ESTIMATION OF TRAFFIC STATES OF AN UNBAN ROAD NETWORK USING A KALMAN FILTERING TECHNIQUE

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1. INTRODUCTION

So far many simulation models are used to estimate traffic states on an urban network. However, most of them are feed-forward models, in other words, they do not consider the potential errors and biases between the estimated traffic conditions and the actual traffic conditions measured by detectors. In Japan, different from western countries, traffic detectors are densely installed on arterial roads. Although the traffic data measured by detectors reflect the change of traffic situations immediately and exactly, very few traffic simulation models utilize the detector data successfully in the current surveillance systems.

Kalman filter is one of feedback Approaches [1, 2] that can extract the information that traffic detectors have more efficiently and effectively. Cremer [1] proposed a well-known Kalman filtering method to estimate the traffic states on freeway by defining a macroscopic traffic flow model as state equations and describing a relationship between the state variables and measurement variables as observation equations. However, the method that was developed focussing on freeway systems cannot be applied to urban networks because the traffic flows on urban roads are quite different from those on freeways due to the existence of intersections. The state equations should be modified so as to take turning movements and signal timings at intersections into account.

Another promising aspect of Kalman filtering approach is that it can account for the influence of traffic flow phenomena, which are specific to urban traffic, such as in-and out-flows to/from side roads, parking vehicles, and pedestrians, indirectly though noise terms in state and observation equations. These traffic phenomena are too complicated to be integrated directly into a macroscopic model that is oriented to the application for large road network.

This study aims to develop a feedback model to estimate traffic states on an urban road network by using a Kalman filtering method:

1) Modify an existing continuum model that was developed for freeway to describe urban traffic more accurately. In particular, incorporate the traffic phenomena at intersections into the traffic state estimation equations.
2) Define the observation equation for urban traffic that relates the state variables to measurement variables. In particular, the observation equation should represent the relationship between signal timings and traffic flow variables.
3) Propose how model parameters should be calibrated and how they should be validated.
4) Investigate the sensitivity of noise-covariance matrix of system equation and observation equation using in Kalman filter method to correct the estimated traffic states.

2. MODEL FORMULATION

2.1 Macroscopic Traffic Flow Model

The urban networks in this study are two-way streets with different turning movements as shown in Figure 1. The urban street is decomposed into a set of unidirectional links to represent the streets between two successive intersections and nodes to represent the intersections. Each link is subdivided into a few segments with suitable length. Define the traffic variables of each segment as follows:

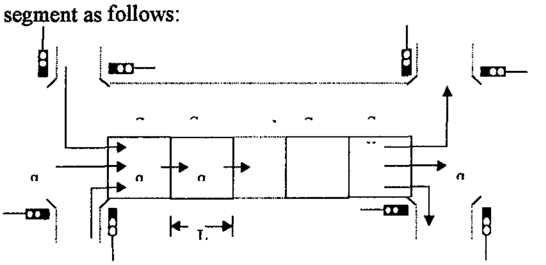

Figure 1. Urban Road Network

Notation

$c_i^l(k)$ traffic density in segment i link l at time k

$v_i^l(k)$ space mean speed in segment i link l at time k

$q_i^l(k)$ traffic volume leaving segment i link l

$w_i^l(k)$ time mean speed in segment i link l

ΔL_i^l length of segment i in link l

Δt time interval

State Equation

According to Cremer [3], suppose that the density and space mean speed are described as

$$
\begin{aligned}
c_i^l(k+1) &= c_i^l(k) + \frac{\Delta T}{\Delta L_i^l}\left[q_{i-1}^l(k) - q_i^l(k)\right] \\
v_i^l(k+1) &= \beta v_i^l(k) + (1-\beta) V\left(\bar{c}(k)\right) \\
\text{where } \bar{c}(k) &= \alpha c_i^l(k) + (1-\alpha) c_{i+1}^l(k+1)
\end{aligned}
\tag{1}
$$

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Observation Equation

The relation between the state variables and the measurement variables are assumed to be

$$\begin{aligned} q_i^l(k+1) &= c_i^l(k+1)v_i^l(k+1) \\ w_i^l(k+1) &= v_i^l(k+1) \end{aligned} \tag{2}$$

Different from freeway networks, the flow rate in the state and observation equations for urban network have to be modified in accordance with where the segment is:

- First Segment of Entry Link
  The flow rate $q_0^{\cdot}(k)$ entering to this segment has to be the inflow volume of the network.
- Last Segment of Entry and Intermediate Links
  The flow rate $q_n^{\cdot}(k)$ leaving from this segment has to be a function of signal timing to account for the flow interruption at signalized intersections.
- First Segment of Intermediate Link and Exit Link
  The entering flow rate of this segment is composed of the flow rate from three different direction (i.e. left, through and right) of intersection segments which enter link l.
- Last Segment of Exit Links
  The flow rate at exit is not influenced by traffic signals and the capacity of downstream segment is large enough to store the outflows.
- Segments in Intersections
  One segment will be allocated to represent the traffic flow within intersections in accordance with the turning movements as shown Figure 2. To simplify the analysis of the macroscopic model, traffic phenomena, such as on-street parking, in and out-flows from/to side roads, and bus/contra-flow lanes, are not considered.

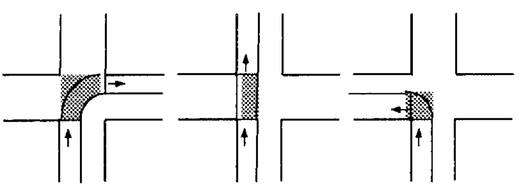


Figure 2. Segments for Turning Movements

Shock Wave

To represent the interfering between queues and flow from upstream, the concept of shock wave is used. It describes how the queues propagate and dissipate after traffic signal turns green.

Spill-Over and Under-Flow Estimates

In actual numerical calculation, it can happen that the density is greater than the jam density or less than zero. In fact, the density that exceeds the maximum must be adjusted to upstream segments so that the spill-over occurs and the density that is less than zero must be adjusted so as to be absorbed in the downstream segments:

$$\begin{aligned} \text{if} \quad & c_i^{l_i}(k) > c_{max} \\ & c_{i-1}^{l_i}(k) = c_{i-1}^{l_i}(k) + \left(c_i^{l_i}(k) - c_{max}\right) \\ & c_i^{l_i}(k) = c_{max} \end{aligned} \tag{3}$$

$$\begin{aligned} \text{if} \quad & c_i^{l_i}(k) < 0 \\ & c_{i+1}^{l_i}(k) = c_{i+1}^{l_i}(k) + c_i^{l_i}(k) \\ & c_i^{l_i}(k) = 0 \end{aligned} \tag{4}$$

2.2 Kalman Filter

By choosing density and space mean speed as state variable vector $x(k)$ and flow rate and time mean speed as observation variable vector $y(k)$, the macroscopic model can be rewritten as follows:

State equation

$$x(k+1)=f(x(k), u(k))+Qq(k)+\Gamma\varphi(k) \tag{5}$$

Observation equation

$$y(k)=g(x(k), u(k))+\zeta(k) \tag{6}$$

where

$$\begin{aligned} x^T(k) = [&c_1^{l_1}, v_1^{l_1}, \ldots, c_n^{l_1}, v_n^{l_1}, {}_i c_r^{l_1 l_i}, {}_i v_r^{l_1 l_i}, {}_i c_{th}^{l_1 l_i}, {}_i v_{th}^{l_1 l_i}, \ldots, \\ &c_n^{l_n}, v_n^{l_n}, {}_i c_r^{l_n l_{ii}}, {}_i v_r^{l_n l_{ii}}, {}_i c_{th}^{l_n l_{ii}}, {}_i v_{th}^{l_n l_{ii}}]_{(k)\ [1xN]} \end{aligned} \tag{7}$$

$$\begin{aligned} y^T(k) = [&q_i^{l_i}, w_i^{l_i}, q_i^{l_{ii}}, v_i^{l_{ii}}, \ldots, \\ &q_i^{l_{n-1}}, w_i^{l_{n-1}}, q_i^{l_n}, w_i^{l_n}]_{(k)\ [1xn]} \end{aligned} \tag{8}$$

$$\begin{aligned} \Gamma = \mathrm{diag}[&\frac{\Delta t}{\Delta L_1^{l_1}}, 1, \ldots, \frac{\Delta t}{\Delta L_n^{l_1}}, 1, \ldots, \\ &\frac{\Delta t}{\Delta L_n^{l_n}}, 1, \frac{\Delta t}{{}_i\Delta L_r^{l_n l_{ii}}}, 1, \frac{\Delta t}{{}_i\Delta L_{th}^{l_n l_{ii}}}, 1]_{[NxN]} \end{aligned} \tag{9}$$

$u(k)$ control vector assigned by traffic signal light
$q(k)$ inflow volume from inflow links
$\mathbf{Q}$ coefficient matrix of inflow volume
$\Gamma\varphi(k)$ noise matrix representing modeling errors
$\zeta(k)$ noise matrix representing measurement errors
N total number of segments of whole network
n number of observation points of whole network

In this study, the extended Kalman Filtering technique is applied to develop a non-linear estimation model. By linearizing Eqs. (7) and (8) around the nominal solution $x^*(k)$, the state equation and observation equation reduce to:

$$\begin{aligned} \tilde{x}(k) = &\mathbf{A}(k)\hat{x}(k-1) + \mathbf{B}(k-1)u(k-1) \\ &+ d(k-1) + \mathbf{Q}q(k-1) + \Gamma\varphi(k-1) \end{aligned} \tag{10}$$

$$\tilde{y}(k) = \mathbf{C}(k)\tilde{x}(k) + \mathbf{D}(k)u(k) + e(k) + \zeta(k) \tag{11}$$

where

$\tilde{x}(k)$, $\tilde{y}(k)$ one step predictor of $x(k)$, $y(k)$
$\hat{x}(k-1)$ filtered estimate of $x(k-1)$

$$A(k)=\frac{\partial f}{\partial x}\bigg|_{\substack{x(k)=\bar{x}(k)\\ u(k)=u_0(k)}} \qquad B(k)=\frac{\partial f}{\partial u}\bigg|_{\substack{x(k)=\hat{x}(k)\\ u(k)=\hat{u}_0(k)}}$$

$$C(k)=\frac{\partial g}{\partial x}\bigg|_{\substack{x(k)=\bar{x}(k)\\ u(k)=\bar{u}_0(k)}} \qquad D(k)=\frac{\partial g}{\partial u}\bigg|_{\substack{x(k)=\bar{x}(k)\\ u(k)=\bar{u}_0(k)}} \tag{12}$$

It should be noted here that the derivation requires lots of carefulness because different state and observation equations have to be chosen in accordance with signal timing, lane configuration, and whether detectors are installed or not. Calculating $\mathbf{A}(k)$, $\mathbf{B}(k)$, $\mathbf{C}(k)$ and $\mathbf{D}(k)$ step by step, state variables can be corrected as follows:

$$\hat{x}(k+1)=\tilde{x}(k)+\mathbf{K}(k)[y(k)-\tilde{y}(k)] \tag{13}$$

where

$$\begin{aligned} \mathbf{K}(k) &= \mathbf{M}(k)\mathbf{C}^{T}(k)\left[\mathbf{C}(k)\mathbf{M}(k)\mathbf{C}^{T}(k)+\mathbf{W}(k)\right]^{-1} \\ \mathbf{P}(k) &= \mathbf{M}(k)\text{-}\mathbf{K}(k)\mathbf{C}(k)\mathbf{M}(k) \\ \mathbf{M}(k+1) &= \mathbf{A}(k)\mathbf{P}(k)\mathbf{A}^{T}(k)-\mathbf{V}(k) \end{aligned} \tag{14}$$

$\mathbf{V}(k)$, $\mathbf{W}(k)$ are noise-covariance matrix of system equation and observation equation, which can be defined as follows:

$$\begin{aligned} \mathbf{V}(k) &= \Gamma\Phi(k)\Gamma' \\ \mathbf{W}(k) &= \mathbf{E}\left[\zeta(k)\zeta^{T}(k)\right] \\ \Phi(k) &= \mathbf{E}\left[\varphi(k)\varphi^{T}(k)\right] \end{aligned} \tag{15}$$

The one-step prediction, $\mathbf{x}(k)$ and $\mathbf{y}(k)$, are directly calculated by using the macroscopic model. $\mathbf{V}(k)$ and $\mathbf{W}(k)$ were assumed constant over the whole time period. Also, the signal control scheme have no influence on derivative matrixes $\mathbf{A}(k)$ and $\mathbf{C}(k)$ since it is assumed to be pre-time.

3. MODEL CALIBRATION

Taking an isolated intersection as example, the performance of Kalman filtering method was examined. The data in this study was produced by TRAF-NETSIM simulation software package [4]. Traffic detectors were virtually installed at all entrances and exits. The flow rate measure at entrances were used as inflow volumes and the spot speed as well as the flow rate measured at exits were used as measurement variables in the method.

Firstly, the influential model parameters, such as free flow speed, saturation headway, and start-up lost time, of TRAF-NETSIM were calibrated by traffic data actually measured at an intersection in Bangkok [5]. Then, two sets of simulation data were created by TRAF-NETSIM; one for calibration of the macroscopic model and another for verification of the new model. The inflow volumes at entrances were varied from 1100 to 1300 vph. For simplicity, the inflow volume assumed to be same for all entrances. Since the percentage of heavy vehicle was less than 4% in the actual traffic data, only passenger cars were considered in the simulation by TRAF-NETSIM.

Table 1 shows the simulation conditions and the parameters calibrated by a set of traffic data. A t-test of stop delay for another data set justified those parameters.

Table 1. Simulation Conditions and TRAF-NETSIM Parameters Calibrated

Parameter	Remarks
Simulation Period	30 minutes
Inflow Volume	1100, 1200, 1300 vph.
Traffic Composition	Passenger car 100%
Traffic Signal Scheme	Pre-timed with 4 phases
Free Flow Speed	64 kph
Saturation Headway	2.2 sec.
Start-up Lost Time	2 sec.

The density vs. speed curve and weighting factors α and β in Eq.(1) are significant model parameters in the macroscopic model. Figire 3 depicts the density vs. speed curve identified by the first simulated data. Both α and β were optimized as 0.4 in this case.

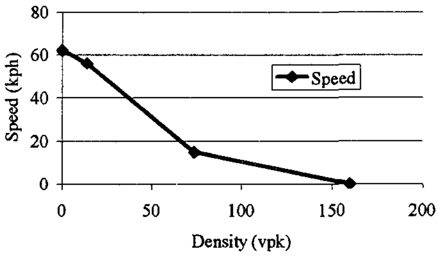


Figure 3. Density vs. Speed Curve Identified

4. NUMERICAL EXPERIMENTS

Using the second simulated data produced by TRAF-NETSIM, which were not used for calibrating the model parameters, it was investigated how Kalman filter contributed to improve the estimation precision and how sensitive the noises in both state and observation equations are on the estimation precision.

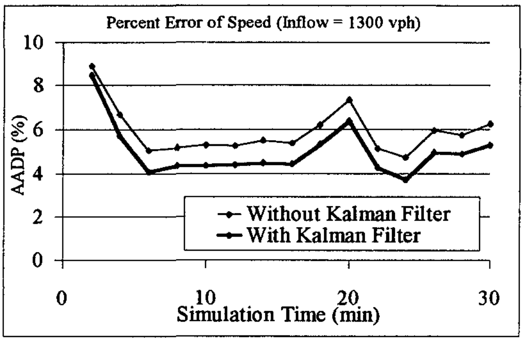
4.1 Model Verification

The road network of isolated intersection analyzed in this study has eight exit links. The traffic flow rate and spot speed estimated by the Kalman filtering method were compared with those simulated by TRAF-NETSIM at the exit points of those links. An index of the average absolute percentage difference (AAPD %) is effective to evaluate the overall estimation precision:

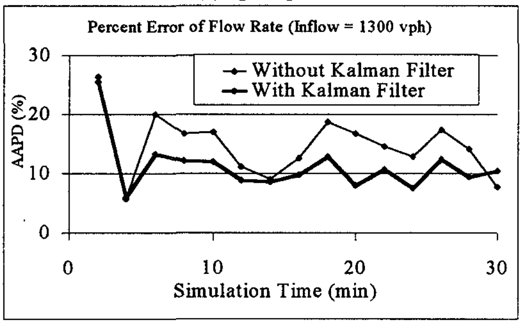
$$\text{AAPD(k)}=\frac{1}{\text{N}}*\sum_{\text{i=1}}^{\text{N}}\frac{\left|\text{Measured}_{\text{i,k}}-\text{Estimated}_{\text{i,k}}\right|}{\text{Measured}_{\text{i,k}}}\times 100(\%) \tag{15}$$

where N is the number of measurement points. The flow and was accumulated for 2 minute and the spot speed was also averaged over 2 minutes.

Figure 4 illustrates AAPD Error of a) spot speed and b) flow rate for inflow volume of 1300 vph. The Kalman Filtering technique is effective in improving the precision for both variables. For other inflow volumes of 1100 and 1200 vph,

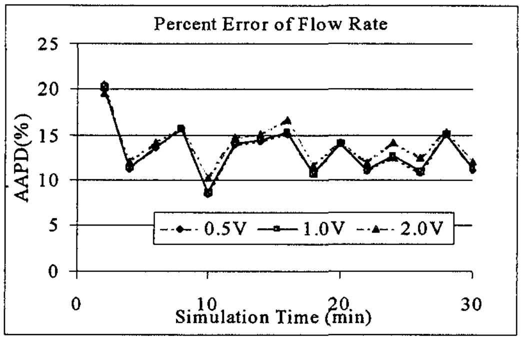


(a) Spot Speed

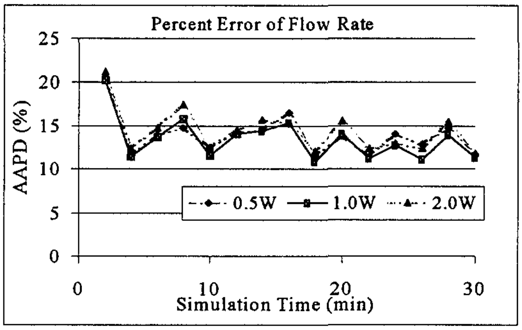


(b) Flow Rate

Figure 4. With and Without Kalman Filter



(a) State Variable Noise



(b) Measurement Variable Noise

Figure 5. Effects of Noise on Estimation Precision.

4.2 Sensitivity of Noises

In the previous analyses, the elements of noise matrices were determined using the calibration data so that the difference between the estimated and simulated variables should be minimized on a trail and error basis. Figure 5 exhibits how the noises in a) the state and b) the observation equations affect the estimation precision of flow rate. For simplicity, all elements in both **V** and **W** matrices were multiplied by 0.5 or 2.0. They show that the noises have little influence on the precision. This is because the road network analyzed here is very small, isolated intersection, and the detector data used here are coming from a software package, NETSIM, which is not contaminated with noises.

5. CONCLUSIONS

To improve the accuracy of estimation of traffic states on an urban road network, a new method, which is based upon the feedback concept of Kalman filtering technique, was proposed. First, a macroscopic model was modified to represent traffic flows at intersection. Next, the state and observation matrices that are necessary in the linearization process of Extended Kalman filter. The new method was applied to an isolated intersection. The comparison between the estimated and actual values for both traffic flow rate and spot speed justified the new method to be effective in improving the estimation precision by several percent. It was also analyzed how much the noises in the macroscopic model and the measurement equation affect on the precision.

The estimation method proposed here is still far from practical implementation. A lot of subjects remain unsolved: The method should be verified with actual field data. It should be applied to a large road network. The macroscopic model also should be modified so as to represent other traffic phenomena, such as lane changing, parking.

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