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CHAOS IN TRAFFIC FLOW DYNAMICS OF A CAR FOLLOWING MODEL

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1. INTRODUCTION

Traffic flow is a complex dynamical system because of its nature to under go sudden changes especially in dense traffic conditions due to irregular stop-start situation and many other contributing factors such as drivers' behavior, vehicles' performance, traffic flow conditions, driving environment etc. Chaos theory has been proved very useful in exploring many complex dynamical systems e.g. weather forecasting, fluid dynamics etc. For a system in chaotic state, long-term predictions is not possible due to omnipresent uncertainty in determining initial state that grows exponentially fast in time. The system is still deterministic in a sense that if the initial conditions can be determined exactly the future behaviour can be predicted by integrating the time evolution equations of the system. As there is always some imprecision in specifying initial conditions of chaotic systems, the long-term behavior becomes unpredictable while short-term predictions can be made more reliable under certain circumstances. This theory may also contribute to provide insight into the complexity in the system.

Car following models attempt to mimic the microscopic behavior of individual vehicles in a platoon. In this paper, Collision Avoidance car-following model is used to simulate motion of vehicles. It was observed that a periodic perturbation to the Equilibrium State of car following model produces chaotic motion in some of the following vehicles for some particular initial conditions. The predictability of motion of vehicles is also measured based on the calculated value for KS entropy.

2. CAR FOLLOWING MODEL

Collision Avoidance car following model was developed by Gipps[1] in 1981. The main attractiveness of this model is that it may be calibrated using common sense assumptions about the behavior of driver, needs only the maximal braking rates that a driver will wish to use, and predicts other drivers will use, to allow it to fully function. Besides this, the parameters used in this model correspond to the obvious characteristics of driver and vehicle and already validated with real traffic data and the results produced are acceptable.

This model was derived to calculate a safe speed with respect to the preceding vehicle by setting limits on the performance of driver and vehicle. This model assumes that the driver of following vehicle selects his speed to ensure that he can bring his vehicle to safe stop if the vehicle ahead come to a sudden stop.

$$v_n(t+\tau)=\min\left\{\begin{array}{l} v_n(t)+2.5a_n\tau(1-v_n(t)/V_n)(0.025+v_n(t)/V_n)1/2, \\ b_n\tau+\sqrt{b_n{}^2\tau^2-b_n\left[2[x_{n-1}(t)-s_{n-1}-x_n(t)]-v_n(t)\tau-v_{n-1}(t)^2/\hat{b}\right]} \end{array}\right\} \quad (1)$$

where,

- a_n is the maximum acceleration which the driver of vehicle *n* wishes to undertake.
- b_n is the most severe braking that the driver of vehicle n wishes to undertake ($b_n<0$).
- s_n is the effective size of vehicle n, that is, the physical length plus a margin into which the following vehicle is not willing to intrude even when at rest.
- V_n is the speed at which the driver of vehicle n wishes to travel.
- $x_n(t)$ is the location of the front of vehicle n at time t.
- $v_n(t)$ is the speed of vehicle n at time t, and
- τ is the apparent reaction time, a constant for all vehicles.

This model have two terms as shown above, the first one limits a substantial proportion of the vehicles in free traffic flow conditions while the second one is the limiting condition for almost all vehicles. We assume that transition between these two terms occurs smoothly.

Gipps had verified his model parameters and suggested that the following values can be assigned to simulate vehicles behavior to represent real world traffic cases.

- a_n normal population, $N(1.7,0.3^2)$ m/sec^2,
- b_n equated to $-2a_n$,
- s_n normal population, $N(6.5,0.3^2)$ m,
- V_n normal population, $N(20,3.2^2)$ m/sec,
- τ 2/3 second, and
- $\hat{b}$ minimum of -3.0 and $(b_n-3.0)/2$ m/sec^2

3. VEHICULAR MOTION IN A PLATOON

A platoon consists of a lead vehicle and N following vehicles was simulated using linearized equation of motion. It was assumed that prior to time t=0, each vehicle was traveling at a constant speed v and distance headway d while at t=0, this equilibrium state was perturbed, introducing a small sinusoidal variation to the lead vehicle's speed.

$$v_0(t)=v+A\sin\Omega t \quad (2)$$

where A, Ω, v>0. The effects of this disturbance in lead vehicle's speed on other following vehicles were

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analyzed systematically using simulation program. The above equations of motion were solved numerically to calculate position of each vehicle in platoon using Runga-Kutta Gill algorithm.

4. RESULTS AND ANALYSIS

The computer simulation program was extended to produce time series diagrams, post-transient phase diagrams, Poincare sections, power spectrum, bifurcation diagram and spectrum of Lyapunov exponent from simulated data of motion of vehic les.

These diagrams are popular tools to analyze presence of chaos in a system. The value of Lyapunov exponent and KS entropy were calculated to characterize vehicular motion and to measure the predictability of the system respectively. Figure 1 and 2 show examples of time series and phase diagrams/Poincare Sections, respectively.

4.1 Sensitivity Analysis

Sensitive dependence on initial conditions is an important characteristic of a chaotic system. The sensitivity analysis of model parameters were conducted systematically changing values of each of model parameters and observing their effect on output results of simulation program. It is established that reaction time τ, braking rate b_n and speed V_n are sensitive parameters of this model as significant variations in results were observed with small variation in the values of these parameters. While for other parameters no such variations were noted so all other parameters were treated as insensitive parameters and those parameters were assigned with some reliable values as verified by Gipps.

The effect of V_n is dominant only when the vehicles are accelerating, as higher is the value of V_n the acceleration rate would be higher. The reaction time (τ) was sensitive in a sense that the response of vehicle would be quicker if its reaction time is smaller. The braking rate b influences the amlitude of disturbance that is if b is less than b_{n-1} the disturbance will damp, while if b is greater than b_{n-1}, disturbance will amplify.

Observing results shown in Figure 2, we can say these results do not match exactly with aperiodic motion while it also not looks like chaotic so it may need further investigation calculating the value of Lyapunov exponent and plotting power spectrum using these initial conditions.

4.2 Power Spectrum and Lyapunov Exponent

Although a broad-banded power spectrum does not guarantee the occurrence of chaotic motion but it definitely is a reliable indicator of chaos. Power spectrum for each vehicles in the platoon is shown in Figure 3 for τ = 0.4 sec, b = -2.5 m/sec^2 which shows all vehicles except leading vehicle have broad-banded power spectrum so those vehicles may have chaotic motion.

Lyapunov exponent is one of the most effective and popular tools to characterize a chaotic system. The equation of motion was linearized using Wolf et al.[2] method to calculate the value of Lyapunov exponent.

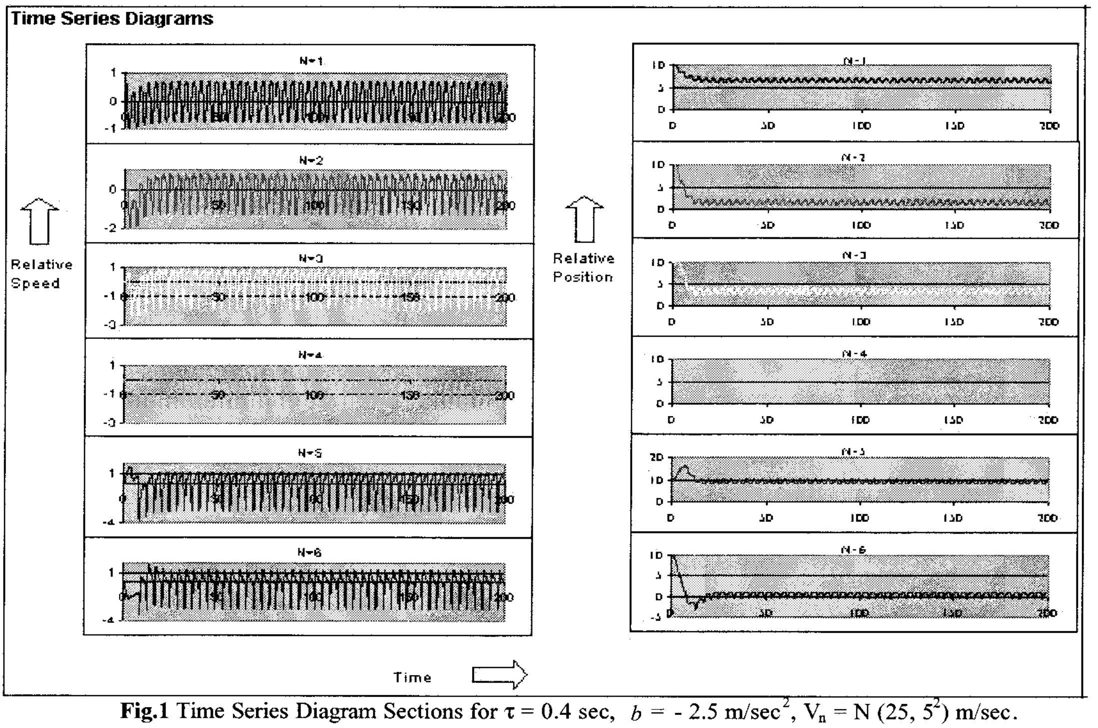
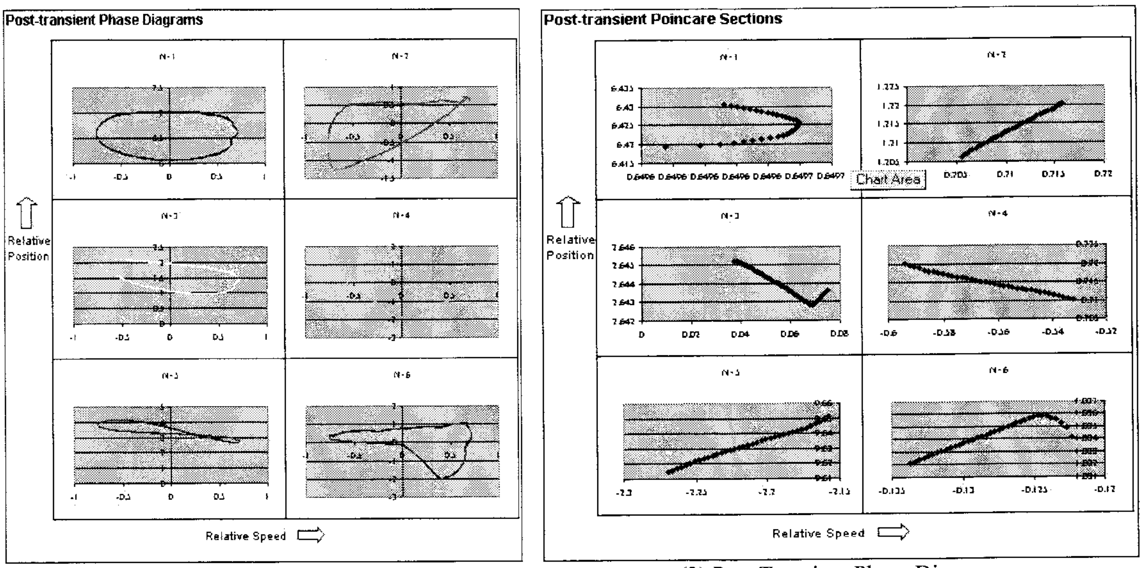


Fig.1 Time Series Diagram Sections for τ = 0.4 sec, b = - 2.5 m/sec^2, V_n = N (25, 5^2) m/sec.



(1) Post Transient Phase Diagrams (2) Post Transient Phase Diagram

Fig.2 Post-transient Phase Diagram and Poincare Sections for $\tau = 0.4$ sec, $b = -2.5$ m/sec^2, $V_n = N(25, 5^2)$ m/sec.

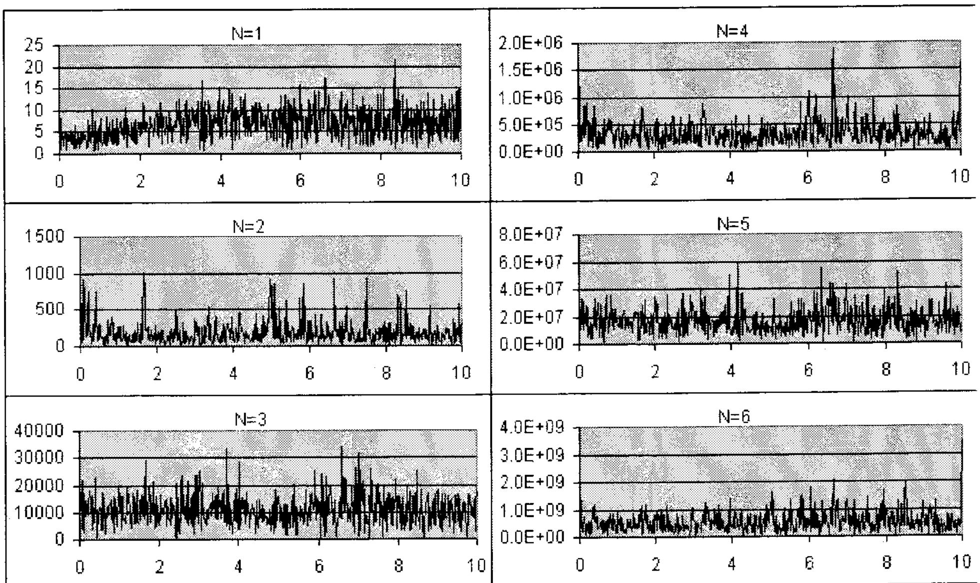


Fig.3. Power Spectrum for $\tau = 0.4$ sec, $b = -2.5$ m/sec^2

After linearization, the first term becomes

$$\delta v_n(t) = \left[1 + \frac{2.5 a_n \tau}{V_n}\{(0.025 + v_n(t-\tau)/2V_n)^{-1/2}\} - 1\right]\delta v_n(t-\tau) \quad (2)$$

While the second term becomes

$$\delta_n(t) = \frac{b_n v_{n-1}(t-\tau)\delta_{n-1}(t-\tau) + b_n \tau \delta v_n(t-\tau)}{\sqrt{b_n^2 \tau 2 - b_n\left\{2(x_{n-1}(t-\tau) - s_{n-1} - x_n(t-\tau)) - v_n(t-\tau)\tau - v_{n-1}^2(t-\tau)/\hat{b}\right\}}} \quad (3)$$

The value of Lyapunov exponent were calculated for some of the initial conditions for which chaos were expected after analyzing time series diagrams, phase diagrams and Poincare sections. The spectrum of Lyapunov exponent computed from simulation program shown in Figure 4. The value of Lyapunov exponent for each vehicle in the platoon was calculated using Wolf et. al method. For $\tau = 0.4$ sec, $b = -2.5$ m/sec^2, N=7 and time=300 sec, the values of Lyapunov exponent were calculated to be zero for leading vehicle and 0.011, 0.012, 0.014, 0.016, 0.017, 0.019 for 1st, 2nd, 3rd, 4th, 5th, 6th

following vehicles. Whereas for τ = 0.4 sec, $\hat{b}$ = -3.5 m/sec^2, N=7 and time=300 sec, the values of Lyapunov exponent were calculated to be zero for all vehicles. This indicates that in former case, the vehicles exhibit chaotic behavior to some extent as the value of this exponent is just above zero. It also indicates that the following vehicle behaves more chaotic than the vehicle in the front in platoon.

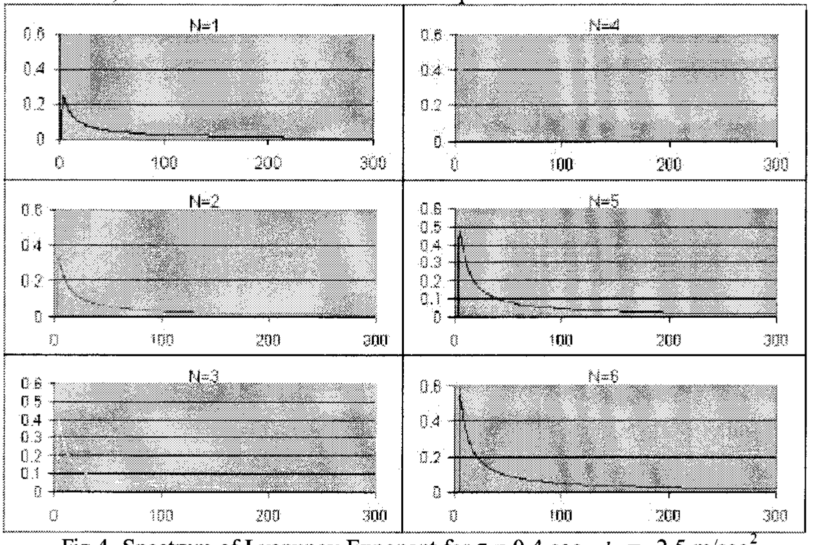


Fig.4. Spectrum of Lyapunov Exponent for τ = 0.4 sec, $\hat{b}$ = -2.5 m/sec^2

4.3 KS Entropy and Predictability of Motion

The numerical approximation to the value of KS entropy can be made by adding values of all positive Lyapunov exponents of the system. As car following model is a one-dimensional system and has only one Lyapunov exponent the value of KS entropy shall be approximately equals to the value of Lyapunov exponent. Following relationship can be used to calculate the prediction time for the motion of vehicles.

$$T \sim (1/\lambda_+) \log_e(L / \varepsilon) \quad (4)$$

For example, for τ = 0.4 sec, $\hat{b}$ = -2.5 m/sec^2, the 6th following vehicle had $\lambda_+ = 0.019$ if the precision of velocity data is 1 in 10^8 then the prediction time can be calculated as

$$T \sim (1/0.019) \log_e(10^8) \quad (5)$$

That is prediction about the motion of vehicles is reliable only for approximately 969.5 time units.

5. DISCUSSION

It was observed that for certain parameter values, a regular periodic perturbation to the Equilibrium State of the car following model could generate chaotic oscillations in following vehicles to some extent. The motion of vehicles in platoon is predictable even when chaotic behavior exists but such prediction is valid only for a short duration of time. This paper presents a theoretical approach to explore the complexity in traffic flow dynamics. The scope of this paper is limited by the performance of Collision Avoidance car following model. As a matter of fact, no model is perfect to explain real-world traffic cases so there is alway some noise coming from the model itself. Further study is recommended using time series data coming from real-world traffic flow. This paper may contribute to understand platoon dispersion phenomenon and can be applicable in a number of Intelligent Transportation Systems.

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