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Simple Spatial Interaction in Health-Care Facility Location-Allocation Model *

By Fauzy AMMARI**, Toshihiko MIYAGI***

1. INTRODUCTION

The costly and inefficient producing of completely services for health care in all demand nodes and its patient flows motivate us to study the facility location as a hierarchy. This study aims at developing a formulation and a simple solution procedure for location-allocation of hierarchical health care facilities. Generally, health service facilities with the large coverage are the health centers or the village clinics and the hospitals. We concerned with hospital (or general hospital) as an upper level and health center as a lower level. The key issue in health care facility that we are concerned with are: (1) where the facility should be located, (2) how to allocate demand nodes with a set of facility type, and (3) how the patients behavior based on spatial interaction effect works. Gore (1991) describes that it would involve specifying rules of access regarding eligibility, priority, and location of the service and relating those to the person's actual position in the system, including endowment, location, and a wide range of other phenomena relevant to access.

The problem is formulated as a minimization of total patients-weighted distance. Basically, this model is based on the hypothesis that the location and allocation are controlled by different decision makers. The locational configuration of hospitals and health centers are determined in relation to the consequences of hierarchy. The consequence to which we refer, in this case, is the ability of hospitals site to cover health centers site. Since the most important thing is to save patients as quickly as possible, critical coverage distance has sense of critical travel time for referral movements to the next treatment in a higher facility in emergency time. Thus, we are concerned not with facility placement disperse, but with the resulting coverage patterns of those placements.

The allocation of patients (nonemergency) is working based on user-attracting system. The user-attracting system denote the choice of facility is made by the user. Such systems should have a spatial interaction model to represent user choice behavior. In doing so, Leonardi (1981) describes two subsystems as follows. The first subsystem is the accessibility-sensitive demand mechanism. It depends on the location size, and proximity of the facilities. Usually, the demand will increase with the accessibility of the location. The second subsystem is the congestion-sensitive demand mechanism. Leonardi (1981) identifies that the congestion-sensitive demand mechanism receives as inputs the actual demand from each location, the location and size of facilities in each location, and the transport cost between demand and facility locations.

The results we expected in our model show the performance of users' behavior based on the accessibility criterion of spatial interaction (gravity) effect in choosing a site of hospital or health center to minimize the total patients weighted distance in a network. It will affect the precision of locating the state of these facilities in node. In application to a given network, in case of discrete candidate site and continuos candidate site, it needs a conditioned conventional heuristic algorithm because of the two type of facilities.

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** Graduate Student member of JSCE, Graduate School of Engineering, Gifu University, e-mail: fauzy@cc.gifu-u.ac.jp
*** Professor, Faculty of Regional Studies and Graduate School of Eng., Gifu University,1-1, Yanagido, Gifu-City 501-1193, Japan, FAX: +81-58-230-1528 e-mail: miyagi@cc.gifu-u.ac.jp

2. MODEL

The background of basic model is median model with an adjustment of the decision of allocation variable. This is done by changing the nature of allocation variable from a 0-1 to a stochastic variable, that is, the probability that the patients at node i attracts to a facility at j. The form for user-attracting model is as follows:

$$U_{kj} = G_k \frac{f(\beta, c_{kj}) H_j X_j}{\sum_j \left(f(\beta, c_{kj}) H_j X_j \right)} \qquad \forall k, j \tag{1}$$

where

k, j = *subscripts labeling the locations of health center site and hospital site;*

G_k = *total referral patients from health center site k;*

D_j = *attracted demand in hospital site j;*

H_j = *a measure of attractiveness of hospital in site j;*

c_{kj} = *the total cost associated with a displacement from health center site k to hospital site j, measured in appropriate units;*

β = *a spatial discount parameter* (≥ 0) *to be valued empirically;*

$f(\beta, c_{kj})$ = *a space discount function such as* $\exp(-\beta c_{kj})$ *(as used here) or* $c_{kj}^{-\beta}$, *which is strictly monotonically declining;*

U_{kj} = *the number of referral patients in health center site k allocate to hospital site j.*

Accessibility is a measure of user's benefit consistent with a spatial interaction behavior. Leonardi (1981) describes that according to most spatial interaction data, users seem to apply a definite distance-decreasing discount factor on facilities, the most natural measure of accessibility seems to be a sum of the capacity (or attractiveness) of all service facilities, each one discounted with its own space-discount factor. Accessibility is measured as:

$$A_i = \sum_j f(\beta, C_{ij}) H_j \ . \tag{2}$$

The simplest congestion-sensitive demand model is a linear feedback signal that is proportional to the difference between total capacity and attracted demand, which changes the value of the attractiveness weights for each facility. Its weight are evaluated according to the formula,

$$H_j = \alpha_j (Q_j - D_j) \tag{3}$$

where

Q_j = *the capacity of hospital in site j,*

D_j = *the total demand attracted in hospital site j, and*

α_j = *given constants, typical of each location.*

We are concerned with three kinds of patient flow, that is, the allocation to hospital or health center and the referral allocation from health center to hospital. Based on the definition of subsystems above, the overall location-allocation system can be assembled. It is an adjustment of Leonardi (1981) framework is shown in figure 1.

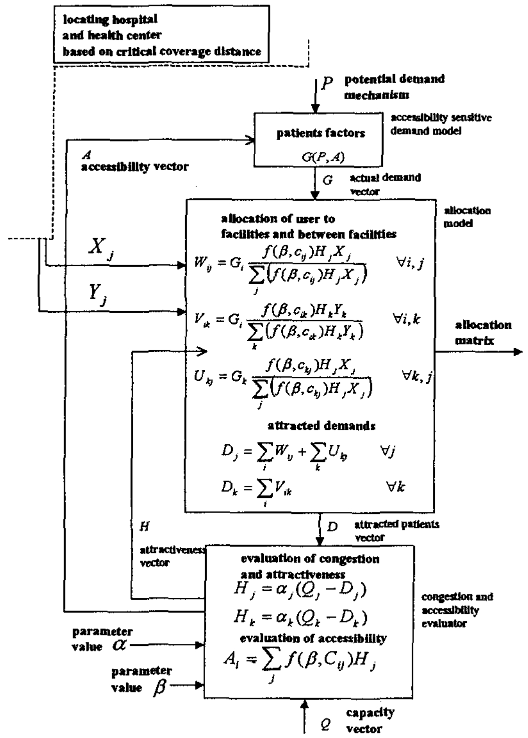

Figure 1. The structure of location-allocation model.

To formulate the model the following assumptions have been made in order to achieve a tractable problem structure. (1) We consider that a health care facility system consists of hierarchically coordinated two level facilities capable of exchanging patients, personnel, equipment and services. (2) each health center must be located within a unit critical coverage distance of the hospital. (3) The facility can be located anywhere on the network. (4) Users' behavior is based on spatial interaction (gravity) model. (6) It is use zero-one variables to model whether or not a discrete location is executed. Mathematical formulation and notations are written as follows:

P_h = *number of hospital to be located;*

P_c = *number of health center to be located;*

d_{ij} = *minimum total travel distance between node i and candidate site of hospital j;*

d_{ik} = *minimum total travel distance between node i and candidate site of health center k;*

l_{ik} = *minimum total travel distance between candidate site of health center k and hospital site j;*

θ = *rate of referral patients;*

G_i = *total patients approximatly in node i;*

G_k = *total referral patients from health center site k;*

D_j = *attracted demand in hospital site j;*

D_k = *attracted demand in health center site k;*

Input Variable

$$a_{kj} = \begin{cases} 1: \text{if candidate health center } k \text{ is within } D_{hc} \text{ distance unit of candidate site hospital } j \\ 0: \text{if not} \end{cases}$$

Decision Variables

$$X_j = \begin{cases} 1: \text{if candidate site of hospital is selected} \\ 0: \text{if not} \end{cases}$$

$$Y_k = \begin{cases} 1: \text{if candidate site of health center is selected} \\ 0: \text{if not} \end{cases}$$

W_{ij} = *allocation of patients at node i to hospital site j;*

V_{ik} = *allocation of patients at node i to health center site j;*

U_{kj} = *allocation of referralpatients at healthcenter to hospital site j.*

$$\text{MINIMIZE} \quad \sum_i \sum_k d_{ik} V_{ik} + \sum_i \sum_j d_{ij} W_{ij} + \sum_j \sum_k l_{kj} U_{kj} \quad (4)$$

$$\text{SUBJECT TO} \quad \sum_j X_j \le P_h \quad (5)$$

$$\sum_k Y_k \le P_c \quad (6)$$

$$l_{kj} a_{kj} \le D_{hc} \qquad \forall k,j \quad (7)$$

$$Y_k \le \sum_j a_{kj} X_j \qquad \forall k \quad (8)$$

$$W_{ij} - X_j G_i \le 0 \qquad \forall i,j \quad (9)$$

$$V_{ik} - Y_k G_i \le 0 \qquad \forall i,k \quad (10)$$

$$2U_{kj} - (Y_k + X_j) G_k \le 0 \qquad \forall j,k \quad (11)$$

$$W_{ij} = G_i \frac{f(\beta, c_{ij}) H_j X_j}{\sum_j \left(f(\beta, c_{ij}) H_j X_j \right)} \qquad \forall i,j \quad (12)$$

$$V_{ik} = G_i \frac{f(\beta, c_{ik}) H_k Y_k}{\sum_k \left(f(\beta, c_{ik}) H_k Y_k \right)} \qquad \forall i,k \quad (13)$$

$$U_{kj} = G_k \frac{f(\beta, c_{kj}) H_j X_j}{\sum_j \left(f(\beta, c_{kj}) H_j X_j \right)} \qquad \forall k,j \quad (14)$$

$$G_k = \theta G_i \qquad \forall i,k \quad (15)$$

$$Q_j \ge \sum_i W_{ij} + \sum_k U_{kj} \qquad \forall j \quad (16)$$

$$Q_k \ge \sum_i V_{ik} \qquad \forall k \quad (17)$$

$$X_j, Y_k = 0,1 \qquad \forall j,k \quad (18)$$

$$W_{ij}, V_{ik}, U_{kj} \ge 0 \qquad \forall i,j,k \quad (19)$$

The objective function (4) minimize the total patient (demand) weighted distance from a demand node to the health center or hospital and referral patients from health center to hospital. Constraint (5) stipulates that the hospital must be located no more than H hospitals. Constraint (6) states that no more than C health centers are to be located. Constraints (7) state the guarantee of critical coverage distance for the distance between the health center and the hospital. Constraints (8) state that at candidate health center k can be covered by more than one selected hospital. Constraints (9) state demand at node i can only be assigned to a hospital at candidate site j if we locate a hospital at candidate site j. Constraints (10) similar to constraints (9) state demand at node i can only be assigned to a health center at candidate site k if we locate a health center at candidate site k. Constraint (9) and (10) link the location variables and the allocation variables. Constraints (11) state the link of referral patients from a health center at candidate site k to a hospital at candidate site j. Constraints (12) state that allocation of patients at node i to hospital site j based on spatial interaction model. Constraints (13) are identical to constraints (12), ensure the allocation value to health center with spatial interaction. Constraints (14), which are identical to constraints (12) and (13), evaluate the allocation of referral patients at health center site k to hospital site j. Constraints (15) state the number of referral patients. Constraints (16) state the balance of capacity between hospital site j and the total allocation. Constraints (17) are similar to constraints (16) for health center. Constraints (18) and (19) are the integrality constraints, respectively.

3. NUMERICAL EXAMPLE

To illustrate the model, we consider a given network with total population 2000 as shown in figure 2. Network with 9 nodes and each node has demand in 100 unit. The number of hospital and health center to be located equal 2, respectively. The facilities will be located at the demand nodes, where one candidate site for locating one facility. The critical coverage distance is 35. The rate of patients in average are equal 0.5%. The referral patients rate is approximately equal 10% of the patients coming from each demand node.

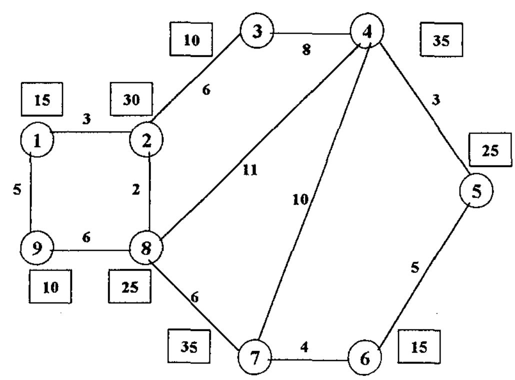

Figure 2. A given network.

Concern with the calculation of allocation variables, we assume the space discount function as exponential as follows

$$f(c_{ij}) = \exp(-\beta c_{ij}) \quad . \qquad (20)$$

It is similar to the space discount function of i to k and k to j. The parameter β is assume equal 0.1. The evaluation of congestion and attractiveness values are calculated based on equation (3) with the total attracted demand for hospital and health center in each candidate site is approximately equal 25 patients per day, respectively. The capacity of hospital and health center in each site are estimated equal 35 patients per day and 30 patients per day, respectively. The parameter α in each site of hospital and health center are equal 0.8 and 0.9, respectively.

The measure of accessibility to the service from a given demand location are dealt with equation (2). in order to evaluate the effect of accessibility, we establish three type value as shown as follows,

$$\text{Type I} \;<\; \text{type II} \;<\; \text{type III}. \qquad (21)$$

The procedure to calculate the model may be can be described as follows, at the first step, we concern with the problem of locating hospital and health center based on critical coverage distance, the second step, deal with calculation of allocation variables, that is, the decision is based on the evaluation of congestion and attractiveness, evaluation of accessibility, and depend to other parameter. Because this is one of a type of NP-Complete problem, the iteration work until find a criterion. Figure 3shows the outline of the procedure.

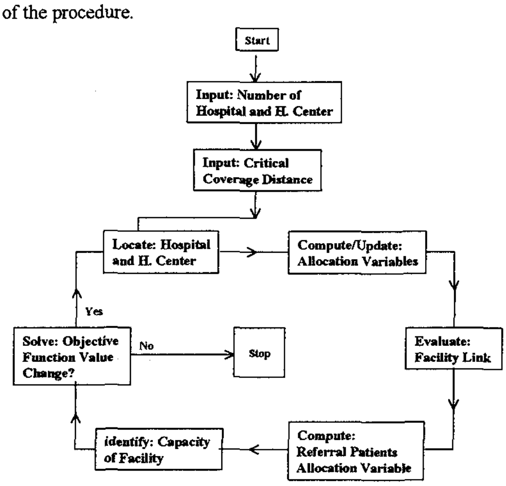

Figure 3. The outline of calculation procedure.

Figure 4 shows the three type allocations value for hospitals located at node 2 and node 5 and health centers located at node 1 and 8. The performance of accessibility type I is trial 4 time with the change of hospital site and health center site. For convenience, the candidate site of each type of facility could be described as a zero chain, which we can change the zero with X or Y.

Accesibility Type I

Trial No.	Site of Facility	Xj, Yk	to Hospital	to Health Center	Referral Patient	Total Obj.Value
1	oxooxoooo yoooooooyo	X2=1 X5=1 Y1=1 Y8=1	d22W22=15.84 d35W35=7.58 d45W45=27.33 d55W55=13.20 d65W65=19.24	d11V11=7.58 d78V78=53.08 d88V88=12.69 d91V91=12.55	l12U12=1.99 l82U82=3.15	174.80
2	oxooooxoo yoooooooyo	X2=1 X7=1 Y1=1 Y8=1	d22W22=15.34 d32W32=15.45 d47W47=88.81 d57W57=57.94 d67W67=15.49 d77W77=17.89	d11V11=7.58 d88V88=12.64 d91V91=12.55	l12U12=1.94 l82U82=1.27	246.86
3	oooxooxoo yoooooooyo	X4=1 X7=1 Y1=1 Y8=1	d44W44=18.23 d54W54=19.17 d67W67=15.23 d77W77=18.03	d11V11=7.58 d21V21=22.42 d38V38=20.08 d88V88=12.64 d91V91=12.55	l17U17=16.81 l87U87=5.35	167.87
4	oooxooxoo oyoooooyo	X4=1 X7=1 Y2=1 Y8=1	d44W44=18.03 d54W54=19.17 d67W67=15.23 d77W77=18.03	d12V12=11.33 d22V22=15.00 d32V32=15.13 d88V88=12.50 d98V98=15.11	l27U27=11.25 l87U87=5.35	156.10

Figure 4. The results for accessibility type I.

Accesibility Type II

Trial No.	Site of Facility	Xj, Yk	to Hospital	to Health Center	Referral Patient	Total Obj. Value
1	oxooxoooo yoooooyo	X2=1 X5=1 Y1=1 Y8=1	d22W22=16.12 d35W35=7.61 d45W45=27.69 d55W55=13.44 d65W65=19.41	d11V11=7.61 d78V78=54.08 d88V88=12.69 d91V91=12.56	l12U12=2.03 l82U82=3.20	176.44
2	oxooooxoo yoooooyo	X2=1 X7=1 Y1=1 Y8=1	d22W22=15.49 d32W32=15.60 d47W47=89.25 d57W57=58.50 d67W67=15.60 d77W77=18.02	d11V11=7.61 d88V88=12.69 d91V91=12.56	l12U12=1.96 l82U82=1.28	248.52
3	oooxooxoo yoooooyo	X4=1 X7=1 Y1=1 Y8=1	d44W44=18.20 d54W54=19.31 d67W67=15.30 d77W77=18.20	d11V11=7.61 d21V21=22.38 d38V38=20.10 d88V88=12.69 d91V91=12.56	l17U17=16.91 l87U87=5.38	168.66
4	oooxooxoo oyoooooyo	X4=1 X7=1 Y2=1 Y8=1	d44W44=18.20 d54W54=19.31 d67W67=15.30 d77W77=18.20	d12V12=11.36 d22V22=15.00 d32V32=15.15 d88V88=12.50 d98V98=15.15	l27U27=11.33 l87U87=5.38	156.89

Accesibility Type III

Trial No.	Site of Facility	Xj, Yk	to Hospital	to Health Center	Referral Patient	Total Obj. Value
1	oxooxoooo yoooooyo	X2=1 X5=1 Y1=1 Y8=1	d22W22=20.37 d35W35=8.06 d45W45=33.29 d55W55=16.97 d65W65=22.00	d11V11=7.64 d78V78=60.32 d88V88=13.43 d91V91=12.81	l12U12=2.07 l82U82=3.94	200.23
2	oxooooxoo yoooooyo	X2=1 X7=1 Y1=1 Y8=1	d22W22=15.56 d32W32=15.75 d47W47=89.69 d57W57=59.06 d67W67=15.75 d77W77=18.16	d11V11=7.64 d88V88=12.73 d91V91=12.58	l12U12=1.98 l82U82=1.28	250.18
3	oooxooxoo yoooooyo	X4=1 X7=1 Y1=1 Y8=1	d34W34=20.13 d44W44=18.37 d54W54=19.45 d67W67=15.38 d77W77=18.38	d11V11=7.64 d21V21=22.35 d88V88=12.73 d91V91=12.58	l12U12=4.36 l82U82=1.28	152.65
4	oooxooxoo oyoooooyo	X4=1 X7=1 Y2=1 Y8=1	d44W44=18.37 d54W54=19.45 d67W67=15.38 d77W77=18.37	d12V12=11.39 d22V22=15.00 d32V32=15.19 d88V88=12.50 d98V98=15.19	l27U27=11.41 l87U87=5.41	157.67

Figure 5. The results for accessibility type II and III .

Figure 5 shows the trial for accessibility type II and type III with site of hospital and health center same in each trial. In order to know the effect of accessibility, we record the values of allocation based on facility type and referral patients.

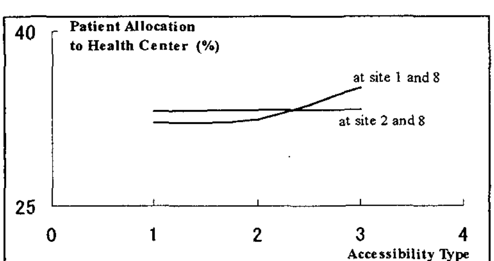

Figure 6. Patients allocated to health center Vs accessibility type.

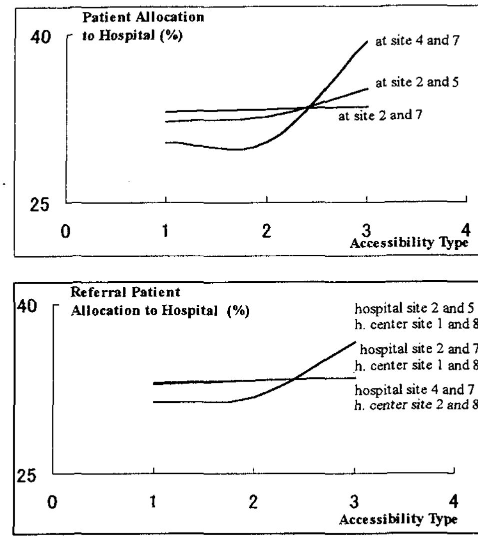

Figure 7. The performance of patients allocated to hospital and referral patients in each type of accessibility.

Figure 6 and 7 show that in some cases the flow of patients going to health care facilities increase in the high value of accessibility. This performance also shows that if the configuration of location site change the of the accessibility may be has no effect. Although, in this illustration, we are not search the minimum value of patient-weighted distance, but in changing configuration facility site the objective function values could be compared.

4. CONCLUSION

A spatial interaction model in choosing a site of hospital or health center has been combined with a simple hierarchical location model. Our model could be showing the performance of users-attracting based on the accessibility criterion of spatial interaction effect to minimize the total patients weighted distance. We find some effects in a simple numerical illustration with a simple algorithm. It is this observation that motivates the searching optimal solution that represent compromises between location decision maker and the behavior of users and their willingness to choose access.

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