Travel Behavior Models by Combing Randomness and Vagueness Uncertainty*

By Backjin Lee**, Akimasa Fujiwara***, Yoriyasu Sugie****, Moon Namgung*****

1. Introduction

Uncertainty is associated with the definition of the criteria and goals, the values of individuals and society, the lack of knowledge about the system behavior, and the quality of information and data. However, transportation researchers cannot avoid the uncertainties, because these are essentially related with the future state and human behavior (Kikuchi, 1998). Travel behavior models can be divided into two main parts: The first model is the random utility model such as logit model and probit model. This model is based on the randomness of traveler's perception, and probability density distribution is applicable for measuring uncertainty. The other one is the fuzzy reasoning model and it is based on the vagueness of traveler's perception. In this case, possibility distribution is suitable for measuring uncertainty. The two models have developed individually. However, randomness and vagueness must be considered simultaneously but not independently, because travelers will incorporate these two characteristics in choice situation.

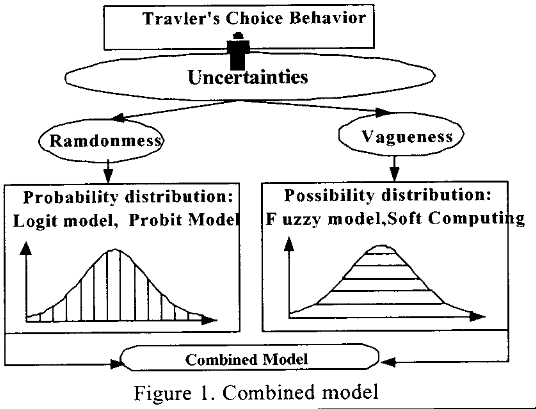

Figure 1. Combined model

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**Student Member, Graduate School for IDEC, Hiroshima Univ.
***Member, Graduate School for IDEC, Hiroshima Univ.
****Member, Graduate School for IDEC, Hiroshima Univ.
*****Non-Member, Dept. of Civil Eng., Wonkwang Univ.
(1-5-1 Kagamiyama Higashi-Hiroshima, 739-8529, TEL0824-24-6922 FAX0824-24-6922)

Therefore, we need a combined model to consider the vagueness and randomness together as shown in Figure 1. This model can also contribute to enhance the forecast accuracy of the travel demand. Two obstacles, however, should be considered for the combined model: One is how can divide traveler perceptions into randomness and vagueness? Another one is what is the relationship between randomness and vagueness?. Therefore, three methodologies for the combined model are suggested by considering the two obstacles. It is noticed that the detailed explanation on the framework of fuzzy model, for instance, membership functions, fuzzy inference rules, and defuzzification method are omitted to the simplicity of the paper.

2. Combined Models of Randomness and Vagueness

(1) The data

In 1996, the survey was conducted at two intercity roads, Honnam highway and No.22 local way among driving commuters from Sunchon city to Kwangju metropolitan in Korea with an objective to examine received perception levels of travel time on the alternative routes and ordinary commute route. The number of 504 sheets was totally distributed at the both roadsides and collected by examiner. To survey the received perception levels of travel time, the questionnaire was designed to ask three levels of travel time of each alternative route. For instance, "How long time does the route may take to travel, if the route takes a short time, a moderate time, and a long time, respectively". Note that during the survey period, Honam highway was constructing for road widening. The construction could cause making the traveler perceptions on travel time more vague, since the travel time of Honam highway might be unstably increased due to the construction.

(2) Latent class clustering method

The fundamental assumption of latent class clustering method is based on the statement of Lotan and Koutsopulos (1993). They stated an example to explain how randomness and vagueness are associated with traveler perceptions of travel time. They suggested that under similar conditions, a traveler familiar with a certain link is able to derive a distribution of travel times. Therefore, probability measures can be used to model the perceptions of the very familiar user. On the other hand a traveler unfamiliar with a certain link has very little idea on the actual travel time of that link. Fuzzy sets can be used in this case to model traveler perceptions, and incorporate poor knowledge, and lack of experience or familiarity with the network. Based on the perception, the latent class clustering method by applying Expectation- Maximization (EM) algorithm, see Appendix, is employed to divide the travelers into familiar travelers and unfamiliar travelers, as shown in Figure 2. The reason why EM algorithm is adopted in this method is due to the expected probability with which the travelers are partitioned into two groups. Because the expected probability means the membership degree to join each group, it is regarded as the index representing the relationship between randomness and vagueness uncertainty. Therefore, the combined model can be established by weighting the expected probability. More specially, the first step is the travelers of the study are divided into familiar and unfamiliar travelers by the latent class clustering method. The second step is a standard logit model is employed to analyze the familiar traveler's perception, while a fuzzy model is applied for the unfamiliar traveler's perception. The last step is the estimated probabilities of the second step models are combined with weighting the expected probability. For the latent class clustering method, the number of latent classes is decided by BIC index. The results of BIC values of latent class clustering are shown in Table 1. When the latent class is divided into 2-latent classes, the lowest BIC value is obtained. Therefore, the result of 2-latent class clustering as shown in Table 2 is employed for the analysis.

Table 1. BIC Values

Num. Of Class	1-Latent class	2-Latent classes	3-Latent classes	4-Latent classes
BIC	3081.9	2614.7	2808.2	7732.9

Table 2. 2-Latent Classes Clustering

Latent class	Latent class 1		Latent class 2	
Variables	μ_1	σ_1	μ_2	σ_2
|Hs-Ls|	3.143	1.512	1.595	0.525
|Hm-Lm|	3.179	1.451	1.449	0.523
|Hl-Ll|	3.930	1.629	1.569	0.527

Note) Hs, Hm and Hl denote short, moderate, and long travel time of Highway, Ls, Lm and Ll denote short, moderate, and long travel time of Local way

As shown in Table 2, latent class 1 has the bigger means and standard deviations than those of latent class 2. This result represents that the travelers of latent class 1 have more distinct perception levels on the difference of travel time of alternative routes so that the latent class 1 is regarded as randomness group, namely familiar group, and vice versa. A logit and a fuzzy model are estimated for whole data, latent class 1 and latent class 2, respectively.

Table 3. Logit, Fuzzy and Combined models

Logit models			
	Whole data	Latent class 1	Latent class 2
Variables	Coefficient (t-value)		
Travel time	-0.044(-4.32)	-0.045(-3.95)	-0.055(-2.01)
Age	-0.024(-2.59)	-0.021(-1.51)	-0.025(-1.97)
Switch*	0.640(2.48)	0.714 (1.77)	0.570 (1.62)
# Of sample	284	139	145
$L(o)-L(\beta)$	-16.414	-11.816	-5.599
ρ^2	0.083	0.123	0.056
Adjusted ρ^2	0.068	0.092	0.026
% of Right	64.79%	66.91%	62.76%
Note) Switch*: Willingness to switch a present route by traffic situation.			
Fuzzy Models			
Variables	Short, Moderate and Long travel time		
% of right	62.676%	61.871%	63.448%
Combined model (% of Right)		**67.64%**	**64.63%**
		66.14%	

The estimated results are shown in Table 3. It is noticed that the mean value of short, moderate and long travel time is used as the travel time variable for the logit model because of multicollinearity of the model. The highest % of right of the logit model is outputted at the latent class 1, while the lowest % of right results is at the latent class 2. The opposite phenomenon is resulted to fuzzy models, that is, the result of the latent class 2 is better than that of the latent class 1. These results represent that the vagueness of uncertainty is well considered in the fuzzy model, while the logit model (random utility model) is applicable to treat the randomness of uncertainty. All the results of the combined models for the latent class 1 and latent class 2 are better than those of logit and fuzzy models. Therefore, the results support the fact that the suggested combined approach can be contributed to enhance the accuracy of the travel behavior model.

(3) Relationship index method

The relationship index method is to find an index that represents a relationship between randomness and vagueness. If the relationship index (RI) is known, then the combined probability is obtained by weighting the relationship index to the results of the logit and fuzzy models. In randomness-based model, a probability of traveler n choosing alternative i can be written as follow.

$$P_n(i) = \Pr(U_i \geq U_j) = \Pr(V_{in} + \varepsilon_{jn} \geq V_{jn} + \varepsilon_{in}) \\ = \Pr(\varepsilon_{jn} - \varepsilon_{in} \leq V_{in} - V_{jn}) = \Pr(\varepsilon_n \leq V_{in} - V_{jn}) \quad (1)$$

Where V_i and V_j are the systematic components of utility function U_i and U_i for alternative i and j, respectively. ε_{in} and ε_{jn} are the disturbances (or random components). If ε_n is assumed normally distributed with mean zero but with $\sigma = 1$, the binary probit function can be drawn like **I** in Figure 3. Now consider a relationship between the probability function and vagueness uncertainty. The vagueness uncertainty, **II** in Figure 3, will be increased as the probability approaches to 0.5, while the vagueness is reduced as the probability is becoming higher or lower than 0.5. Specially, the difference of utilities of i and j is big, the vagueness is reduced. Based on this conception, the relationship index between probability function and vagueness uncertainty can be obtained from Equation 2.

$$RI = \frac{1}{1 + [x - a/b]^2} \quad (2)$$

Where x is $V_{in} - V_{jn}$, and a and b correspond with mean and standard deviation of the probability distribution, respectively. Furthermore, the relationship index between vagueness and randomness uncertainty, **III** in Figure 3, can be intrinsically obtained like (1-RI).

Therefore, the combined model can be obtained by weighting the relationship index to the results of the logit model and fuzzy model.

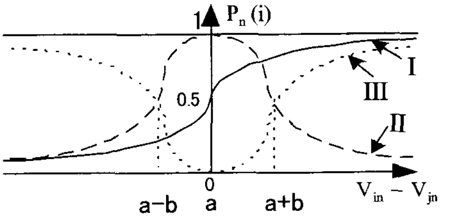

Figure 3. Probability(**I**), vagueness(**II**) and randomness(**III**)

The whole data in Table 3 is used to estimate the combined model by employing the relationship index. The coefficient value of the logit and the fuzzy model are omitted since the results already shown in Table 3. The % of right of the combined model is higher than those of the logit and the fuzzy model. Therefore, it is known that the relationship index method is available for the combined model.

Table 4. Estimation results of the relationship index method

Whole data (% of Right)	Logit model	Fuzzy model	Combined model
	64.79%	62.676%	**66.55%**

(4) Fuzzy probability method

In this paper, only the methodological framework for the fuzzy probability method is suggested. If the random component ε_n of Equation (1) is normally distributed with mean μ and σ, Equation (4), and the believe function based on possibility distribution is assumed like Figure 4, Equation (5). The fuzzy probability can be obtained in Equation (6). The estimation procedure is suggested: The first step is initial values of the possibility parameters in Equation 5 are assumed. The second step is Equation 6 is estimated based on maximum likelihood estimation method. The last step is the initial values of the first step are changed by trial and error method until the maximum likelihood value is reached.

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\} \quad (4)$$

$$S(x;\ a, b) = \begin{cases} 0 & for \quad x < a \\ 2(\frac{x-a}{b-a})^2 & for \quad a \le x < \frac{a+b}{2} \\ 1-2(\frac{x-b}{b-a})^2 & for \quad \frac{a+b}{2} \le x < b \\ 1 & for \quad x \ge b \end{cases} \quad (5)$$

$$P_n(i) = \int_R \mu(x) f(x) dx = \int_{-\infty}^{x<a} 0 \times f(x)dx + \int_{a \le x}^{x<\frac{a+b}{2}} 2(\frac{x-a}{b-a})^2 \times f(x)dx + \int_{\frac{a+b}{2} \le x}^{x<b} \left\{1-2(\frac{x-b}{b-a})^2\right\} \times f(x)dx + \int_{b \le x}^{x<\infty} 1 \times f(x)dx \quad (6)$$

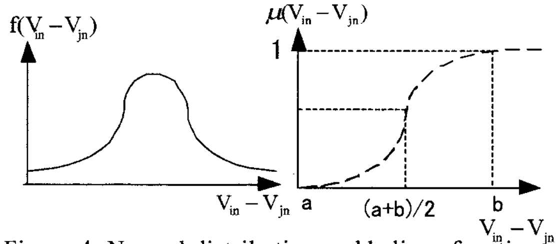

Figure 4. Normal distribution and believe function

3. Conclusions

The paper aims to suggest some possible approaches to combine randomness and vagueness. However, the combined model of randomness and vagueness is difficult due to the difference of theoretical backgrounds of each uncertainty, poor criterions and others. Three methodologies are suggested and some practical results are also shown in this paper. By considering the results, it can be insisted that the combined model can give a chance to improve the prediction accuracy of travel behavior models more exact. Because all methods in the paper are suggested based on the probability theory, other combined methods based on the possibility theory have to be explored in the future.

References

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Appendix

1. Latent Class Clustering Method

Suppose that the probability density function of a random vector W has a finite mixture of k latent class distribution f_k, then $f(w\phi) = \sum_{k=1}^{K} \pi_k \prod_{l=1}^{L} f_{kl}(w_{il};\theta_{kl})$.

We assume the k latent class distributions come from multivariate normal densities with unknown means $\mu_{1l},\dots,\mu_{Kl}$ and variances $\sigma_{1l}^2,\dots\sigma_{Kl}^2$ of the distribution f_{kl}. The complete-data log likelihood for ϕ has the multinomial form. (Mclachlan and Krishnan, 1997)

$$\log L_c(\phi) = \log\left(\prod_{i=1}^{I}\prod_{k=1}^{K}\left[\pi_k^{z_{ik}}\left\{\prod_{l=1}^{L} f_{kl}(w_{il};\theta_{kl})\right\}^{z_{ik}}\right]\right) \quad (1)$$

EM algorithm is applied to solve the log likelihood function, and the iteration between E and M step continues until the maximum likelihood value is obtained.

<E-STEP>

$$E_{\phi^t}(z_{ik} \setminus w) = z_{ik}^{(t)} = \frac{\pi_k^{(t)} \prod_{l=1}^{L} f_{kl}(w_{il};\theta_{kl}^{(t)})}{\sum_{k=1}^{K} \pi_k^{(t)} \prod_{l=1}^{L} f_{kl}(w_{il};\theta_{kl}^{(t)})} \quad (2)$$

<M-STEP>

$$\pi_k^{(t+1)} = \sum_{i=1}^{I} z_{ik}^{(t)} / I, \quad (3)$$

$$\sigma_k^{(t+1)} = \sum_{i=1}^{I} z_{ik}^{(t)} (w_{il} - \mu_k^{(t+1)})^2 / \sum_{i=1}^{I} z_{ik}^{(t)} \quad (4)$$

The number of latent classes is decided based on Bayesian Information Criterion: BIC = -2ln(L) + pln(I)

Where, p and I are a number of parameters and samples, respectively.