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# Network Evolution with Cost-Benefit Evaluation Rules*

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## 1. INTRODUCTION:

Transportation networks that provide contacts, communications, and trade are an indispensable part of economies and our daily lives. The provision of higher quality transportation connections between economic regions changes the accessibility of cities to each other. As a result the change in the communication patterns between the cities, in terms of business trips, effects the overall economic geography of the whole city system. The investments of transportation, by their nature, are expensive, and short run and long run effects of the transportation investments on the society and the economy are complex and difficult to measure.

Cost-benefit analysis stands out as a simple yardstick that can be used to make decisions about the transportation investments. A higher quality transportation link provided between two nodes will lead to new opportunities of trade and other kinds of contacts. The purpose of this study is to investigate the evolution pattern of a network of cities under the cost-benefit evaluation rule.

## 2. SCOPE OF THE STUDY:

Networks are the media of contacts, communications, transport, and trade. The improvements in the transportation and communication networks, providing faster and cheaper movement of goods, people, and information, increase the interdependency among the regional economies. In this study we propose a multi-regional equilibrium model in order to provide some insights about the effects of cost-benefit rules on the evolution pattern of the network. Evolution of the network refers to the successive construction of new links, or improvement of the existing links that connect the cities. The cities are located at the nodes of the network and they are connected with transportation links to each other.

The improvement of a link between the two nodes will, of course, provide directly new opportunities and advantages of trade and other kinks of communications for those nodes, but also, indirectly, for the other pair of cities that can use this link for transportation. As a result, the improvement of a link will not only change the accessibility of these two nodes, but the accessibility of all of the cities and effect the overall economy of the network.

The construction cost of the link refers to the investment on the link improvement. The benefit provided by the link is calculated by the increase in the consumer surplus over the whole system. The new OD patterns due to the improvement of a link is calculated for all of the potential links and compared to the exiting OD pattern, and the difference between the two is assumed to be the benefit achieved from the improved link. The model is based on the city system equilibrium model with perfect population mobility and competition. Furthermore, land is incorporated into the growth model as a factor for housing consumption. The process of link improvement is carried out until all of the links are improved observing the evolution pattern of the network.

## 3. THE MODEL:

An economic system of $n$ cities, indexed by $i=1,2,..,n$, is considered, where the cities are connected by a transportation network. The economy produces one fixed product bundle consumed by the population. Perfect competition is assumed to prevail in good markets both within each city and between cities, and commodities are traded without any barriers such as transport costs. The population is homogenous and freely mobile among cities, whereas multi-habitation and inter-city commuting are not allowed. Furthermore, the total population of the whole system is given exogenously at any point in time, and is constant.

Each city is geographically monocentric, and consists of two parts, the central business district (CBD) and residential area. The city residents commute to the single CBD by intra-city railway systems, and pay for the commuting. For simplicity, we assume that each CBD is a point and all production activities are concentrated in the DBD of the respective cities. Production agents employ only the labor force, and local labor force is fully employed in their respective markets. Land is assumed to be owned publicly. There is no agricultural land available, which means that the land price is zero at the edge of the city. Residential area is divided into land lots, whose areas are fixes to the same size regardless of their location. The households achieve the same utility levels regardless of the location in which they live.

The central government controls the travel costs between cities by improving the existing transportation connections according to cost-benefit analysis. The government does not levy taxes on households and firms when doing this. The model depicts (a) the urban economics model to describe urban land use patterns of the respective cities, (b) the general equilibrium model to characterize the whole economy of the city system. The size, land use patterns, and production capacities of respective cities, the so-called economic geography, are endogenously calculated at each step given the spatial distribution of population of the whole economy [1].

### 3.1. The Urban Economic Model:

For the representative household, living at a distance $u_i$ from the CBD of city $i$, the composite commodity consumption, $x_i(u_i)$, is given by

$$x_i(u_i) = y_i - p_i(u_i) - c_i u_i \tag{1}$$

where $u_i$ is income $p_i(u_i)$ is the land rent per unit lot size at point $u_i$, and $c_i$ is the cost of commuting per unit distance

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that is constant everywhere in the city. The indirect utility function is supposed to be

$$V_i(u_i) = y_i - p_i(u_i) - c_i u_i \tag{2}$$

Since all households will get the same utility regardless of their location, the spatial conditions gives us $\partial V(u_i)/\partial u_i=0$. From equation (2) it can be seen that increased transport costs with increased commuting costs are offset by reduced rents, thus we have $\partial p_i(u_i)/\partial u_i=-c_i$. From the assumption, that there is no agricultural land use, there holds $p_i(L_i)=C_0-c_iL_i=0$ at the edge of the city where $u_i=L_i$. Integrating the equation $\partial p(u_i)/\partial u_i=-c_i$ we have the land gradient given by:

$$p_i(u_i) = c_i(L_i - u_i) \tag{3}$$

The utility level of the household at the city edge, $u_i=L_i$ is

$$V_i = y_i - c_i L_i \tag{4}$$

which is also equal to the utility level for all households regardless of their location within the city.

Given the fixed lot size over the economy, the size of city $i$ can be defined by the area of urban land use. Thus,

$$N_i = \int_0^{L_i} 2\pi u_i \, du_i = \pi L_i^2 \tag{5}$$

By integrating the individual consumption over the city population, the aggregate demand function, $F_i$, can be described by

$$F_i = \int_0^{L_i} 2\pi u_i x_i(u_i) du_i = N_i(y_i - c_i \pi^{-\frac{1}{2}} N_i^{\frac{1}{2}}) \tag{6}$$

Similarly we can get the aggregated land rents, $P_i$, and transportation costs, $T_i$, over the population of city $i$ by

$$P_i = \int_0^{L_i} 2\pi u_i p_i(u_i) du_i = \frac{1}{3} c_i \pi^{-\frac{1}{2}} N_i^{\frac{3}{2}} \tag{7}$$

$$T_i = \int_0^{L_i} 2\pi u_i^2 du_i = \frac{2}{3} c_i \pi^{-\frac{1}{2}} N_i^{\frac{3}{2}} \tag{8}$$

The equilibrium utility levels of the representative households can be fully characterized by the three parameters, $y_i$, $N_i$, and $c_i$:

$$V_i = y_i - c_i \pi^{-\frac{1}{2}} N^{\frac{1}{2}} \tag{9}$$

Let us next define the behavior of the firms. We will assume a constant return to scale technology and use the production function, $Y_i$, of the form

$$Y_i = N_i^{\alpha} \left\{ \sum_{j \neq i} N_j \left( \frac{R_{ij}}{N_j} \right)^{\xi} \right\}^{\gamma} \tag{10}$$

where $Y_i$ is total output, $R_{ij}$ is the inter-city communication between city $i$ and $j$, and $\alpha$, $\xi$, $\gamma$ are parameters satisfying $\alpha+\xi\gamma=1$. The production function, (10), indicates that the cities have identical production technology but different potential for human contacts. The frequencies of interaction among cities are endogenous in the model. Since the factor demands for production are determined by perfect competition, equating the marginal products of the labor force and the frequency of inter-city communication respectively to wage rent, $\omega_i$, and transportation cost between node $i$ and $j$, $d_{ij}$, we get the following conditions:

$$\omega_i = \alpha \frac{Y_i}{N_i} \tag{11.a}$$

$$d_{ij} = \gamma \xi Y_i \frac{N_j d_{ij}^{-\frac{\xi}{1-\xi}}}{R_{ij} \sum_{k \neq i} N_k d_{ik}^{-\frac{\xi}{1-\xi}}} \tag{11.b}$$

From (7) and (11.a) the household income is given by:

$$y_i = \omega_i + \frac{P_i}{N_i} \tag{12}$$

The inter-city communication frequency, $R_{ij}$, is determined from 11.b as:

$$R_{ij} = \gamma \xi Y_i \frac{N_j d_{ij}^{-\frac{\xi}{1-\xi}}}{d_{ij} \sum_{k \neq i} N_k d_{ik}^{-\frac{\xi}{1-\xi}}} \tag{13}$$

### 3.2. Equilibrium Conditions:

The wage rate of a particular city is determined so that the supply and demand for the labor force is brought into equilibrium. The population distribution among cities can be brought into equilibrium when no household has incentive to move. Thus the equilibrium of population distribution can be characterized by

$$V_1 = V_2 = \cdots = V_n = \overline{V} \tag{14}$$

where $\overline{V}$ is the equilibrium utility level. Finally, the constant total population of the whole system is indicated by the following equation, where $N$ is the exogenously given total population of the system.

$$\sum_{i=1}^{n} N_i = N \tag{15}$$

In the model described above, $N$, $c_i$, and $d_{ij}$ are the exogenous variables whereas $V_i$, $u_i$, $P_i$, $y_i$, $N_i$ and $R_{ij}$ are endogenous variables determined by the equilibrium conditions.

## 4. INVESTMENT RULE:

The investment decision is taken by the central government, which controls the travel costs between the cities by improving the existing transportation links. The investment decisions are taken by considering the cost-benefit analysis.

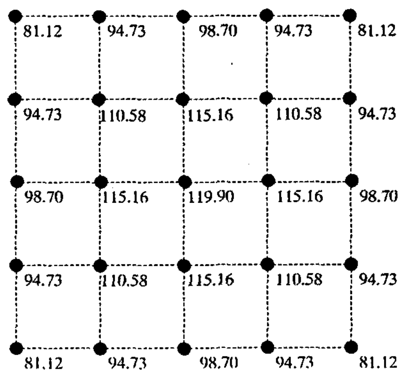



Figure 1. The 5x5 Network with Initial Equilibrium Populations

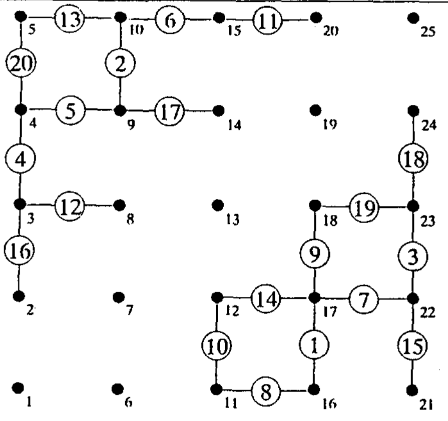



Figure 2. The Evolution Pattern of the Network

We will consider two kinds of cost-benefit rule. One is going to be calculated by considering the new OD patterns of the network after the construction of a new link. In this case the inter-city communication frequencies, $R'_{ij}$, and transportation costs, $d'_{ij}$, are calculated for all links that can potentially constructed in the next stage. Then, by using the existing values of inter-city communication values, $R_{ij}$, and transportation costs, $d_{ij}$, consumer surplus is calculated with the following equation and taken to be the benefit provided by the improved link:

$$B_l = \sum_{i=1}^{n}\sum_{j\neq i}^{n}\frac{1}{2}\left(R'_{ij} - R_{ij}\right)\left(d_{ij} - d'_{ij}\right) \tag{16}$$

where $B_l$ indicated the benefit due to the improvement of link $l$. In the second cost-benefit rule the decision is taken regarding the indirect equilibrium utility level. In this case the link which will provide the highest increase in the equilibrium utility level of the whole system is chosen to be constructed in the next stage of the evolution.

## 5. COMPUTATIONAL SIMULATIONS:

The network of cities simulated is a simple grid system which lies on a flat ground. The network simulated, which consists of 5x5=25 cities, is shown in **Figure 1**. The central government improves the existing transportation connections either by improving the quality of the existing connections or introducing a higher-level transportation system. At one stage of the network evolution the government improves only one link which is selected according to the cost-benefit rule. The links which have been improved once cannot be improved further.

The network evolution is simulated as follows: First, given initial populations, the system is brought into equilibrium on the network with no link improvement. After that, the link (connecting neighboring cities) is improved by decreasing its cost from its initial value of $d_{ij}$=1.0 to $d'_{ij}$=0.7. Following the improvement, the new utility levels of all of the cities are calculated and checked whether they are in equilibrium or not. If the equilibrium is not reached new populations are assigned to each and every city and this process is repeated until the equilibrium is obtained. At every stage of the evolution, the potential benefits due to possible link improvements are calculated by bringing the system into equilibria. Since the all links have the same length, the cost of construction of each links is assumed to be same, so, the link which provides the highest benefit is chosen to be constructed at the respective stage. The same procedure is repeated to determine the remaining links to be constructed.

The convergence to the equilibrium utility level is regulated by the following population dynamics:

$$N_{t+1}(i) = N_t(i) + \lambda(V_i - \bar{V}) \tag{17}$$

which assumes that the population moves toward the locations with above-average utility levels and away from those with below-average. Here $N_{t+1}(i)$ is the population of the $i^{th}$ city to be used in the next iteration step, $\lambda$ is the adaptation parameter reflecting the adjustment speed of convergence, $\bar{V}$ is the average utility level of the whole system calculated by taking the average of the utility levels of all the cities weighted by the population of each city, and $V_i$ is the utility level of the $i^{th}$ city. The second term on the right hand side of the equation (17) shows the change in the population of $i^{th}$ city, which is positive if $V_i$ is greater than $\bar{V}$ and negative if $V_i$ is smaller than $\bar{V}$.

## 6. RESULTS:

The simulation shows that there exist a number of different evolution patterns. The initial equilibrium configuration of the population is decisive in regulating the successive evolution patterns. Given the initial equilibrium as depicted in **Figure 1**, the network evolves as shown in **Figure 2**. The numbers in the circles indicate the order of the links

Figure 3. Cumulative Benefit

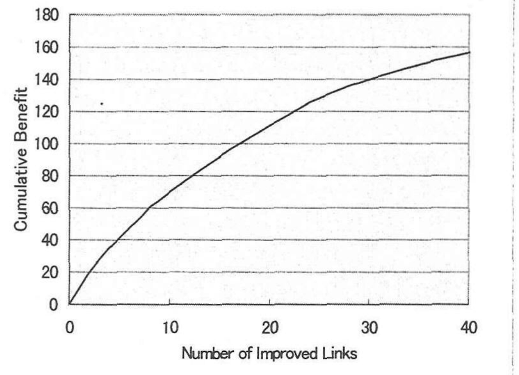



Figure 4. Equilibrium Utility Level

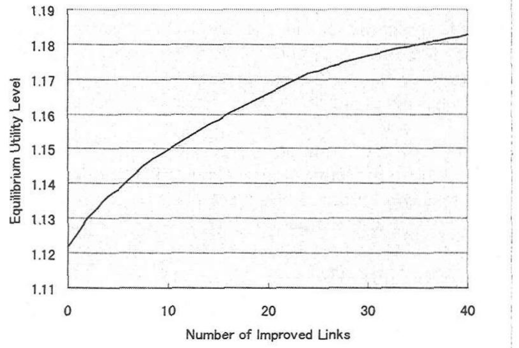



Figure 5. Variance of Population

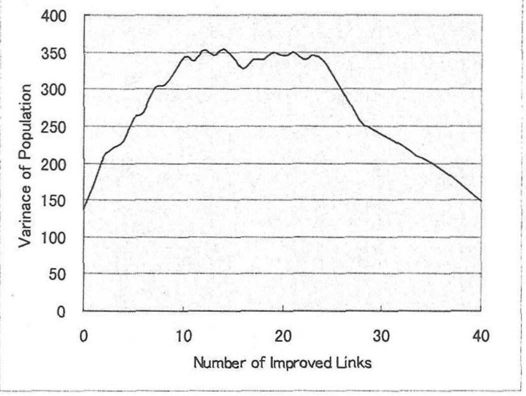
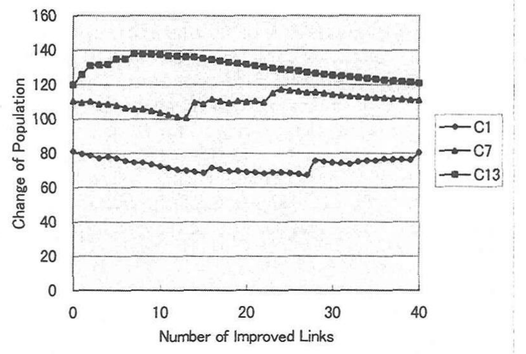



Figure 6. Change of Population in Cities 1, 7 and 13




improved. This evolution pattern corresponds to the most decentralized case where no cities disappear from the network. For this case, both of the cost-benefit rules applied show the same evolution pattern. From the figure it is observed that the network structure is highly decentralized. It starts from the corners of the network and moves towards inside to guarantee the decentralized population allocation. Irrespective of its decentralized features, the central city (city 13) enjoys the largest share of the population. The changes in the cumulative benefit and equilibrium utility level are shown in **Figures** 3 and 4 respectively. Both rules exhibit diminishing returns to link improvement. **Figure 5** indicates the change in the variance of the population over the whole network. The variance increases at the first half of the evolution and then decreases again. As it can be seen from **Figure 6** the migration is mainly from the outer cities towards the centrally located cities. During the second half of the evolution process this movement is reversed and the population changes back to the level which is similar to the initial equilibrium level. This fact can also be seen from the change of the variance of the population, as the variance of the populations becomes almost same as the initial equilibrium level. Due to shortage of space, the details of other evolution patterns are reserved for presentation.

## 7. CONCLUSION:

With the model used in the study, in which inter-city communication frequencies are calculated endogenously, the network shows various evolution patterns. The historical incidences to determine the initial equilibrium configuration of the population are crucial in regulating the evolution patterns. Simulation experiments should be further made under different conditions.

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