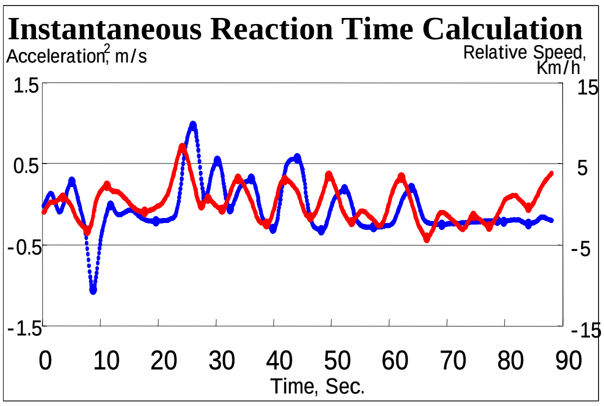

Figure 1: Instantaneous reaction time calculation from acceleration and relative speed time series

Figure 1 presents a technique suggested by Suzuki for calculation of instantaneous reaction time from acceleration and relative speed data considering only the points where drivers changes their acceleration to deceleration and vise versa. He found different reaction time for the same driver for different driving conditions.

(2) Sensitivity Factor

The sensitivity factor λ is the regression constant obtained in the regression of acceleration at time (t+ T) against the relative speed at time (t). The work of Leutzbach shows that the stability of traffic flow is controlled by the stability factor C = λT.

4. Stability Criterion

(1) Local Stability

The conditions for local stability were established numerically using Laplace transforms by Herman et al. (1959) as follows;

- if $0 \le (C = \lambda T) < 1/e = 0.368$, response is non-oscillatory;
- if $1/e \le (C = \lambda T) < \pi/2$, response is oscillatory with damped amplitude;
- if $(C = \lambda T) = \pi/2$, response is oscillatory with constant amplitude;
- if $(C = \lambda T) > \pi/2$, response oscillatory with increasing amplitude.

(2) Asymptotic Stability

Rothery (1964) set $C = \lambda T = 0.5$ as a boundary line to divide the regions of asymptotic stability as shown in figure 2. It was concluded that for asymptotic stability to be stable C must be less than 0.5. The work of Leutzbach (1972) also shows that the stability of traffic flow is controlled by the stability factor C.

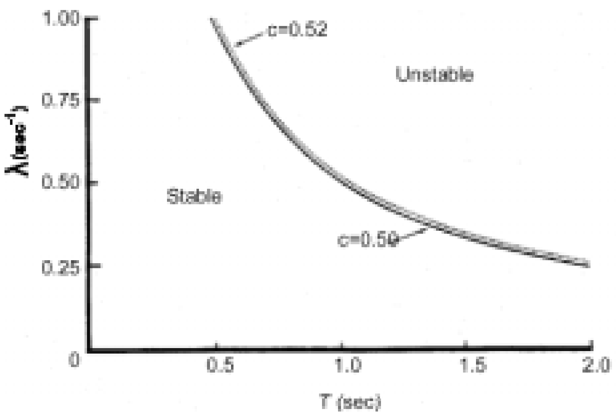

Figure 2: Regions of Asymptotic Stability (Rothery 1964)

More recently, Ferrari (1994) has analyzed the stability of a linear two-car model while Zhang et al. (1997) have analyzed the local and asymptotic stability criterion for classical car-following model both theoretically and numerically considering different reaction time, T and sensitivity factor, λ for individual drivers. E.N. Holland (1997) has summarized stability criterion for several car following models and discussed similarity in stability criterion for these models. He introduced a new concept of anticipation point from which drivers base their decisions to change speed. The anticipation time is the time taken for a wave to travel from the point it is observed to the driver. He concluded that if a driver's reaction time is greater than this anticipation time then perturbation waves can not be supported, leading to instability.

$$\text{Anticipation time} > \text{Reaction time} \Rightarrow \text{Stability} \tag{5}$$

5. Results and Analysis

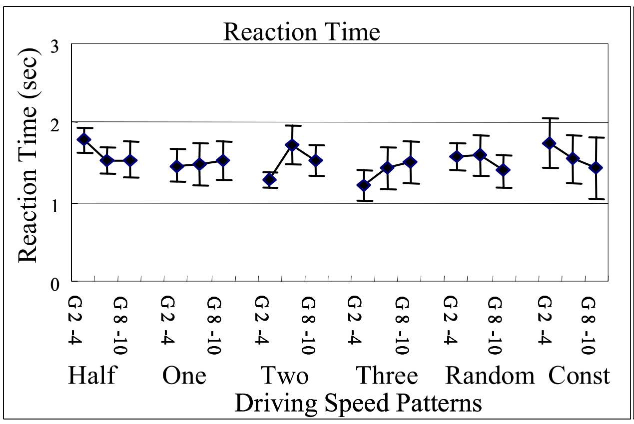

Figure 3: Reaction time variation along the platoon

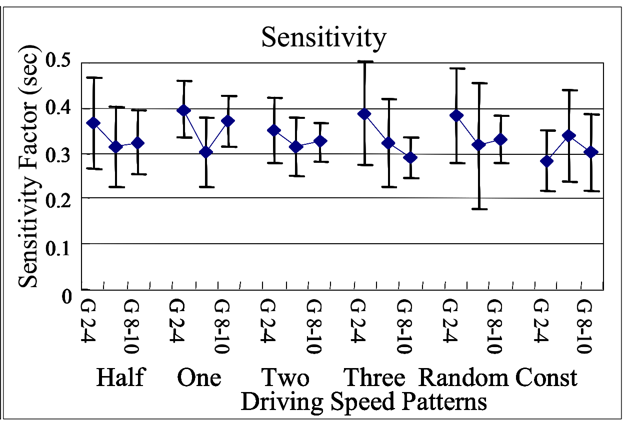

Figure 4: Sensitivity factor variation along the platoon

In order to grasp the averaged drivers' characteristics, the following vehicles are assembled into three groups of every three successive vehicles. The variation of driving characteristics of following vehicles in the platoon are presented for different speed patterns of lead vehicle, where rectangular points represent the mean value while lines above and under indicate biasness from the mean values.

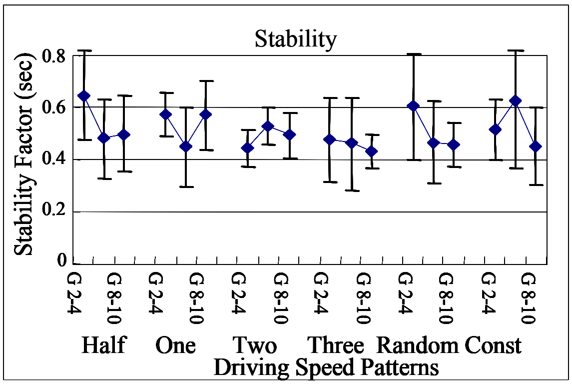

Figure 5: Stability factor variation along the platoon

Figure 3 shows that the mean value for reaction time varies with in the ranges of 1.0 to 2.0 seconds with relatively higher standard deviation values. It can be noticed that half wave and constant speed patterns have decreasing trend along the platoon but no such variation trend can be observed for all other speed patterns. The decreasing reaction time along the platoon can be justified as the drivers behind is generally in better position to perceive the information on driving conditions ahead compared to those in front position as they can see movement of several vehicles down stream to predict the driving conditions ahead. The driving conditions for one-wave, two-wave, three-wave and random speed patterns are comparatively unstable than half-wave and constant speed patterns. In such case the information about the driving conditions in long queue ahead is not much relevant as the drivers need to concentrate on the movement of the vehicle immediately ahead. It means the driving information is not influenced by the position of driver in the platoon in unstable driving condition so the responses are mainly governed by the individual performance of each driver and irrespective of their position in the platoon.

It can be observed from Figure 4 that the mean values for sensitivity factor varies with in the range of 0.3 to 0.4 while the mean value for stability factor varies with in the range of 0.4 to 0.6 as shown in figure 5. Besides this it can be noticed that there is no indication of any particular variation trend from these figures. Based on these results it can be concluded that the driving characteristics are mainly governed by the individual performance of the drivers and is irrespective of their position in the platoon.

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