A heuristic method for solving multilevel hierarchical transportation system *

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1. Introduction

The recent industrialization in the past decade has drastically increased the demand for movement of people, commodity and information. These flows require a complex network of linkages between origins and destinations. Thus, hierarchical network arises in the situation where the traffic must be transported from its origin to destination, but it is expensive or impractical to create direct connections among all the origin-destination (OD) pairs. The economy of scale in the hub-to-hub links also provides a major incentive, since hierarchical network concentrate flows on those links.

The hierarchical network design involves locating the hubs from a given set of nodes and allocating those non-hub nodes to the hubs such that it minimizes the total cots of transport for the whole network. Since the hub location problems were first formulated, several variations of the problems have been well studied including the formulations and relaxing several assumptions[1].

Even though, most of the past research has focused on the hierarchical network with only one level of hierarchy. In practice, however, the transportation networks generally consist of several levels of hierarchy. For example, the national postal service network consists of three distinct hierarchical facilities, which are post boxes, local post office and regional post office[2]. These different types of facilities transport goods via different mode of transportation. Thus, result in difference discount factor for the economy of scale in each link. Without taking this factor into account, we cannot develop an approach that is applicable in solving a realistic network

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The objective of this paper is to develop a method for designing a multilevel hierarchical network. We extend a model of single level hierarchy be a multilevel one then propose heuristic algorithms to solve the model. The model and algorithm is tested on a well-known data set and it is illustrated that the total network cost is reduced from those previously established due to the multilevel hierarchical structure.

2. Model Formulation

The formulation for single level hierarchical network considered in this study is classified as uncapacitated hub location problem (UHLP) with single allocation. The problem characteristic can be described as follow. From a given set of n nodes, identify nodes to act as hubs and allocate those non-hub nodes to the hub. The hubs are assumed to be fully interconnected, but the non-hub nodes can interact only via hubs. The hubs are integrated and uncapacitated which means there is no restriction for the number of nodes assign to a single hub in any hierarchical level. Since the allocation rule is single, any given node can only be assigned to only one hub. These assignments have to be done such that the total cost of transportation for the whole network is minimized.

O'Kelly[3] formulated the single allocation UHLP as a quadratic integer program. Inspired by his formulation, we develop the multilevel hierarchy model as

M-UHLP

Minimize

$$\sum_{K=1}^{K_{\max}-1}\sum_{i_K}^{|N_K|}\sum_{h_K}^{|H_K|} X_{ih}^{K}\beta^{K}C_{ih}\sum_{j_K}^{|N_K|}W_{ij} + \sum_{K=1}^{K_{\max}-1}\sum_{j_K}^{|N_K|}\sum_{m_K}^{|H_K|} X_{jm}^{K}\alpha^{K}C_{jm}\sum_{i_K}^{|N_K|}W_{ij}$$

$$+\sum_{h_{K_{\max}}}^{|H_{K_{\max}}|}\sum_{m_{K_{\max}}}^{|H_{K_{\max}}|} X_{jj}^{K_{\max}}W_{ij}\,C_{ij}\,\delta^{K_{\max}} + \sum_{K=1}^{K\max}\sum_{j} X_{jj}^{K} f_j^{K} \qquad (1)$$

subject to

$$0 \le X_{jj}^{K} \le 1 \text{ and integer for all } j,K \quad (2)$$

$$0 \le X_{ij}^{K} \le 1 \text{ and integer for all } i,j,K \quad (3)$$

$$\sum_{h} X_{ih}^{K} = 1 \text{ for all } i,K \quad (4)$$

$$X_{ij}^{K} \le X_{jj}^{K} \text{ for all } i,j,K \quad (5)$$

$$|H_K| = |N_{K-1}| \text{ for all } K \quad (6)$$

The superscript K represent the level of Hierarchy $K \in \{1,2,3,...K_{\max}\}$, variable X_{ij}^{K} is equal to 1 if node i is assigned to hub h at hierarchical level K and zero otherwise. If node j is a hub at level K then X_{jj}^{K} is equal to 1. F_j^K is the incremental fix cost for establishing a hub at node j and $\alpha^K, \beta^K, \delta^K$ represent the factor for economy of scale, cost for collection and distribution respectively. $|N_K|$ and $|H_K|$ is the dimension of the set of nodes and hubs at level K.

The first and the third term in the objective function represent the cost for collection and distribution. The second term represent the cost for sending goods on a hub-to-hub link. And the fourth term, represent the fix cost for operation of hubs, which includes handling, sorting, transshipment, etc. Constraints (2),(3) restrict X_{ij}^{K} and X_{jj}^{K} to 0 and 1. Constraint (4) ensures that the each node at each level is assigned to one and only one hub. Constraint (5) enforce that any node may not be assign to location j unless it is a hub of the same level of hierarchy K. Constraint (6) requires that the set of nodes at level K+1 is the same as the set of hubs at level K.

3. M-GATS heuristic

M-GATS(Multilevel-Genetic Algorithm and Tabu Search) is a hybrid heuristic that we adopted to solve the proposed model. M-GATS consists of two main components, which are Genetic algorithm (GA) and Tabu search (TS). The GA component is used for determine the optimal number of hubs as well as its location; while TS is used for determine the allocation of non-hub nodes to the hubs. The algorithm of M-GATS is described in section 3.1 and brief description of GA and TS are given in section 3.2 and 3.3. For principles of GA and TS in a broader sense, please refer to Abdinnour[4] and Goldberg[5].

3.1 M-GATS scheme

M-GATS starts form acquiring the nodal flow and cost as well as the coordinates. Then it calls GA to evaluate the hub locations and the number of hubs and the number of nodes. Then we use TS to evaluate the allocation for the best individual in each generation of population in GA. After the best hub location and allocation are obtained from GA and TS, we then consolidate the traffic to the hub nodes and consider these hub nodes as regular nodes in the second level hierarchy. At this point, we start GA and TS algorithm again to obtain a network of second level hubs. The consolidation and evaluation will continue to the higher level until there is no improvement form consolidates the nodes further, and then the algorithm will stop. The Flowchart for M-GATS is described in Fig. 1.

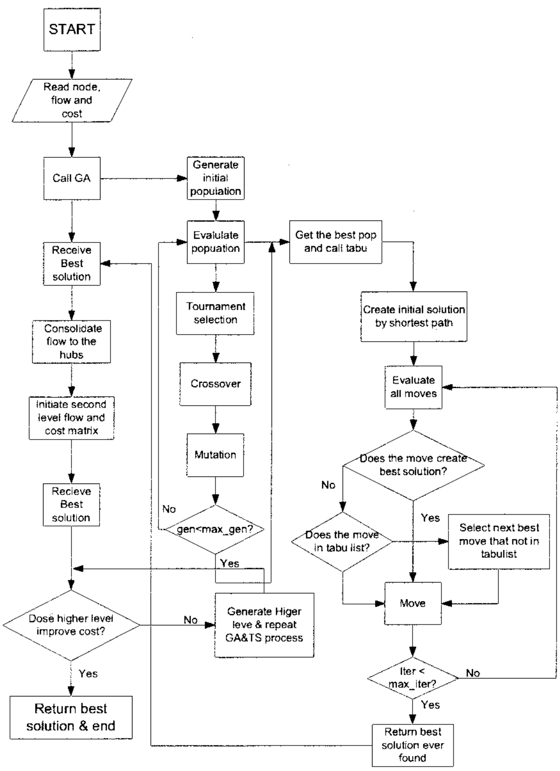

Fig. 1 Flowchart for M-GATS heuristic

3.2 Genetic Algorithm

Genetic algorithm is a search algorithm that employs the process of natural selection to evolve the population of feasible solutions. Our algorithm starts from generate a set of random initial solutions called "population" as a string of 1 and 0 to represent hub node and non-hub node. Each solution in the population is referred to as individual. From this population, the fitness of each individual will be evaluated by allocating the nodes to the hubs according to least cost rule. Then, a pair of individual will be randomly picked from the population pool and the individual with higher fitness will win with some probability and transferred to a mating pool. From this mating pool, a pair of individual will be randomly selected to "mate". The offspring is formed by taking the bits after the random crossover point in the first parent string and combine it with the bits before the crossover point form the second parent string, and vise versa. After population for the next generation is formed, it will go through a process of mutation (randomly change form 0 to 1 and 1 to 0) with some low probability.

By the process mentioned above, the individuals with low fitness will eventually die off and new individual with good fitness will be encountered. After the population pass thru several iterations, the optimal solution will survive in the population as in the nature rules of survival of the fittest.

3.3 Tabu search

Tabu search is a heuristic approach to overcome local optimality entrapment in optimization problems with non-convex objective function. TS guides the search to continue exploring the feasible region even after a local optimum has been reached, and tries to prevent falling back to the same local optimum by using tabulists and aspiration criteria.

In our algorithm, the TS components start from the hub location obtained form the best solutions in each generation of GA. From the hub location, each node will initially assigned to the hubs by least cost rule. Then all neighborhoods of solution that can be obtained by reallocate one node to other hubs will be evaluated. The size of this neighborhood is equal to $(n-p)(p-1)$. The solution will move to the neighborhood with the maximum improvement in objective value or the least non-improvement one. After a move, the moved node will be banned from relocation for a given iterations (stored in tabulist) to prevent cycling. However, the tabulist can be override if the movement of that node can lead the solution to that better than it ever encountered before. Then the search will continue by evaluate and reallocate the best non-tabu nodes.

4. Computation results

In this section, we present the results of computation experiments to justify M-GATS. The computational comparisons are based on two well-known benchmark problems namely CAB dataset and AP dataset.

4.1 CAB dataset

The CAB dataset is based on airline passenger flow between 25 US cities in 1970. The problem set consists of 25 nodes along with cost and flow between each node. The solutions form M-GATS was compared to the optimal result obtained by Skorin-Kapov[6]. The results are shown in Table 1.

It is noted that in M-GATS, GA determines the optimal number of hubs by taking into account the trade off between the fix hub cost and reduction in cost of transportation. But in the study[6] the optimal solutions was derived base on a predetermined number of hubs ranging form 1-4 hubs with no cost for establishing a hub. Therefore in order to make a direct comparison, we add the term (no of hub*F_j) to the solutions and compare them to obtain the optimal one. In Table 1, it is shown that our M-GATS heuristic was able to obtain the optimal solutions in all cases. This proved the validity of our algorithm. But in all cases, M-GATS was not able to reduce the cost further by establishing second level hubs. This is because the number of nodes available in the problem set is too small.

Table 1
Computation result for CAB dataset with 25 nodes

α	F_j	M-GATS hubs	Opt. hubs	MGATS	Opt. OV. (1lev.)	%dev
0.8	200	5	5	1690.57	1690.57	0
	150	12,20	12,20	1594.08	1594.08	0
	100	2,4,12	2,4,12	1458.83	1458.83	0
0.6	200	12,20	12,20	1601.20	1601.20	0
	150	2,4,12	2,4,12	1483.56	1483.56	0
	100	2,4,12	2,4,12	1333.56	1333.56	0
0.4	200	12,20	12,21	1501.62	1501.62	0
	150	4,12,18	4,12,19	1351.69	1351.69	0
	100	1,4,12,17	1,4,12,18	1187.51	1187.51	0

Table 2
Computation result for AP dataset with 50 nodes

α^1	α^2	F_j^1	F_j^2	no. of M-GATS 1st level hub	no. of M-GATS 2nd level hub	M-GATS O.V.	M-GATS 1st Level O.V.	% diff.
0.75	0.2	2000	3000	20	6	110244.1	115359.6	-4.64016
0.75	0.2	3000	4000	14	6	124634.2	131567.1	-5.5626
0.75	0.2	4000	5000	11	5	137413.9	142580.3	-3.75974
0.75	0.2	5000	6000	8	0	153492.2	153492.2	0
0.75	0.2	6000	7000	7	0	161184.8	161184.8	0
0.75	0.1	2000	3000	19	6	105843.1	115605.2	-9.22318
0.75	0.1	3000	4000	14	5	122843.5	132072.2	-7.51257
0.75	0.1	4000	5000	10	5	132426.2	142867.8	-7.88484
0.75	0.1	5000	6000	9	4	145212.8	152293.1	-4.87581
0.75	0.1	6000	7000	6	0	161231.0	161231.0	0

4.2 AP dataset

The AP dataset is based on the mail flows in an Australian city. The dataset contains 200 nodes represent postal districts. The nodes can be combined to obtain a smaller dataset of 10,20,40, or 50 nodes. The optimal solution from M-GATS is compared to the optimal results obtained by Ernst et. al[7] in the same fashion as in 4.1 and shown in Table 2. But noted that the results from Ernst et. al[7] in the 50 nodes problem were available for up to 5 hubs, while the solutions from M-GATS contain at least 7 hubs. Since the direct comparison is not possible, therefore the comparisons are done based on the results of M-GATS for 1 level of hierarchy compare with M-GATS with 2 level of hierarchy. From the calculation results, it is shown that in most of the case the existence of 2nd level hierarchy can reduce the total cost of transportation by up to 10%. This is because the multilevel hierarchy arrangement consolidates the flow more efficiently and gains more advantage from the economy of scale.

5. Conclusion

In this paper, we have proposed a formulation for multilevel hierarchical transportation network and developed a heuristic model based on GA and TS namely M-GATS to solve the proposed model. From the computational experiments, the M-GATS has appeared to by quite efficient. It was able to find the optimum solutions to all benchmark problems and also able to solve large size problems (up to 200 nodes). The calculation results also have shown that the establishment of network in multilevel hierarchical manner can reduce the total network cost further. Thus multilevel formulation is not only more realistic, it also results in more cost efficient network as well.

However, the real transportation network problem consists of thousands of nodes. It appears to be a difficult to apply the proposed method to solve these very large size problems. This is because the calculation time and memory requirement increase dramatically as the number of nodes increase. Therefore, we need to develop an efficient method to cluster the given set of nodes to a manageable size and solve them separately.

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