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SENSITIVITY OF MACROSCOPIC TRAFFIC FLOW SIMULATION MODEL PARAMETERS

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1. INTRODUCTION

Macroscopic traffic flow models, which are able to handle large size of freeway systems with the fast simulation time, provide an economical and effective way to evaluate alternatives of geometric design and control strategy. Such models contain a set of parameters, which have to be estimated according to real traffic data. Since model parameters have significant effects on the performance of the simulation, they have to be identified carefully. In general, the identification procedure is formulated as a parameter optimization problem, which can be solved based on iterative comparison of model estimates with real traffic variables. Various techniques can be used for this purpose. However, the choice primarily depends on the nature of the parameters: If they are insensitive to traffic situation, a static approach can be used. If they posses non-linearity, a random search technique will be effective to reach the optimum value. If they are sensitive to traffic condition, a dynamic method should be adopted. This study focuses on the comparison of those method concerning with the parameter estimation of a particular macroscopic model. Three methods were selected; Nonlinear Least Square technique (NLT) as static one, Box technique (BCT) as random search, and Kalman filtering technique (KFT) as dynamic one.

2. MACROSCOPIC TRAFFIC FLOW MODEL

2.1 Macroscopic Model

Traffic variables for the macroscopic traffic flow simulation model, which was first derived by Payne and later modified by Cremer [1] are

$\rho_j(k)$: density of segment j at time k

$v_j(k)$: space mean speed of segment j at time k

$q_j(k)$: flow rate at a point of boundary between segment j and $j+1$ at time k

$w_j(k)$: time mean speed at a point of boundary between segment j and $j+1$ at time k

$r_j(k)$: ramp entry flow rate of segment j at time k

$s_j(k)$: ramp exit flow rate of segment j at time k

The first equation is the continuity equation that describes how density varies with time:

$$\rho_j(k+1) = \rho_j(k) + \frac{\Delta t}{\Delta L_j}\left(q_{j-1} - q_j + r_j - s_j\right)_{(k)} \quad (1)$$

where Δt is time increment, and ΔL_j length of j^{th} section. The second equation, which is so called the momentum, defines the variation of space mean speed over time:

$$v_j(k+1) = v_j(k) + \frac{\Delta t}{\tau}\left\{v_e\left[\rho_j(k)\right] - v_j(k)\right\} + \frac{\Delta t}{\Delta L_j} v_j(k)\left[v_{j-1}(k) - v_j(k)\right] - \frac{\nu \cdot \Delta t}{\tau \Delta L_j}\frac{\rho_{j+1}(k) - \rho_j(k)}{\rho_j(k) + \kappa} \quad (2)$$

where τ = time constant,
κ = density constant,
ν = anticipation constant, and
v_e = speed at equilibrium state.

The third equation is the fundamental relationship among traffic volume, speed, and density:

$$q_j(k) = \alpha\left(v_j(k) * \rho_j(k)\right) + (1-\alpha)\left(v_{j+1}(k) * \rho_{j+1}(k)\right) \quad (3)$$

where α=weighting parameter ranging $0 \le \alpha \le 1$.

The parameters identified in this study are τ, ν, κ, and α.

2.2 Equilibrium Speed-Density Relationship

The second term on the right side of Eq. 2, which is referred to as relaxation term, defines the relationship between speed and density at equilibrium states. A general expression proposed by May and Keller [4] was adopted in this study as follow:

$$v_e(\rho) = v_f\left[1 - \left(\frac{\rho}{\rho_{jam}}\right)^a\right]^b, \quad (4)$$

where ρ_{jam} is the jam density, v_f is free-flow speed and a, b are sensitivity factors which are positive numbers. In this study, the identification of these parameters was treated as another problem and estimated separately from the macroscopic parameters. The values of v_f = 113 kph, ρ_{jam} = 160 vpk/lane, a = 1.46, and b = 6.03, were accepted with 10 percent of significance level.

2.3 Study Road Section

Traffic data in this study were collected from outbound direction of the Second Stage Expressway of Bangkok between Vichaiyut Hospital and Kasemrat Hospital on Wednesday, December 3, 1998. The total length of study area is about 6.0 Kilometers. The traffic volume and spot speed at the entrance and the exit were observed by video camera, whereas the ramp data were observed manually. Two sets of data were collected during both peak hour and off-peak (14:00 to 15:30 hrs and 16:30 to 18:00hrs) to cover the traffic situations of both peak and off-peak periods. As shown in Fig.1, the road section with three lanes was divided into 9 subsections ranging from 300 meters to 800 meters.

Keywords: Parameter Identification, Macroscopic Model, FRESIM, Box Complex, Kalman Filtering

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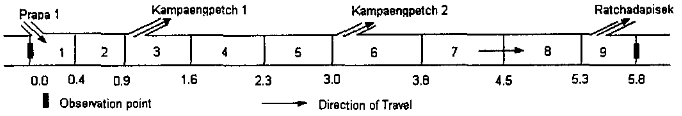


FIG.1 Model of Study Road Section

3. PARAMETER ESTIMATION TECHNIQUE

3.1 Nonlinear Least Square Method (NLT)

The objective function J was set as the error between observed variables and model outputs:

$$J = \sum_{i=1}^{n}\left(\gamma_q \cdot (q_i - \hat{q}_i)^2 + \gamma_w \cdot (w_i - \hat{w}_i)^2\right) \tag{5}$$

To minimize the error $\frac{\partial J}{\partial \beta_m} = 0$,

must be satisfied, where β_m is the model parameter of τ, ν, κ, and α, and γ_q, γ_w are the adjustment factors of both volume and the speed errors. Normally, the reciprocals, $1/\sigma_q^2$ and $1/\sigma_w^2$, are used. The objective function is nonlinear with respect to the model parameters. Applying the Taylor Expansion to the objective function,

$$\hat{\beta} = \hat{\beta}_0 - \left[\nabla^2 \hat{\beta}_0\right]^{-1} \nabla \hat{\beta}_0 \tag{6}$$

or

$$\begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \vdots \\ \beta_p \end{bmatrix} = \begin{bmatrix} \beta_{10} \\ \beta_{20} \\ \beta_{30} \\ \vdots \\ \beta_{p0} \end{bmatrix} - \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1p} \\ S_{21} & S_{22} & & \\ \vdots & & \ddots & \\ \vdots & & & \\ S_{p1} & \cdots & \cdots & S_{pp} \end{bmatrix}^{-1} * \begin{bmatrix} s_{1y} \\ s_{2y} \\ \vdots \\ \vdots \\ s_{py} \end{bmatrix} \tag{7}$$

where

$$s_{my} = \frac{\partial J}{\partial \hat{\beta}_m} = \sum_{i=1}^{n}\left(-2\gamma_q \cdot (q_i - \hat{q}_i) \cdot \frac{\partial q_i}{\partial \beta_m} - 2\gamma_w \cdot (w_i - \hat{w}_i) \cdot \frac{\partial w_i}{\partial \beta_m}\right) \tag{8}$$

$$S_{ml} = \frac{\partial^2 J}{\partial \hat{\beta}_m \partial \beta_l} = \sum_{i=1}^{n}\begin{pmatrix} 2\gamma_q \cdot \left(\frac{\partial q_i}{\partial \beta_m} \cdot \frac{\partial q_i}{\partial \beta_l} - (q_i - \hat{q}_i) \cdot \frac{\partial^2 q_i}{\partial \beta_m \partial \beta_l}\right) \\ + 2\gamma_w \cdot \left(\frac{\partial w_i}{\partial \beta_m} \cdot \frac{\partial w_i}{\partial \beta_l} - (w_i - \hat{w}_i) \cdot \frac{\partial^2 w_i}{\partial \beta_m \partial \beta_l}\right) \end{pmatrix} \tag{9}$$

i represents observation data at each time step and m, l *indicate the individual unknown parameters. Iterations* are repeated until the changes of unknown parameters are small enough or no more improvement in the correlation between model variables.

3.2 Box Complex Technique (BCT)

This method is a random search technique, which has proven effective in solving problem with nonlinear objective function subject to non-linear inequality constraints. The procedure should tend to find the global maximum due to the fact that the initial set of points is randomly scattered throughout the feasible region [3].

Unlike NLT, Box Complex algorithm does not require any derivatives. Even so, it is still not easy to decide whether the global optimum is reached. It needs to repeat the procedure while changing initial values.

3.3 Kalman Filter Technique (KFT)

KFT has a potential for future traffic control systems because it can estimate traffic states in real time based on a feedback concept without any driver's behavior model. The parameter identification problem can be integrated into state estimation problem [6] by treating the parameters as another set of state variables in the state equation. First, formulate Kalman filter, Eqs. 1 and 2 were treated as state equations, while Eq. 3 as the observation equation along with the following equation:

$$w_j(k) = \alpha v_j(k) + (1-\alpha) v_{j+1}(k) \tag{10}$$

where parameter α is the same as Eq. 1. In addition, the white noise errors were induced in both macroscopic model formula and measurement process. Thus, the state equation becomes as follow:

$$\mathrm{x}(k+1) = \mathrm{f}[\mathrm{x}(k)] + \Gamma\varphi(k) \tag{11}$$

For the observation equation,

$$\mathrm{y}(k) = \mathrm{g}[\mathrm{x}(k)] + \psi(k) \tag{12}$$

where

$$\mathrm{x}(k) = (\rho_1, v_1, \ldots \rho_i, v_i, \ldots \rho_n, v_n, \tau, \nu, \kappa, \alpha)^T_{(k)} \tag{13}$$

$$\mathrm{y}(k) = (q_0, w_0, q_n, w_n)^T_{(k)} \tag{14}$$

$$\varphi(k) = (\varphi_1^\rho, \varphi_1^v, \ldots\ldots, \varphi_n^\rho, \varphi_n^v)^T_{(k)} \tag{15}$$

$$\psi(k) = (\psi_0^q, \psi_0^w, \psi_n^q, \psi_n^w)^T_{(k)} \tag{16}$$

$$\Gamma = diag\left(\frac{\Delta t}{\Delta L_1}, 1, \ldots\ldots, \frac{\Delta t}{\Delta L_n}, 1\right)^T_{(k)} \tag{17}$$

$\varphi(k)$ and $\varphi(k)$ are referred as to modeling errors and measurement errors, respectively. Finally linearize the state and observation equation around the nominal solution, $\bar{x}(k)$ using Taylor's expansion.

$$\bar{\mathrm{x}}(k+1) = \mathrm{f}[\hat{\mathrm{x}}(k)] + \frac{\partial \mathrm{f}}{\partial \mathrm{x}}(\mathrm{x}(\mathrm{k}) - \hat{\mathrm{x}}(k)) + \Gamma\varphi(k) = \mathrm{A}(k)\mathrm{x}(k) + \mathrm{b}(k) + \Gamma\varphi(k) \tag{18}$$

$$\bar{\mathrm{y}}(k) = \mathrm{g}[\mathrm{x}(k)] + \frac{\partial \mathrm{g}}{\partial \mathrm{x}}(\mathrm{x}(\mathrm{k}) - \bar{\mathrm{x}}(k)) + \psi(k) = \mathrm{C}(k)\mathrm{x}(k) + \mathrm{d}(k) + \psi(k) \tag{19}$$

where

$$\mathrm{b}(k) = \mathrm{f}[\hat{\mathrm{x}}(k)] - \frac{\partial \mathrm{f}}{\partial \mathrm{x}}\hat{\mathrm{x}}(k) \tag{20}$$

$$\mathrm{d}(k) = \mathrm{g}[\bar{\mathrm{x}}(k)] - \frac{\partial \mathrm{g}}{\partial \mathrm{x}}\bar{\mathrm{x}}(k) \tag{21}$$

$$A(k)=\frac{\partial f}{\partial x}=\left[\frac{\partial(\rho_1,v_1,\ldots\rho_n,v_n)_{(k+1)}}{\partial(\rho_1,v_1,\ldots\rho_n,v_n,\tau,\nu,\kappa,\alpha)_{(k)}}\right]_{(2n+4)\times(2n+4)} \quad (22)$$

$$C(k)=\frac{\partial g}{\partial x}=\left[\frac{\partial(q_0,w_0,q_n,w_n)_{(k)}}{\partial(\rho_1,v_1,\ldots\rho_n,v_n,\tau,\nu,\kappa,\alpha)_{(k)}}\right]_{(4)\times(2n+4)} \quad (23)$$

$\tilde{x}(k)$ is the estimated state vector before observing new data, $\mathbf{y}(k)$. $\hat{x}(k)$ is the updated vector after obtaining actaul measurement variables, $\mathbf{y}(k)$. Φ and Ψ are the error covariance matrix of state equations and of observation equations, respectively.

4. CALIBRATION AND VALIDATION

To obtain reliable model parameters robust for various traffic conditions, the parameters have to be identified for extensive traffic situations. It is almost impossible to obtain such data only from actual fields. It requires vast efforts for data collection and compilation. In this study, the traffic data were generated by TRAF-FRESIM. The field data collected at the study area during a time period were used as input for FRESIM. The outflow volume, and spot speed at the exits were used to calibrate the influential parameters of FRESIM. The parameters calibrated are:

- Free-flow speed: 112 kph
- Parameter for collision avoidance time period: 1
- Minimum separation for generation of vehicles: 1.7 tenths of a second

After the parameters were justified for the traffic data measured during another time period, extensive traffic data were produced using FRESIM by changing inflow volumes at entrances.

5. NUMERICAL EXPERIMENTS

Three cases are examined:

1) Case 1: Off-peak period: Similar to traffic data observed during 14:10 to 14:30 hrs.
2) Case 2: Peak period: Similar to traffic data observed during 16:40 to 17:00 hrs.
3) Case 3: Smooth traffic situation with inflow volume between 4800 to 5200 vph.

The results by simulation runs of macroscopic model with a certain parameter sets estimated by the three techniques were compared with the real. As the statistics to evaluate each method quantitatively, the objective function J and root mean square of error (RMSE) of speed and volume were calculated.

5.1 Comparison between NLS and BCT

In NLT, the model parameters were estimated with changing the initial values randomly in 50 sets. τ and ν were constrained in the range from 0 to 9999, κ was from 0 to 200, and α was from 0 to 1, respectively. In BCT, three sets of initial parameters shown in Tables 1 were first applied. They are coming from Cremer [2], Papageorgiou [5], and Cremer and May [1], respectively. Furthermore, different numbers of complex points, which yielded 10, 25, and 50 sets of initial values, were also examined. The estimated parameters for each case are summarized in Tables 2.

TABLE 1 Initial Value for BCT

No.	τ	ν	κ	α
1	34.0	21.6	20.0	0.80
2	72.0	28.0	40.0	0.80
3	20.4	23.9	40.0	0.95

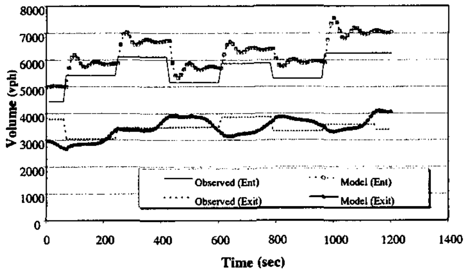
TABLE 2 Average Parameter Estimated by NLT and BCT

	Case 1		Case 2		Case 3	
Technique	NLT	BCT	NLT	BCT	NLT	BCT
τ (sec)	20.41	8.12	518.47	242.51	301.26	419.93
ν (km^2/hr)	189.41	19.53	1565.40	630.12	1662.08	740.00
κ (vkp)	200.0	17.3	200.0	150.9	200.0	13.9
α	1.000	0.888	0.961	1.000	0.963	0.989
J	60.57	55.50	153.60	140.27	13.99	10.13
$RMSE_q$ (vph)	323.23	312.51	572.07	544.04	68.18	45.27
$RMSE_w$ (kph)	3.85	3.65	5.59	5.37	1.29	1.15

In NLT, different initial values often resulted in absolutely different solutions for all three cases. In other words, the parameters strongly depended on the initial values. Furthermore, little improvement was gained even if the program started with different initial values. This seems to be because the objective function is nonlinear and has a lot of extreme values. Due to the nature of NLT based on derivatives, it is very difficult to escape from a local minimum once entrapped.

As shown in Table 2, BCT produced better estimates for all 3 indices, including objective function (J), RMSE of volume and spot speed, than NLT in all cases. The initial values had small effect on the final solutions because BCT has such a mechanism that generates a number of random points automatically with avoiding a local minimum. Consequently, the method successfully yielded the parameters that were substantially different from the initial values. The calculation process of BCT is much simpler than NLT because it does not require any derivative and matrix operations as NLT. In addition, Table 2 indicates that BCT was effective in estimating the parameters for the off-peak of Case 1 and smooth traffic state of Case 3.

However, the estimation is not successful in Case 2. Fig.2 shows the variation of traffic volume and spot speed predicted by the macroscopic model with the parameters by BCT for case 2. Although the estimated speed and volume approximately follow observed one on the average, the variation in the short term is still large. As shown in the spot speed of Fig.2, there was a sudden speed drop around 600 seconds. In other words, traffic situations became congested after the time point. To treat this phenomenon more precisely, the data set of Case 2 was divided into two parts; before and after the abrupt change of speed. And then the parameters were identified separately. Table 3 exhibits the parameters for each time period. With being aggregated for both periods, the separation was effective in improving both the objective function and the RMSE of spot speed.



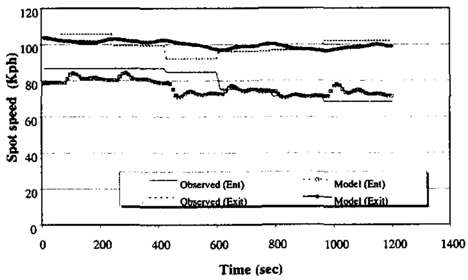


FIG. 2 Volume and Spot Speed Estimated by Optimum Parameters Using BCT for Case 2

TABLE 3 Parameters before and after Speed Change (Case2)

Period	first 600 sec	last 600 sec	Aggegate
τ (sec)	637.62	95	
ν (km^2/hr)	1901.79	340.93	
κ (vkp)	200.0	200.0	
α	1.000	1.000	
J	78.27	49.28	127.55
$RMSE_q$ (vph)	521.53	568.30	545.42
$RMSE_w$ (kph)	6.17	2.96	4.840

5.2 Kalman Filtering Technique

The results in the previous section suggest that the parameter should be identified in accordance with traffic situations. That is, the real time estimation of model parameters may work well in the ultimate sense. The KFT simultaneously estimates model parameters as well as traffic state variables in real time manner. The state variables and model parameters are adjusted every time step so that the difference between estimated and observed measurement variables should be minimized.

Table 4 presents the value of objective function and the RMSE for Case 2 using the KFT with the initial values estimated by BCT, as shown in Table 2. Unfortunately, KFT was not effective in improving any indices. The deficiency may have arisen from some causes. First of all, it may be caused by the spatial variation of the traffic situation. In this study, the observation points are only at the entrance and the exit of the road section with locating far away each other. The traffic situations in the intermediate points were not considered. Next, generally in KFT, the statistics of noise variables have significant effect on the estimation precision. In this study, arbitrary values were assigned to them without the validation of actual data.

TABLE 4 Performance Indices for Both BCT and KFT

Parameters	J	$RMSE_q$	$RMSE_w$
Initial Condition (Optimized by Box Complex)	140.23	544.13	5.37
Estimated by Kalman filtering	170.28	558.49	6.31

6. CONCLUSIONS

The BCT was superior to the NLT in estimating the macroscopic model parameters. It provided better results and required less computation effort. The NLT failed to estimate the parameters if appropriate initial values were not given. The BCT provided further improved outcomes for the transient situation by identifying the parameters depending on the traffic condition. The dynamic technique such as KFT should be applied to this problem. Anyhow, the program in this study has to be modified in its formulation: Consideration should be made on what parameters should be included into the state and observation equations, how to adjust the error matrix, and how to deal with the parameter constraints. And the traffic data from intermediate segments should be considered. Furthermore, the mathematical relationship of the model parameters with traffic condition, such as density, and the relationship among model parameters should be investigated.

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