Social Welfare Maximization of Health-Care Facilities under Multilevel Programming *

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1. INTRODUCTION

The main constraints face by many countries in efforts to improve physical infrastructure of health services are the problem of dispersing population site and the problem of isolated area. The country that consist of thousand islands and mountains always face with the problem of location decisions. Since, the limited funds allocated to the health sector brings the decision maker to the problem of choosing a best location. The strategy to handle this problems are developed in the application of location theory. The infrastructure of health care system is a special case concern with the public facility location problem. Here, many nonmonetary cost and benefits must be considered. In many countries, efficiently and loss costly provision of health care facilities is an important public policy. Also, health care facility locations are almost concerned with long-term decision.

One way to solve the user benefit and the supplier benefit in health care facility problems is to hypothesize the existence of some social welfare function. The most reasonable interpretation of such a function is that it represents a user decision maker's and a supplier decision maker's about how to trade off the utilities of these facilities and the total costs. To build such model, we consider a decentralized noncooperative decision system in which one leader and several follower of equal status are involved. We assume that the location as a leader and the choice models as followers may have their own decision variables and objective functions. The leader as an upper level can only influence the reactions of followers through his own decision variables, while the followers as a lower level have full authority to decide how to optimize their objective functions in view of the decisions of the leader and other followers. This problems bring us to the multi-step optimization. A powerful tool dealing with decentralized noncooperative decision system is the multilevel programming.

2. MODELING THE SOCIAL WELFARE MAXIMIZATION

We now try to construct a social welfare maximization model based on multilevel programming. The upper level of the programming concerns with location variables. The task of upper level is to provide location combination based on fixed land costs and fixed building cost as physicals aspect. In the hierarchical of health care services aspect, the location combinations of two facility types is based on critical coverage distance. On the other side, the lower level of the programming deals with three sub-optimization models, they are the utility functions produced by supplier, the user choice optimization, and the supplier management functions. However, the user optimization model stands as an outer model, his inner model is accessibility-facility congestion-sensitive user allocation model. Figure 1 shows the outline of the main model.

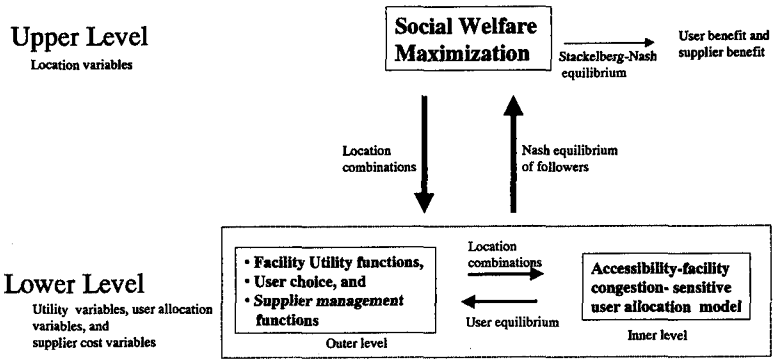

Figure 1. The social welfare maximization model.

We deal with two types of hospital as an upper level facility and health center as a lower level facility. The locational configuration of hospitals and health centers are determined in relation to the consequences of hierarchy, that is, the ability of

* Keywords: health care; location problem; user benefit; supplier benefit; regional planning; land use.
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hospitals site to cover health centers site. Thus, we are concerned not with facilities placement disperse, but with the resulting coverage patterns of those placements, illustrated in figure 2. The mathematical formulation of locations decision is linear and use 0-1 variables as a location decision variables.

$$(\psi^1+\xi^1 r^1)\sum_j x_j^1 \le B_{hospital}$$
$$(\psi^2+\xi^2 r^2)\sum_k x_k^2 \le B_{health\ center}$$
$$x_k^2 \le \sum_j a_{kj} x_j^1 \qquad \forall k \in K$$
$$\sum_j x_j^1 \ge 1,\quad \sum_k x_k^2 \ge 1,\quad x_j^1 \in \{x_{jA}^1, x_{jB}^1\}$$
$$x_j^1 = \{0,1\},\quad x_k^2 = \{0,1\} \quad J \cap K = \phi, \forall j \in J, k \in K$$

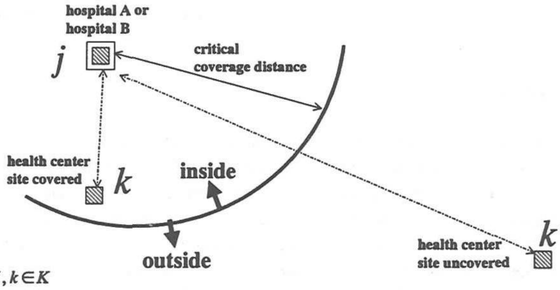

Figure 2. Coverage patterns of hospitals and health centers site.

In the lower level, firstly, we consider the user choice model. Here, the location-user choice linking constraints work based on the accessibility-facility congestion-sensitive user allocation model. Figure 3 shows the linking constraints. Here, the allocation of patients (non-emergency) is based on user-attracting system. The user-attracting system denote the user choice of facility. Such systems is formulated by a spatial interaction model to represent user choice behavior.

$$y_{ij}^1 - x_j^1 G_i^1 \le 0 \qquad \forall i \in I, j \in J$$
$$y_{ik}^2 - x_k^2 G_i^2 \le 0 \qquad \forall i \in I, k \in K$$
$$2y_{kj}^3 - (x_k^2 + x_j^1)\varphi G_i^2 \le 0 \qquad \forall k \in K, j \in J$$
$$y_{ij}^1 \ge 0, y_{ik}^2 \ge 0, y_{kj}^3 \ge 0 \quad \forall i \in I, k \in K, j \in J$$
$$0 < \varphi \le 1$$

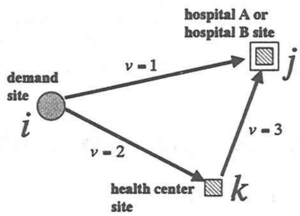

Figure 3. The location-user linking constraints.

Leonardi (1981) describes two subsystems as follows. The first subsystem is the accessibility-sensitive demand mechanism. Usually, the demand will increase with the increasing accessibility to the location. The second subsystem is the facility congestion-sensitive demand mechanism. Leonardi (1981) identifies that the congestion-sensitive demand mechanism based on inputs of actual demand and accessibility between demand and facility locations. Figure 4 shows this sub-system.

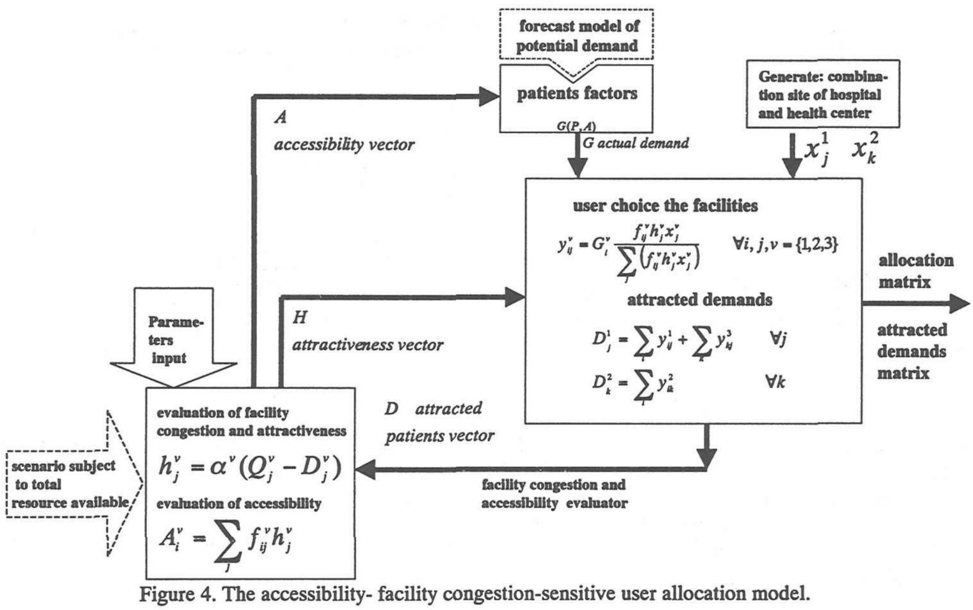

Figure 4. The accessibility- facility congestion-sensitive user allocation model.

The space discount function is assumed as exponential as follows:

$$f_{ij}^{v} = \exp(\beta_1^{v} u_{(1)j}^{v} + (-\beta_2^{v}) u_{(2)ij}^{v} + (-\beta_3^{v}) u_{(3)j}^{v} + \beta_4^{v} u_{(4)j}^{v} + \beta_5^{v} u_{(5)j}^{v}) . \tag{1}$$

Thus, the user allocations based on logit model are as follows,

$$y_{ij}^{v} = G_i^{v} \frac{f_{ij}^{v} h_j^{v} x_j^{v}}{\sum_j f_{ij}^{v} h_j^{v} x_j^{v}} \qquad \forall i \in I, j \in J \tag{2}$$

where $v = 1 \rightarrow i = i \ and \ j = j; \quad v = 2 \rightarrow i = i \ and \ j = k; \quad v = 3 \rightarrow i = k \ and \ j = j.$

Next, we consider facility utility function produced by suppliers within the purpose of provided patients satisfactions. The facility utility functions in order are as follows.
Information service linear function, we deal with how the basic information as well as the item of addresses, service time schedule, specialists type, and so on. The information service variables offer a means of the method to inform to the user.
Travel cost function, this function deal with the utility of how much the user has to pay for one trip from his place to the hospital or the health center and also as referral patients have to move from a health center to a hospital. We use conventional linear function like those function concerns with total travel time.
Health care cost, we use a linear function describe the total treatment cost of one patient. Actually, it is very difficult to make generalization, since it is depend on the illness and the treatment way. But, basically, the aggregated total cost of treatment can be measure by limited common illness case treatments. Here, also we can adopt such payment system like health insurance.
Parking service, this utility function deals with the probability of rejection. Where each parking area of the hospitals or the health centers have the parameter concern with how many minutes a car of user park and the variable of fixed number of bays. The rejection parking probabilities are used as the parking service utility in each facility site.
Frequency of mass-transit, this is concerned with the route of mass-transit arrangement by local government and may be in any case hospital service bus arrangement by hospital. The utility values are taken in the number of mass-transit service within one hour in the working day.

And the last lower model is the supplier management functions deal with hospital or health center operation strategic planning. Economically, suppliers are always faced with the problem of operation costs and personnel costs. The ***operation costs*** basically derive from purchasing drugs, to keep foods in stock, electricity in operationalize medical equipment's, room maintenance's, and so on. These items work as variables to calculate the total operation costs. In case of ***personnel or staffing costs,*** deal with salary of physicians, nurses, specialists, cleaning service staffs, receptionist staffs, and so on. Generally, the personnel costs are always measured based on average working time per month. Also in this lower model, we directly insert a ***subsidy function***. In any case, the subsidy function works based on local government policy in supporting health care facilities.

The model formulations are described in equation (3)-(6) as follows.

$$\left.\begin{array}{ll}
\textbf{Max Welfare} & W = \sum_{v=1}^{v=3} W_1^{v} + \sum_{v=1}^{v=2} W_2^{v} \\
\quad\textbf{where} & W_1^{v} = \sum_i \sum_j y_{ij}^{v} x_j^{v} (u_{(1)j}^{v} + (-u_{(2)ij}^{v}) + (-u_{(3)j}^{v}) + u_{(4)j}^{v} + u_{(5)j}^{v}) \\
 & W_2^{v} = \sum_i \sum_j y_{ij}^{v} x_j^{v} ((-s_{(1)ij}^{v}) + (-s_{(2)j}^{v}) + u_{(3)j}^{v} - \psi^{v} - \xi^{v}) \\
\textbf{S.T.} & (\psi^{1} + \xi^{1} r^{1}) \sum_j x_j^{1} \le B_{hospital} \\
 & (\psi^{2} + \xi^{2} r^{2}) \sum_k x_k^{2} \le B_{health\ center} \\
 & x_k^{2} \le \sum_j a_{kj} x_j^{1} \qquad \forall k \in K \\
 & \sum_j x_j^{1} \ge 1, \quad \sum_k x_k^{2} \ge 1 \quad x_j^{1} \in \{x_{jA}^{1}, x_{jB}^{1}\} \\
 & x_j^{1} = \{0,1\}, \quad x_k^{2} = \{0,1\} \quad \forall j \in J, k \in K
\end{array}\right\} \textbf{Upper level model} \tag{3}$$

The upper level works based on the limited allocated funds for deciding the number of hospitals and the number of health centers. In the location combinations steps, after locating basic size (type A) of hospital, it is still evaluated based on local conditions to change the hospital type A with hospital type B. The objective function contains the function of user benefit and supplier benefit, respectively. The social welfare value means the balance value of the user benefit and the supplier benefit.

The followers in the lower level work in finding his optimum value based on location combination decided by leader. In other words, the sub-problem optimizes his reaction because the location decision revealed by the leader. The Nash equilibrium of the followers will be the solution of the system.

$$\text{Max.} \quad f_1(y) = \sum_i \sum_j y_{ij}^1 + \sum_i \sum_k y_{ik}^2 + \sum_k \sum_j y_{kj}^3$$

$$\begin{aligned} \text{S.T.} \quad & y_{ij}^1 - x_j^1 G_i^1 \le 0 \quad \forall i \in I, j \in J \\ & y_{ik}^2 - x_k^2 G_i^2 \le 0 \quad \forall i \in I, k \in K \\ & 2y_{kj}^3 - (x_k^2 + x_j^1)\varphi G_i^2 \le 0 \quad \forall k \in K, j \in J \\ & y_{ij}^1 \ge 0, y_{ik}^2 \ge 0, y_{kj}^3 \ge 0 \quad \forall i \in I, k \in K, j \in J \\ & 0 < \varphi \le 1 \end{aligned}$$

Lower level model (4)

$$\text{Max} \quad f_2(u)^v = \sum_i \sum_j (-u_{(2)ij}^v) + \sum_j \{u_{(1)j}^v + (-u_{(3)j}^v) + u_{(4)j}^v + u_{(5)j}^v\}$$

$$\begin{aligned} \text{S.T} \quad & 1 \le \sum_j u_{(1)j}^v x_j^v \quad v = \{1,2,3\} \\ & \sum_i \sum_j u_{(2)ij}^v x_j^v \le C \quad v = \{1,2,3\} \\ & \sum_j u_{(3)j}^v x_j^v \le IP \quad v = \{1,2,3\} \\ & 1 \le \sum_j u_{(4)j}^v x_j^v \quad v = \{1,2,3\} \\ & 1 \le \sum_j u_{(5)j}^v x_j^v \quad v = \{1,2,3\} \\ & u_{(1)}^v > 0, u_{(2)ij}^v > 0, u_{(3)j}^v > 0, u_{(4)j}^v > 0, u_{(5)j}^v > 0 \quad \forall i \in I, j \in J \end{aligned}$$

Lower level model (5)

$$\text{Max.} \quad f_3(s)^v = \sum_j \{(-s_{(1)j}^v) + (-s_{(2)j}^v) + s_{(3)j}^v\}$$

$$\begin{aligned} \text{S.T.} \quad & \sum_j s_{(1)j}^v x_j^v \le OC \quad v = \{1,2\} \\ & \sum_j s_{(2)j}^v x_j^v \le PC \quad v = \{1,2\} \\ & 0 \le \sum_j s_{(3)j}^v x_j^v \quad v = \{1,2\} \\ & s_{(1)j}^v > 0, s_{(2)j}^v > 0, s_{(3)j}^v \ge 0 \quad \forall j \in J \end{aligned}$$

Lower level model (6)

3. CONCLUSION

We present a development of mathematical model for user benefit and supplier benefit of health-care facilities as a social welfare maximization. The multilevel programming offers a means of studying decentralized noncooperative decision system. For next study, the nonconvexity and the NP-hardness of the problem bring us to adopt heuristic calculation procedure such as genetic algorithm as a solution procedure for an application in a network.

References

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Main Notations : k, j = *subscripts labeling the candidate site of health center and candidate site hospital*; G_i^v = *total patients in node i(in case* $v = 1$); Q_j^v = *facility capacity*; β_l^v = *a spatial discount parameter* (≥ 0) *to be valued empirically* ($l = (1,2,...5)$); $B_{hospital}, B_{h.center}$ = *budget for hospital and h.center*; ψ^1, ψ^2 = *basic construction cost*; ξ^1, ξ^2 = *cost coeficient*; r, r' = *size of facility*; p_i = *potential demand*; $a_{kj} = 1$ *if candidate health center k is within* D_{kc} *travel time unit of candidate site hospital j*, 0 *otherwise*; C = *total travel cost allocated by users*; IP = *total health care cost allocated by users*; OC = *operating cost budget*; PC = *personnel cost budget*; $u_{(1)j}^v$ = *information service function*; $u_{(2)ij}^v$ = *travel cost function*; $u_{(3)j}^v$ = *health care cost*; $u_{(4)j}^v$ = *parking service function*; $u_{(5)j}^v$ = *frequency of masstransit*; $s_{(1)j}^v$ = *operation cost function*; $s_{(2)j}^v$ = *personnel cost function*; $s_{(3)j}^v$ = *local government subsidy function*; W_1^v = *user benefit*; W_2^v = *supplier benefit*; $x_j^1 = 1$ *if candidate site of hospital is selected*, 0 *if not*; $x_k^2 = 1$ *if candidate site of h.center is selected*, 0 *if not*; y_{ij}^1 = *user at node i choose hospital j*; y_{ik}^2 = *user at node i choose health center k*; y_{kj}^3 = *referral patients from health center site k to hospital site j*;