TO WAIT OR NOT TO WAIT: THE IMPACT OF PROJECT SYNERGY ON OPTIMAL TIMING OF INVESTMENT*

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1. INTRODUCTION:

Coordination is defined as the 'harmonious combination or interaction'[1] of events or situations. When undertaken for projects, it is supposed to increase the efficiency and benefit of the undertaking. Yet despite the known complementary effect of urban development and infrastructure development projects, instances of non-coordination of such projects abound due to a variety of causes. One source is the independent planning of various government agencies. This usually occurs when the responsibility of implementing various kinds of projects (i.e., transport, housing, urban development) is given to different government divisions or sections. Another source is the lack of effective policy instrument to synchronize such projects, especially when implemented by the private sectors. A case in point is the development of urban sub-centers along the coastal area of Metro Manila, the capital of the Philippines. Since land development is perceived to be more profitable, a number of these sub-centers are being developed by private entities with very little government control. Compounding the rise of such sub-centers is the lack of parallel transportation infrastructure development.

This paper seeks to investigate the impact on optimal timing of complementary projects using concept of non-cooperative games. Due to the interactive nature of complementary projects, the choice of optimal timing for each project is affected by the optimal timing of the other project. Possible equilibrium scenarios are established using the reaction function as expressed in the choice of optimal timing of each project.

2. SOCIAL OPTIMAL TIMING: Pure Timing Problem

In this section, the Social Net Present Value of an independent project as well as its optimal timing will be defined. This is known as a pure timing problem[2].

The Social Net Present Value of a project *V(T)* may be expressed as[3]:

$$V(T) = -I\exp(-\rho T) + \int_T^\infty \{b(t) - c(t)\}\exp(-\rho t)dt \tag{1}$$

where *V(T)* – net present value of the project, *I*-Investment cost, *b(t)* – annual benefit, *c(t)* – annual running costs, ρ-social discount rate, *T*-Timing of opening of service.

However, the annual growth of net benefit, incurred only after the project has been implemented, may be expressed as:

$$b(t) - c(t) = (\bar{b} - \bar{c})\exp(\omega t) \tag{2}$$

where $(\bar{b} - \bar{c})$ is the initial value of the annual net benefit at t=0. Thus, equation (1) may be re-written as:

$$V(T) = -I\exp(-\rho T) + (\bar{b} - \bar{c})\int_T^\infty \exp\{(\omega - \rho)t\}dt \tag{3}$$

The Social Net Present Value is optimized at Optimal Timing T^*. In symbol, we have:

$$T^* = \arg\max_T V(T) \tag{4}$$

Keywords: Prior implementation, subsequent implementation, synchronized implementation, optimal timing reaction, coordination strategies

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3. CONDITIONAL SOCIAL OPTIMAL TIMING: Two-Complementary Project Scenario

In this section the definition of Social Net Present Value in the previous section will be extended to a two-complementary project scenario. When a complementary project is implemented (represented as Project B), the Social Net Present Value of Project A is a function of optimal timing of Projects A and B and may be written as:

$$V_A(T_A,T_B) = -I_A \exp(-\rho T_A) + \int_{T_A}^{\infty} \Phi_A(t,T_B)\exp(-\rho t)dt \tag{5}$$

In equation *(5)*, $\Phi_A(t,T_B) = b(t,T_B) - c(t,T_B)$ and represents the net benefit term of Project A. It must be noted that in the formulation for the interactive condition, the net benefit term is a function of its own optimal timing as well as the timing of implementation of Project B.

Taking the partial derivative of $V_A(T_A,T_B)$ with respect to T_A yields the marginal cost for delaying the opening of service, T_A:

$$\frac{\partial V_A(T_A,T_B)}{\partial T_A} = \rho I_A \exp(-\rho T_A) - \Phi_A(T_A,T_A)\exp(-\rho T_A) \tag{6a}$$

$$= \left[\rho I_A - \Phi_A(T_A,T_B)\right]\exp((-\rho T_A) \tag{6b}$$

Thus, the objective would be to determine the optimal timing T_A^* such that the condition for Global Maximum, including non-differentiable case, would be:

$$V_A(T_A^* - h,T_B) \le V_A(T_A^*,T_B) \text{ and } V_A(T_A^* + h,T_B) \le V_A(T_A^*,T_B) \tag{7a}$$

Likewise, the conditions for Local Maximum, if differentiable, will be:

$$\frac{\partial V_A(T_A^* - h,T_B)}{\partial T_A} > 0 \text{ and } \frac{\partial V_A(T_A^* + h,T_B)}{\partial T_A} < 0 \tag{7b}$$

or such that

$$\rho I_A - \Phi_A(T_A^* - h,T_B) > 0 \tag{7c}$$

and

$$\rho I_A - \Phi_A(T_A^* + h,T_B) < 0 \tag{7d}$$

where h is the displacement in time from the optimal timing T_A^*.

Equations *(7b)-(7d)* indicate that at any time h before T_A^*, the marginal change in opportunity cost is greater than the marginal net benefit of implementing the project, thus there is merit to delaying the opening of service. On the other hand, at any time h after T_A^*, the marginal net benefit of implementing the project is greater than the marginal increase in opportunity cost of capital investment, implying that it is no longer beneficial to delay the opening of service.

The expression for net benefit of Project A, $\Phi_A(t,T_B)$, conditional upon T_B, may be decomposed into:

$$\Phi_A(t,T_B) = \begin{bmatrix} \overline{\Phi}_A \exp(\omega_A t) \text{ for } t < T_B \\ \overline{\Phi}_A \exp(\omega_A^\Delta t) \text{ for } t \ge T_B \end{bmatrix} \tag{8}$$

where the complementary effect of the projects is expressed in an enhanced growth rate of net benefit for Project A, ω_A^Δ. Likewise, using the appropriate expression from Equation *(8)*, equation *(6b)* may be transformed to:

$$\rho I_A - \Phi_A(T_A,T_B) = \begin{bmatrix} \rho I_A - \overline{\Phi}_A \exp(\omega_A T_A) \text{ for } T_A < T_B \\ \rho I_A - \overline{\Phi}_A \exp(\omega_A^\Delta T_A) \text{ for } T_A > T_B \end{bmatrix} \tag{9}$$

When $T_A<T_B$, Project A is called the *Prior Implementer* and when $T_A>T_B$, Project A is called the *Subsequent Implementer*.

There are four potential local optimal timing, T_A^*, as shown in figures 1-4.

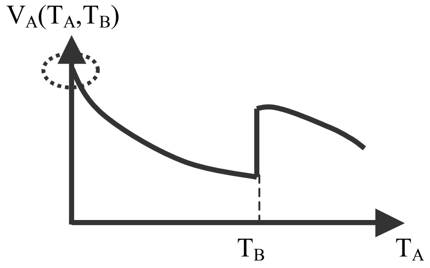

Fig. 1. Implement Now

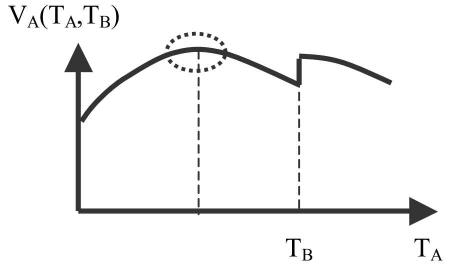

Fig. 2. Prior Implementation

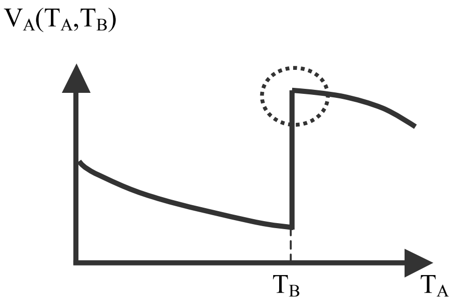

Fig. 3. Synchronized Implementation

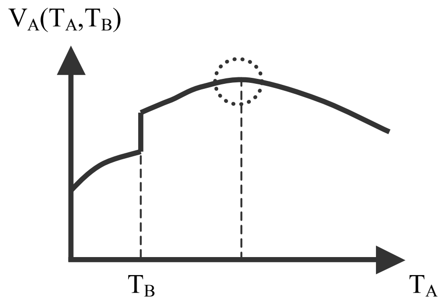

Fig. 4. Subsequent Implementation

With $h>0$, the local maximum points shown may be described as follows, subject to the conditions stated in equations *(7c)* and *(7d)*:

For *implement now*:

$$\text{If } \rho I_A - \overline{\Phi}_A \exp\{\omega_A(0+h)\} < 0, \text{ then } T_A^* = 0 \tag{10}$$

For *prior implementation* of Project A to Project B:

$$\text{If } \rho I_A - \overline{\Phi}_A \exp\left[\omega_A\left\{\left(\frac{1}{\omega_A}\right)\ln\left(\frac{\rho I_A}{\overline{\Phi}_A}\right)\right\} - h\right] > 0 \text{ and } \rho I_A - \overline{\Phi}_A \exp\left[\omega_A\left\{\left(\frac{1}{\omega_A}\right)\ln\left(\frac{\rho I_A}{\overline{\Phi}_A}\right)\right\} + h\right] < 0,$$
$$\text{then } T_A^* = \left(\frac{1}{\omega_A}\right)\ln\left(\frac{\rho I_A}{\overline{\Phi}_A}\right) \tag{11}$$

In this case, its annual growth rate of net benefit remains at ω_A.

For *synchronized* opening of both projects:

$$\text{If } V_A(T_B - h, T_B) \le V_A(T_B, T_B) \text{ and } V_A(T_B + h, T_B) \le V_A(T_B, T_B),$$
$$\text{then } T_A^* = T_B \tag{12}$$

When Project A is opened after project B (*subsequent implementation*):

$$\text{If } \rho I_A - \overline{\Phi}_A \exp\left[\omega_A^\Delta\left\{\left(\frac{1}{\omega_A}\right)\ln\left(\frac{\rho I_A}{\overline{\Phi}_A}\right)\right\} - h\right] > 0 \text{ and } \rho I_A - \overline{\Phi}_A \exp\left[\omega_A^\Delta\left\{\left(\frac{1}{\omega_A}\right)\ln\left(\frac{\rho I_A}{\overline{\Phi}_A}\right)\right\} + h\right] < 0,$$
$$\text{then } T_A^* = \left(\frac{1}{\omega_A^\Delta}\right)\ln\left(\frac{\rho I_A}{\overline{\Phi}_A}\right) \tag{13}$$

All the equations derived are also applicable to Project B, with the corresponding change in project characteristics as denoted by the subscript.

4. REACTION FUNCTIONS:

The objective of the proponents of Project A is to choose the optimal timing T_A^* that will maximize its Net Present Value, in reaction to the opening of service of Project B. In symbol this reaction function may be defined as:

$$T_A^*(T_B) = \arg\max_{T_A} V_A(T_A, T_B) \tag{14a}$$

Likewise, the reaction function of Project B may be described as:

$$T_B^*(T_A) = \arg\max_{T_B} V_B(T_A, T_B) \tag{14b}$$

Corresponding to the four potential local maximum points, using equation *(5)*, Net Present Value may be defined as follows:

For Implement Now Scenario when $T_A^* = 0$,

$$V_A(0,T_B) = -I_A + \frac{\overline{\Phi}_A}{(\rho-\omega_A)} + \overline{\Phi}_A \left\{ \frac{\exp(\omega_A^\Delta - \rho)T_B}{(\rho-\omega_A^\Delta)} - \frac{\exp(\omega_A - \rho)T_B}{(\rho-\omega_A)} \right\} \tag{15}$$

For Synchronized Implementation ($T_A^* = T_B$),

$$V_A(T_B,T_B) = -I_A \exp(-\rho T_B) + \frac{\overline{\Phi}_A}{(\rho-\omega_A^\Delta)} \exp\left(\omega_A^\Delta - \rho\right) T_B \tag{16}$$

For *Prior Implementation* ($T_A^* = \frac{1}{\omega_A} \ln \frac{\rho I_A}{\overline{\Phi}_A}$),

$$V_A\left(\tfrac{1}{\omega_A}\ln\tfrac{\rho I_A}{\overline{\Phi}_A}, T_B\right) = I_A \left\{ \frac{\overline{\Phi}_A}{\rho I_A} \right\}^{\frac{\rho}{\omega_A}} \left(\frac{\omega_A}{\rho-\omega_A} \right) + \overline{\Phi}_A \left[\frac{\exp\left\{\left(\omega_A^\Delta - \rho\right)T_B\right\}}{\left(\rho-\omega_A^\Delta\right)} - \frac{\exp\left\{\left(\omega_A - \rho\right)T_B\right\}}{\left(\rho-\omega_A\right)} \right] \tag{17}$$

(Please see Appendix A for the detailed derivation of Equation 17).

For *subsequent Implementation* ($T_A^* = \frac{1}{\omega_A^\Delta} \ln \frac{\rho I_A}{\overline{\Phi}_A}$),

$$V_A\left(\tfrac{1}{\omega_A^\Delta}\ln\tfrac{\rho I_A}{\overline{\Phi}_A}, T_B\right) = I_A \left(\frac{\overline{\Phi}_A}{\rho I_A} \right)^{\frac{\rho}{\omega_A^\Delta}} \left(\frac{\omega_A^\Delta}{\rho-\omega_A^\Delta} \right) \tag{18}$$

Therefore, at any given T_B, Project A will choose its optimal timing T_A^* that will yield the maximum Social Net Present Value:

$$T_A^* = \underset{T_A}{\arg\max} \left\{ V_A(0,T_B), V_A(T_B,T_B), V_A\left(\frac{1}{\omega_A}\ln\frac{\rho I_A}{\overline{\Phi}_A}, T_B\right), V_A\left(\frac{1}{\omega_A^\Delta}\ln\frac{\rho I_A}{\overline{\Phi}_A}, T_B\right) \right\} \tag{19}$$

The equilibrium solution for this interactive optimal timing choice will be:

$$T_A^* = T_A^*(T_B^*) \text{ and } T_B^* = T_B^*(T_A^*) \tag{20}$$

where the solution is located on the reaction curves of both projects. However, for any game, the determination of the equilibrium consists in first identifying the reaction curve of each player. In this paper, when determining the reaction curves for the complementary projects, a simple fact was observed: if a 45-deg. line is drawn through the origin of the graph of the reaction curves, the only valid reactions above this line would be for Project A to *implement now*, *implement prior to*, or *synchronize*, with Project B. In the same space, the valid reactions for Project B, expressed in its choice of optimal timing, would be to *implement now*, *implement subsequently*, or to *synchronize* with Project A. For the space below the 45 deg. line, Project A may choose to *implement now*, *implement subsequent to*, or *synchronize* with Project B. Choice set of Project B includes *implement now*, *implement prior to* Project A, or *synchronize* its opening of service with Project A. There are at least four equilibrium scenarios that may be derived for the optimal timing of complementary projects, as illustrated in Figures 5-8. Figure 5 shows an equilibrium point where Project A is implemented subsequent to Project B. Thus, the equilibrium point is located below the 45-deg. line. Figure 6 illustrates the opposite scenario, where Project A is implemented prior to Project B. In figure 7, the only choice of optimal timing for both projects is to implement immediately. Figure 8 shows the case when there are multiple equilibrium points located on the 45-deg. line.

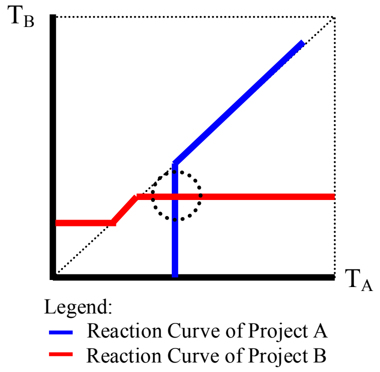

Fig. 5. Equilibrium when Project A is Subsequent Implementer

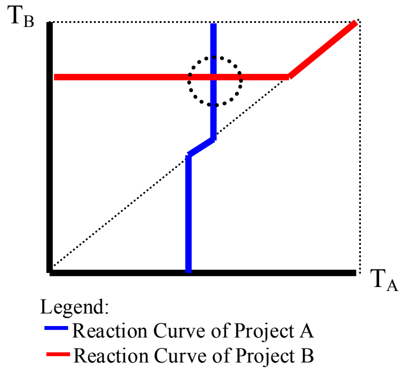

Fig. 6. Equilibrium when Project A is Prior Implementer

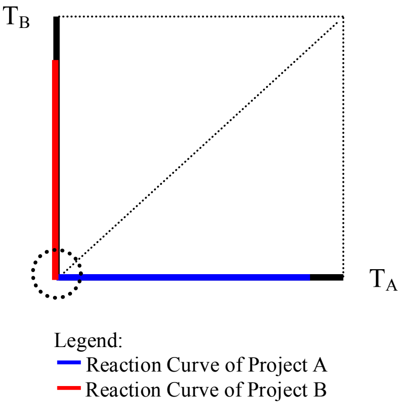

Fig. 7. Equilibrium Point is at T=0

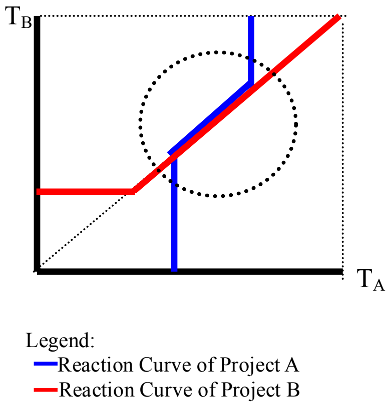

Fig. 8. Multiple Synchronized Equilibrium Points

5. IMPLICATIONS TO SOCIAL COORDINATORS

In the previous section, several scenarios of equilibrium for non-coordinated conditions were shown. However, as has been mentioned at the beginning of this paper, coordination of the optimal timing of the opening of service of complementary projects may increase social surplus. Thus, using the behavior under non-coordinated case, various strategies may be adopted by the entity responsible for overseeing the coordinator, herein referred to as the *social coordinator*. In the following discussion, Project A will be used as reference in the labeling of the equilibrium point.

Figures 5 and 6 show equilibrium points for *Subsequent Implementer-Prior Implementer* and *Prior Implementer-Subsequent Implementer*, respectively. If upon the determination of the increase in social surplus, it is more beneficial to implement the projects earlier than the equilibrium timing, the task of the social coordinator is to offer incentives to the more crucial project. If we consider Project A to be the transportation infrastructure project, in the case where it naturally assumes the *Subsequent Implementer* role (i.e., it will be implemented subsequent to Project B, the Urban Development Project), the options for the social coordinator would be to either subsidize a portion of its Investment cost ***I*** or subsidize part of the annual running cost ***c***. Both strategies will, in effect, hasten the optimal timing of opening of service of Project A[3]. On the other hand, if a later opening of service is more prudent as far as increase of social surplus is considered, the social coordinator may levy taxes on the *Prior Implementer* to delay its optimal timing for opening of service.

For the case where there exist multiple synchronized equilibrium points, if deemed beneficial to encourage both projects to open at the earliest possible synchronized time, the social coordinator may offer a provisional incentive, such as subsidy for investment cost to the more crucial project or to both projects, as the case may be. It is provisional in the sense that it will only be granted if the projects agree to open at the optimal timing from the social net benefit perspective. To delay opening of service, investment cost *I* may be increased through policy instruments such as taxes. Table 1 shows the list of some strategies that are available to social coordinator to affect optimal timing of projects.

Table 1. Some Strategies for Timing Coordination

Objective of Social Coordinator	*Parameters Change*	*Typical Forms of Policy Instruments*
Encourage earlier opening of service	Decrease *I*	Land acquisition incentive;
		Right of way acquisition incentive
	Decrease annual running cost *c*	Subsidize maintenance cost for infrastructure
		Tax Holiday
	Increase annual benefit *b*	Minimum ridership guarantee
		Right-to-develop depot into commercial establishments
Encourage later opening of service	Increase *I*	Asset taxes
	Increase annual running cost *c*	Income tax/sales tax
	Decrease annual benefit *b*	Regulated fare/selling price

Although this is not an exhaustive list, it can give an indication of the capability of the model presented in this paper to assess government policies for project evaluation and coordination. On a final note, however, it must be stressed that in the granting of incentives, the resulting social benefit should be much greater than the incentives offered.

6. FUTURE DIRECTION OF THE RESEARCH:

This paper presented preliminary results of the investigation of the impact of synergy on the behavior of complementary projects as expressed in the optimal timing choice. A more thorough evaluation of the implications as well as formulation of the change in social surplus for various scenarios will be the next step for this research. Finally, to validate the model, it will be applied to a real-world situation in a developing economy.

References

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<u>Appendix A: Derivation of Social Net Present Value for Prior Implementation:</u>

Before substituting optimal timing T_A^* into Equation *(5)*, it must be transformed as follows:

$$T_A^* = \frac{1}{\omega}\ln\left[\frac{\rho I_A}{\overline{\Phi}_A}\right] \tag{a.1}$$

Multiplying both sides by -ρand taking the exponent of both sides yields:

$$\exp(-\rho T_A^*) = \exp\left(-\frac{\rho}{\omega}\ln\left[\frac{\rho I_A}{\overline{\Phi}_A}\right]\right) = \left[\frac{\rho I_A}{\overline{\Phi}_A}\right]^{-\frac{\rho}{\omega}} = \left[\frac{\overline{\Phi}_A}{\rho I_A}\right]^{\frac{\rho}{\omega}} \tag{a.2}$$

By the same method of transformation,

$$\exp\left([\omega_A - \rho]T_A^*\right) = \left[\frac{(\bar{b}-\bar{c})_A}{\rho I_A}\right]^{-\frac{\rho}{\omega}} = \left[\frac{\rho I_A}{\overline{\Phi}_A}\right]^{\frac{\rho}{\omega}} \tag{a.3}$$

Substituting equations *(a.2)* and *(a.3)* into equation *(5)* and simplifying it will result to the Social Net Present Value for *Prior Implementation* as expressed in Equation *(17)*.