Link Based Travel Time Functions for Dynamic Traffic Assignment Models.*

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1. Introduction

Unlike the static models, the distribution in time and space of the vehicles on a link is a matter of concern when considering dynamic traffic assignment models. Therefore, the Bureau of Public Roads (BPR) travel time function is no longer applicable in a time-dependent traffic network. Instead, many recently proposed dynamic assignment models assume different forms of dynamic link travel time functions. Yet, no any well-accepted dynamic link travel time function exists and moreover no consensus have been reached for the suitable form of dynamic link travel time functions. Development of a set of time-dependent link travel time functions for dynamic assignment problems is becoming increasingly important. In order to transform the traffic flow data from roadside detectors into travel times for the purpose of short-term travel forecasting, dynamic link travel functions are also necessary. Link travel time or delay functions, have been extensively studied by traffic engineers. The choice of the dynamic link travel time functions involve several criteria:(1) the desired mathematical properties of the function to satisfy the condition for a unique solution of the model; (2) the cost and limited availability of road data; (3) the computational effort required by the model; and (4) the desired accuracy of the travel time estimates generated by the model. The object of this paper is to review currently available delay models, show some of their properties, identify suitable functions and develop them into dynamic link travel time functions, which would be applicable in dynamic assignment.

2. Background.

(1) Instantaneous versus actual link travel time.

Two are the travel time concepts widely introduced in dynamic traffic assignment modeling; instantaneous travel time and actual travel time. Instantaneous link travel time $\tau_{ij}(t)$ at time t is defined as the travel time that is experienced by vehicles traversing link ij when the prevailing traffic conditions remain unchanged.

Actual travel time is the travel time actually experienced by the vehicles while crossing through the link ij with no traffic preconditions made. In the conditions when the network traffic conditions change over time, the difference between them may be significant.

Accordingly, we notice reactive and predictive dynamic assignment models. The instantaneous travel time concept is often used in the dynamic assignment and it is the one that we mean here when we mention dynamic link travel time in the following.

(2) Explicit and implicit travel time models

Most models proposed in the literature adopt functions simulating explicitly (i.e. travel time functions) or implicitly (i.e. exit functions) the travel time on a link depending on the number of users traveling on it. Travel time functions express link travel time $\tau_{ij}(t)$ as a function of the relevant traffic condition variables. Implicit exit time functions express direct the outflow of a given link as a function of recent "historical" traffic condition variables.

(3) Link entering time.

In all the models considered here, the cost of travel is taken to depend upon the spatial distribution of traffic on each link at the time of entry to it and this will itself depend upon the recent history. Travel costs therefore should not depend upon traffic entering the link at a latter time (Daganzo 1995). It is clear now that our link travel time functions will depend on the traffic conditions at the time that considered vehicles enter the link, or more generally on the link entering time t expressed as; $\tau_{ij} = \tau_{ij}(t)$.

(4) Point queues versus physical queues.

Most of the models adopt the point queue concept to

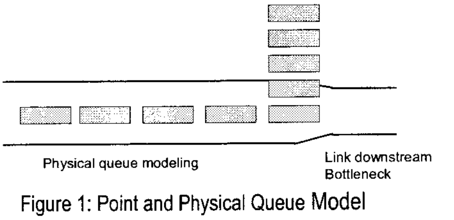

Figure 1: Point and Physical Queue Model

*Key words: traffic phenomena analysis, traffic assignment, dynamic travel time functions

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represent the congested part of the link. It means that the vehicles at the link exit waiting to be served, are arranged in a vertical queue, one at the top of the other. While the physical queue model, arrange the queued vehicles horizontally. The main difference between them consist on the fact that while in the physical queue concept the space occupancy must be taken into consideration (link spillover phenomena can occur, affecting this way the link exit capacity and furthermore link travel time), the point queue concept doesn't consider it at all. Physical queue represent more naturally the flow dynamics but it is more difficult to model.

3. Main features of some most widely used dynamic link travel time models

(1) Whole link models

This type of model considers link travel time to be dependent on the total amount of vehicles $x(t)$ on the whole of the link at each instant t.

$$\tau_{ij}(t) = D_{ij}(x_{ij}(t)) \quad (1)$$

It is more generally expressed by the inclusion of terms of the link inflow $u_{ij}(t)$, exit flow $v_{ij}(t)$ and number of vehicles $x_{ij}(t)$ on the link ij at time t as follows

$$\tau_{ij}(t) = D_{ij}[(u_{ij}(t), v_{ij}(t), x_{ij}(t)] \quad (2)$$

In dynamic networks, the inflow, exit flow and the number of vehicles (figure 2) can be deemed as three different states for the same vehicles.

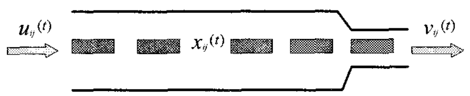

Figure 2: Link State Variables

These models have the advantage that conservation of the traffic can be expressed readily in the form of differential equation:

$$\frac{d\,x_{ij}(t)}{dt} = u_{ij}(t) - v_{ij}(t) \quad (3)$$

The link flow dynamics cannot be represented using this model because it lacks the ability to model the spatial and temporal evolution of the flow within the link itself. Because of its asymmetric property, any dynamic travel choice problem using a dynamic link travel time function in the form of equation (2) has been proven that cannot be expressed as an optimization problem.

In the latest development, formulations expressing exit flow and the number of vehicles as function of link inflow have been proposed. It transforms equation 2 into

$$\tau_{ij}(t) = D_{ij}[(u_{ij}(1), u_{ij}(2), \ldots\ldots u_{ij}(t)] \quad (4)$$

making it more abstract and complicate to be determined. FIFO discipline is vital for this kind of formulation.

(2) Deterministic Queuing models

The main feature here is that the traffic if supposed to freely flow on each link with the possibility of a bottleneck at the downstream end that determines the capacity and hence limits the outflow. Deterministic queuing model is often used to model the phenomena.

Travel time over link ij for vehicles entering link at time t is the sum of two components:

a) a flow-dependent cruise time $d^1_{ij}(t)$ over the first part of the link ij

b) a queuing delay $d^2_{ij}[t + d^1_{ij}(t)]$ at time $[t + d^1_{ij}(t)]$ when the vehicles enter the second part of the link ij

It follows that $\tau_{ij}(t) = d^1_{ij}(t) + d^2_{ij}[t + d^1_{ij}(t)]$ (5)

Since the link is short and cruise time over the first part of the link is short the queuing delay $d^2_{ij}(t)$ can be considered as a good representation of $d^2_{ij}[t + d^1_{ij}(t)]$ simplifying more the model. The queuing delay or the time spent by the vehicles while queued in the front of the link exit can be calculated as $d^2_{ij}(t) = \frac{l_{ij}(t)}{Q}$ where $l_{ij}(t)$ is the queue length at time t and Q is the bottleneck capacity.

Models of this kind have the advantage of being computationally convenient and provide mutually consistent estimates of travel time and outflow. However, because queuing only occurs when flow exceeds capacity, the cost-flow relationship will be constant (based upon the free-flow travel time until inflow exceed capacity.

(3) LWR wave model

This highly detailed model has the advantage of plausible description of the flow dynamics. It can exhibit even the formation and propagation of any congestion through the link. In this model, the instantaneous flow $q_{ij}(x,t)$ at position x on link and time t is determined by the corresponding density $k_{ij}(x,t)$. Key feature of this model is that the propagation of the regions of constant density k along waves with speed $w_{ij}(k) = d\,q_{ij}(k)/d\,k_{ij}$ is determine by the density alone so that

$$w_{ij}(k) = v_{ij}(k) + k_{ij}\frac{d\,v_{ij}}{d\,k_{ij}} \quad (6)$$

Speed decreases with the increasing of the density, so that $\frac{d\,v_{ij}}{d\,k_{ij}} < 0$.

It implies that $w_{ij}(0) = v_{ij}(0) \geq v_{ij}(k) \geq w_{ij}(k)$ with strict inequalities for $k > 0$ meaning that vehicles travel more quickly than do the conditions they encounter, so the trajectory of a vehicle can only be affected by conditions downstream of it.

In the following, two methods for calculating link travel times are shown:

a) Piece-wise linear problem; the three detector problem.

The first method is based on the well-known three-detector problem. We give a very quick review of the key features. First, the flow-density relationship is taken to be a piece-wise function as shown on figure 3. The first part of the graph represents the uncontested state of flow and the second part the congested one. Three detectors problem suggests that the cumulative flow past a middle (M_{ij}) detector can be obtained from the cumulative flow past upstream (A_{ij}) and downstream (D_{ij}) detectors.

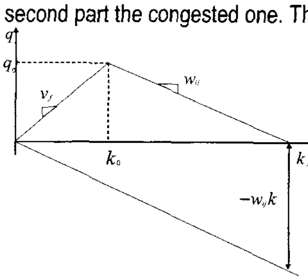

Figure 3: Piece-wise relation

Escaping the details, the method is described in figure 4.

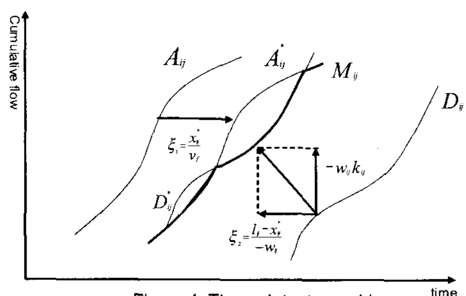

Figure 4: Three-detector problem

The instantaneous link travel time on link (i, j) is defined as

$$\tau_{ij}(t) = \int_0^{l_{ij}} \frac{d\,x_{ij}}{v_{ij}(x,t)} \qquad (7)$$

where $v_{ij}(x,t)$ is the vehicle velocity on link (i, j) at location x and time t. If we choose that the location of the middle detector M_{ij} at time t, coincide with the location $x^*_{ij}(t)$ were a shock arrives at time t than we have

$$A_{ij}\left[t - \frac{x^*_{ij}(t)}{v_f}\right] = D_{ij}\left[t - \frac{l_{ij} - x^*_{ij}(t)}{w_{ij}}\right] + k_{ij}^{\max}\left(l_{ij} - x^*_{ij}\right) \qquad (8)$$

From this relationship the physical queue length can be determined at any time t if the cumulative link entering flow (A_{ij}) and cumulative link exit flow (D_{ij}) are known by time t. Instantaneous link travel time is than calculated as the sum of the two part of the link; the free-flow part and the queued one.

b) Link outflow and exiting shock wave

According to this method, the accumulated flow at exit from the link at any time t can be determined by the accumulated inflow to the link at earlier times and the density of the wave that leaves the link at that time.

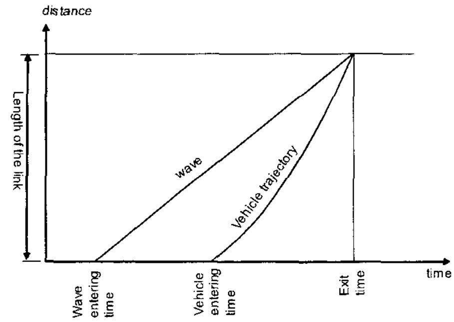

Figure 5: Wave and vehicle leaving the link

After LWR model the flow of traffic past a wave of density k is:

$$q_{ij}(k) = k_{ij}\left[v_{ij}(k) - w_{ij}(k)\right] = -k_{ij}^2 \frac{d\,v_{ij}}{d\,k_{ij}} \qquad (9)$$

If a wave of density k enters a link of length l_{ij} at some time s and propagates to the exit of that link, than it will leave the link at time $t = s + l_{ij}/w_{ij}(k)$

Accordingly the accumulated efflux at that time will be

$$D_{ij}\left[s + \frac{l_{ij}}{w_{ij}(k)}\right] = A_{ij}(s) - \frac{l_{ij}\,k_{ij}^2}{w_{ij}(k)}\frac{d\,v_{ij}}{d\,k_{ij}} \qquad (10)$$

When the Greenshield' linear speed-flow relation is used, the wave speed is given by

$$w_{ij}(k) = v_f\left[1 - k_{ij}/k_{jam}\right] \qquad (11)$$

the accumulated efflux is expressed as;

$$D_{ij}(t) = A_{ij}\left[t - \frac{t}{v_f\left(1 - 2k_{ij}/k_{jam}\right)}\right] + \frac{l_{ij}\,k_{ij}^2}{k_{jaam} - 2k_{ij}} \qquad (12)$$

Because more than one wave can reach the exit link at a given time t we must apply the Newell-Luke minimum principle to select the exact wave. According to this principle we select the wave with give the minimum rase into the cumulative outflow.

This selection rule is expressed according to Newell as

$$s = \arg\min_{\varepsilon} A_{ij}(\varepsilon) - \frac{l_{ij}\,k_{ij}^2}{w_{ij}\left[k_{ij}(\varepsilon)\right]}\frac{d\,v_{ij}}{d\,k_{ij}} \qquad (13)$$

subject to:

$$\varepsilon + \frac{l_{ij}}{w_{ij}[k_{ij}(\varepsilon)]} = t$$

This way we can construct the cumulative link exit flow (D_{ij}) until present time t.

If the cumulative link entering flow (A_{ij}) and cumulative link exit flow (D_{ij}) are known until present time t (figure 6) than travel time can be derivate from the relation

$$A_{ij}(t) = D_{ij}\left[t + \tau_{ij}(t)\right] \qquad (14)$$

When the FIFO discipline is in order then

$$\tau_{ij}(t) = D_{ij}^{-1}\left[A_{ij}(t)\right] - t \qquad (15)$$

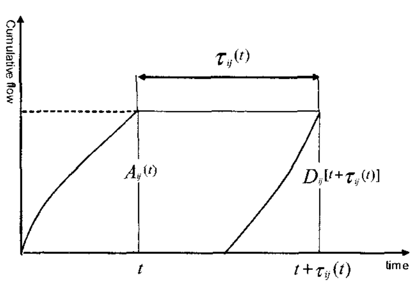

Figure 6: Cumulative arrival and departure

4. Few words on the implicit models (exit functions)

In these types of model a relation between the number of the vehicles leaving the link and the number of vehicles existing on link at time t is supposed to exist and the corresponding function is proposed. In general

$$v_{ij}(t) = v_{ij}[x_{ij}(t), t] \qquad (16)$$

Different types of relation have been proposed but here we will just illustrate the idea here with function proposed by Wie:

$$v_{ij}[x_{ij}(t)] = v_{ij}^{\max}\{1 - \exp[-x_a(t)/\beta_a\} \qquad (17)$$

where $v_{ij}^{\max}$ is the exit capacity and β_{ij} is a parameter that varies with the road type. Implicit functions make the procedure of travel time determination easier but, they lead to a number of theoretical inconsistencies.

5. Conclusions and remarks

As the conclusion to this study we find that the performance of dynamic network traffic assignment models depend heavily on the dynamic travel time functions adopted. Different dynamic link performance models lead to different estimates of cost of travel therefore the network traffic patterns obtained by the assignment differ substantially.

Whole link models can be used when no detailed description of the within link flow dynamic is needed. They lack the ability to spatially describe the within flow dynamics. When a new route comes in use they are discontinues

Deterministic queuing models are relatively simple with assignment proportions mostly determined by link capacities. Assignment proportions are mainly determined by the link exit capacities thus leading to jump discontinuity when new route come in use.

Whole link and deterministic queuing models show discontinuities when a new route comes into use and are asymmetric thus can not be used in the optimization problems.

Wave model enjoy the property of being highly detailed thus giving a better image of the link flow dynamics. By introducing even physical queues they reach to be more realistic but more calculation efforts are necessary. The wave models show small discontinuities when shocks are formed but are continuous when new routes come into use and have plausible interpretations.

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