ESTIMATION OF CAR FOLLOWING MODEL PARAMETERS *

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1. Introduction

A model describing the movement of vehicles in a platoon following each other is an essential part of microscopic simulation of traffic flow. Many car following models have been developed and incorporated in microscopic simulation programmes. The early models developed were based on the *speed-spacing* relationship [1)-3),5),6)]. These models are of the form,

$$S = a + bV + cV^2 \tag{1}$$

where, S = The distance headway between a following vehicle and the leading vehicle at any instant
V= The speed of the following vehicle at the same instant.
a, b and **c** = variable parameters

This applies only to cases where each vehicle in a traffic stream has the same speed and maintains the same spacing[3)] .The speed - spacing model assumes that the speed changes simultaneously as the space changes. However, since the driver's human behaviour is involved in the speed changing action, there is a time lag between space and speed changes.

It was later found that a vehicle follows behind another vehicle trying to maintain a safe distance between them and at the same time maintain the same speed as that of the vehicle in front [1)-4), 6)-9)]. If the relative speed changes (stimulus) the following driver observes the change and takes action by acceleration or deceleration (response) to bring back the relative speed to zero There is a time lag between the stimulus and the response. This gives rise to a *stimulus-response* relationship which is of the form,

$$\ddot{x}_2(t+T) = \lambda[\dot{x}_1(t) - \dot{x}_2(t)] \tag{2}$$

where 1 and 2 denote leading vehicle and following vehicle respectively.
$\dot{x}$ = speed t = time at which the stimulus took place
$\ddot{x}$ = acceleration T = Response time (i.e., time lag between stimulus and response).
λ = a variable coefficient known as *reaction sensitivity* which depends on the driving conditions

The driving conditions which govern λ are: $x_1(t) - x_2(t)$ the distance headway between the leader and the follower at time t and $\dot{x}_2(t)$, the speed of the follower at time t. The general relationship of stimulus-response car following equation thus becomes

$$\ddot{x}_2(t+T) = \frac{\alpha.[\dot{x}_2(t)]^m}{[x_1(t) - x_2(t)]^\ell}.[\dot{x}_1(t) - \dot{x}_2(t)] \tag{3}$$

α, ℓ and m are parameters which can be empirically obtained through regression.

The aim of this study is to determine the parameters α, ℓ and m of the general car following model and to develop a car following model which is applicable to all driving conditions.

2 Previous empirical work on car following models

Newell et al [10)] have hypothesised that there are two speed-spacing relationships, one for acceleration condition and the other for deceleration condition. This hypothesis was not supported with quantitative results. Dijker et al.[5)] proved with experimental work on Dutch roads that the car following models are regime specific. These regimes are the conditions of free-flow, congested flow and transitional flow. The work assumed speed-spacing relationship with a value of 3 for **a** and carried a regression analysis to obtain values of **b** and **c** in the Equation (1). It has been seen from regression that the car following rule is regime dependent. The different traffic situations considered are the passenger cars, light trucks and heavy trucks. The situation of passenger cars was further divided according to the shoulder, median and middle lanes occupied by them. A configuration of eight sets of parameters was thus determined for the model describing the eight traffic regimes. The parameters were used in the FOSIM microscopic traffic simulation of traffic flow at a bottleneck on a Dutch highway. The microscopic simulation has given macroscopic relationships which were reproduction of the actual measurements. The study

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has used average values of distance gaps for each 5 km/h class assuming that this action sufficiently cancels out the influence of acceleration, deceleration and response times on the speed-spacing relationship.

Sinha and Dawson[11] derived a car following relationship by mathematical analysis. The analysis gives the speed at the end of a time interval, the distance moved during the time interval and the time taken for a following vehicle to adjust to the speed of the leader and follow at the desired distance from it. Three different relationships were developed respectively for three driving conditions of the available spacing being equal to, greater than or less than the critical spacing. The model was used to simulate freeway traffic flow in USA and obtained reasonably consistent comparison with actual traffic flow. The analysis has not considered a response time.

Hsu [12] developed a car following model with the concept that drivers use either a speed detecting mode or a distance detecting mode. A memory function was incorporated in the stimulus-response relationship which gave three different forms of the relationship. One, for drivers of distance detecting mode. The second, for drivers with desired spacing headway mode and the third, for drivers of speed detecting mode. A microscopic traffic simulation model was developed deriving a memory function using assumed values for reaction times and parameters. The model was justified by its ability to improve car following simulation when compared with the general car following model and with a model which is a modification of the general model.

Ozaki [9] used the data of car following experiments involving three following cars to establish the general car following model parameters. Instead of using a whole set of time series data, the data was divided into segments of single actions of acceleration or deceleration. The segments of acceleration or deceleration and the corresponding segments of relative speed were traced. The time gap T between the commencement of the two actions was also noted. The regression analysis according to the Equation (3) was restricted to each data set within a segment. From the regression work it was found that for acceleration condition, α=1.1, ℓ=1.0, m=0.9 and for deceleration, α=1.1, ℓ=0.2, m=0.2. Also the response time T was found, through a regression, to be dependent on space and leader acceleration and a linear relationship between the variables was determined. The parameters and response times were used in the simulation of the traffic flow near a sagging vertical alignment on a highway in Japan where deceleration and acceleration waves were encountered. The plot of trajectories has shown that the simulated trajectories closely follow observed trajectories, thus validating the model parameters.

3. The proposed model

A model which accounts for both relative speed requirement as well as the space requirement has been proposed in this study. The new model includes a factor called the *critical speed factor* which is a function of space and current speed as an independent variable in addition to the relative speed, giving acceleration as the dependent variable according to the relationship,

$$\ddot{x}_2(t+T)=\alpha_0+\alpha_1\left[\sqrt{2*f(x_1(t)-x_2(t))}-(\dot{x}_2(t))\right]+\alpha_2[\dot{x}_1(t)-\dot{x}_2(t)] \qquad (4)$$

Where f is the maximum rate of deceleration in m/s^2 that depends mainly on the road and vehicle conditions. α_0, α_1, α_2 are coefficients. The other notations are as per Equations (2) and (3).

$\left[\sqrt{2*f(x_1(t)-x_2(t))}-(\dot{x}_2(t))\right]$ is the critical speed factor. $\sqrt{2*f(x_1(t)-x_2(t))}$ is the maximum speed that can be attained by the follower without colliding in case the leader comes to an instantaneous stop. If this speed is the same as current speed $\dot{x}_2(t)$, the critical speed factor becomes zero and does not contribute to acceleration.

4. Data Collection, Processing and Analysis

A single test car, following vehicles in a stretch of an arterial road in the Takassago prefecture of Japan, was used to collect necessary data .The data collected continuously at time intervals of 0.05 seconds were: (a) the space between the test car and a vehicle being followed, (b) the distance moved by the test car and (c) the acceleration of the test car. The test car was fitted with an electromagnetic distance measuring (E D M) equipment to monitor the distance between itself and a vehicle in front. A fifth wheel recorded the distance travelled by the test car. The acceleration in m/s^2 of the test car was obtained from readings of an accelerometer.

The speed in m/s of the vehicle was calculated as the rate of change of distance. The relative speed in m/s was calculated from the E D M readings as the rate of change of spacing. The E D M readings were found to be erratic in that at times the rays had failed to impinge on the leading vehicle. It was not difficult to trace such incidents as abnormal sudden change of space was indicative of the fault. The initial data processing involved removal of these faulty data lines.

A correlation of acceleration against relative speed was carried out with time lags of T=0.0, 0.5, 1.0, 1.5 and 2.0 between the two variables according to the procedure suggested by Gazis et al.[3] . It was not possible to obtain a single response time T which gave the highest correlation for all the data sets. He correlation analysis gave a mean value of 1.05 for T with a variance of 0.4. Using the response times corresponding to the data sets a regression analysis was carried out using the following logarithmic form of the combination of Equations 2 and 3:

$$\ln\lambda = \ln\alpha + m\ln[\dot{x}_2(t)] - l\ln[x_1(t)-x_2(t)] \qquad (5)$$

The Table 1 shows that the parameters α, ℓ and m obtained differed from one data set to the other with large variations. Therefore this is not a suitable procedure for parameter estimation in the general car following model. Further, there were problems of λ being negative at times which prevents the use of Equation 5 for regression of the respective data lines. This happens when the acceleration and relative speed are of different signs which can occur in the observations.

Since the method of direct analysis of the whole data did not give satisfactory results, a method similar to the one suggested by Ozaki [9] was used as the next approach. The procedure adopted for parameter estimation in this method is as follows:

- Produce graphs of acceleration and relative speed Vs time
- From inspection of the graphs pick the points of beginning or of changing stimulus(relative speed) and corresponding response.(acceleration)
- Subdivide stimulus and response periods into equal parts
- Average acceleration, space, speed and relative speed within the subdivisions.
- Note response time of each pair of stimulus and response.
- Carry out correlation and regression analyses of logarithm of sensitivity λ with logarithms of space and speed and of response time with space, speed and leader acceleration

Table 1. Constant of Regression R^2, T and Parameters (Direct Method)

	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6	Set 7	Set 8	Set 9	Set 10
R^2	0.08	0.07	0.03	0.45	0.13	0.25	0.53	0.01	0.33	0.43
T (sec)	1.00	0.00	1.00	2.00	0.00	1.00	0.00	0.00	1.00	1.50
α	0.16	2410.38	3.41	353.14	157.19	2172.46	82.92	0.99	0.00	0.00
l	-0.64	3.31	1.38	-8.11	-10.84	-1.90	-5.56	-1.18	-5.31	-79.75
m	1.64	1.54	2.80	-12.67	-15.81	-4.95	-8.40	-0.24	2.37	57.90
	Set 11	Set 12	Set 13	Set 14	Set 15	Set 16	Set 17	Set 18	Set 19	Set 20
R^2	0.16	0.57	0.52	0.00	0.65	0.74	0.05	0.19	0.19	0.28
T (sec)	1.50	2.00	0.00	1.50	1.50	1.50	2.00	1.00	1.50	1.50
α	1E+123	1.30	7E+262	2.68	0.00	48.30	1E+21	2.2E+26	0.00	0.00
l	48.24	31.26	109.99	0.52	-6.99	10.39	-0.20	17.22	-27.79	-22.53
m	-65.53	37.70	-132.82	1.78	-0.55	11.42	-22.40	-8.33	-7.27	11.66
	Set 21	Set 22	Set 23	Set 24	Set 25	Set 26	Set 27	Set 28	Set 29	Set 30
R^2	0.66	0.14	0.07	0.30	0.09	0.58	0.02	0.12	0.17	0.13
T (sec)	0.50	0.00	1.50	1.00	1.00	1.00	1.50	1.50	1.00	1.50
α	3E+184	6E-36	3.2E+95	1E-225	0	5E+162	9.5E-08	331.521	0.00987	30104.1
l	199.37	-2.44	50.37	-113.22	-12.03	-8.16	8.46	3.71	-6.41	5.54
m	20.13	38.35	-50.88	130.55	1990.81	-199.58	20.07	3.12	-4.19	3.42
	Set 31	Set 32	Set 33	Set 34	Set 35	Set 36	Set 37	Set 38	Set 39	Set 40
R^2	0.11	0.11	0.26	0.42	0.46	0.89	0.01	0.91	0.02	0.55
T (sec)	0.50	0.50	1.00	1.00	2.00	2.00	1.50	1.00	0.50	0.50
α	1092785	0.00	0.00	119.02	9.73	9E+50	2.7E-06	5.7E-10	2.3E-05	2E+253
l	3.34	2.30	8.70	6.01	-4.05	32.91	-3.09	-28.36	-2.07	48.84
m	-1.13	7.09	16.74	6.87	-4.90	-10.36	3.08	-24.93	2.23	-199.15

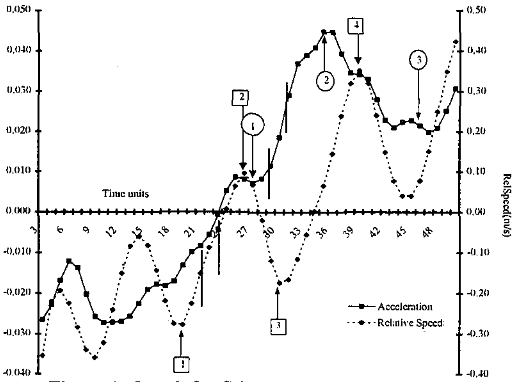

Figure 1. Graph for Stimulus-Response analysis

The Figure 1. is a sample of graphs of acceleration and relative speed Vs time respectively. The time unit is .05 seconds. The curves have been divided into segments of corresponding stimulus and response. The numbers 1, 2, 3, 4 show these divisions. From 1 to 2 is an increase of relative speed (stimulus) from a low to a high. A corresponding low to high is seen from 1 to 2.in the acceleration (response) curve with a time lag (response time) of (29-19)*.05=0.50 seconds. Similarly 2 to 3

Table.2 Summary of regression analysis by driving conditions with regression coefficients

Test	Driving Condition	Acceleration Vs Crit.Speed Factor and Relative Speed				ln Sensitivity Vs ln Space and ln Speed				Resp.Time Vs Space,Speed and Leader Acceleration				
		α_0	α_1	α_2	R^2	α	l	m	R^2	α_0	α_1	α_2	α_3	R^2
North 3	Accelerate	0.032	0.014	0.030	0.69	0.110	-0.095	0.167	0.10	0.833	0.073	-0.097	-0.037	0.27
	Decelerate	-0.038	0.002	0.065	0.47	0.163	0.864	0.678	0.01	0.998	-0.122	0.179	0.002	0.60
	Both	0.022	-0.003	0.092	0.93	0.135	0.172	0.106	0.01	0.707	0.091	-0.076	-0.001	0.21
South 5	Accelerate	-0.093	0.039	0.000	0.73	0.061	-4.668	-6.146	0.35	1.614	-0.061	-0.027	-0.059	0.32
	Decelerate	-0.045	0.014	0.010	0.16	9.148	14.124	13.826	0.60	3.535	0.012	-0.337	0.060	0.79
	Both	-0.099	0.035	0.006	0.81	0.119	-4.229	-5.915	0.34	1.718	-0.072	-0.022	-0.038	0.33
South 6	Accelerate	-0.010	0.010	0.030	0.89	9.149	1.874	-0.763	0.42	0.728	0.060	-0.067	-0.106	0.39
	Decelerate	0.028	-0.010	0.026	0.73	0.000	7.460	34.240	0.60	22.370	0.349	-3.434	-0.719	0.86
	Both	0.004	0.004	0.037	0.90	8.581	1.804	-0.838	0.19	0.882	0.055	-0.075	-0.163	0.26
South 9	Accelerate	-0.068	0.048	0.013	0.95	0.221	0.516	0.171	0.20	1.292	-0.275	0.340	0.003	0.69
	Decelerate													
	Both													
South 10	Accelerate	-0.068	-0.003	-0.003	0.46	749079.2	5.938	-0.103	0.21	-4.876	0.065	0.329	-0.001	0.72
	Decelerate	-0.070	0.003	0.000	0.14	0.000	0.017	2.477	0.10	-2.251	0.018	0.210	-0.019	0.54
	Both	-0.030	0.002	0.005	0.21	0.000	0.283	3.263	0.10	-2.584	0.022	0.224	-0.005	0.49
South 11	Accelerate	-0.043	0.027	0.023	0.82	0.000	-9.018	-7.593	0.31	0.179	0.034	-0.025	-0.003	0.14
	Decelerate	-0.003	-0.007	0.011	0.26	928.713	7.823	3.893	0.48	-0.205	0.180	-0.175	-0.014	0.85
	Both	-0.025	0.020	0.026	0.85	0.005	-4.777	-4.640	0.30	0.110	0.053	-0.048	-0.007	0.32
South 12	Accelerate	0.075	0.089	-0.063	0.74	1.42E+12	21.844	9.191	0.38	-0.302	0.066	-0.031	0.011	0.06
	Decelerate													
	Both													
Overall	Accelerate	0.031	0.001	0.037	0.51	1.683	2.048	0.804	0.12	0.893	-0.010	-0.021	-0.004	0.07
	Decelerate	-0.072	0.003	0.004	0.13	0.142	0.542	-0.205	0.26	1.125	0.012	-0.082	-0.026	0.27
	Both	0.010	0.002	0.034	0.51	0.590	1.328	0.339	0.13	0.982	0.001	-0.048	-0.009	0.10

is a decrease in speed and corresponding decrease in acceleration rate with a response time of (35-26)*.05=.45 seconds. The pairs of stimulus and response segments were picked up manually by inspection The conjugate segments were subdivided into equal number of parts. An example of these divisions between 1 and 2 are shown in the Figure 1. in solid lines. The speed acceleration, space, speed, relative speed and leader acceleration were averaged within the subdivisions for appropriate use in regression. In order to compare the regression of general car following model with that of the proposed model additional information of critical speed factor was calculated and used in the correlation and regression studies of acceleration Vs critical speed factor and relative speed . The results are shown in the Table 2. A value of f =5.0 was used in the calculation of critical speed factor. This was determined as the value of f which gave the highest correlation between acceleration and critical speed.

The regression analysis shows that the proposed model gives higher R^2 values than the general car following model. The regression of T shows that speed is a contributing factor. This contradicts the findings of Ozaki[9)] where speed was ignored as it has insignificant effect on the response time. From the analysis it was found that there exists a relationship,

$$T = \alpha_0 + \alpha_1(x_1(t) - x_2(t)) + \alpha_2(\dot{x}_2(t)) + \alpha_3(\ddot{x}(t)) \tag{6}$$

5. Summary of findings and future improvements.

The proposed model gave good constant of regression R^2 in case of acceleration only. R^2 in the overall case of deceleration is as low as 0.134 individual runs showed high R^2 in some cases. There were 272 data sets in acceleration and only 102 data sets in deceleration. More data need to be analysed to establish the proposed model. The model gives the following two relationships for accelerating and decelerating conditions respectively.

Acceleration
$$\ddot{x}_2(t+T) = 0.031 + 0.001\left[\sqrt{2 * 5(x_1(t) - x_2(t))} - (\dot{x}_2(t))\right] + 0.037[\dot{x}_1(t) - \dot{x}_2(t)] \tag{7}$$

Deceleration
$$\ddot{x}_2(t+T) = -0.072 + 0.003\left[\sqrt{2 * 5(x_1(t) - x_2(t))} - (\dot{x}_2(t))\right] + 0.004[\dot{x}_1(t) - \dot{x}_2(t)] \tag{8}$$

The response time is given by the following relationships:

Acceleration
$$T = 0.893 - 0.010 * (x_1(t) - x_2(t)) - 0.021(\dot{x}_2(t)) - 0.004(\ddot{x}_1(t)) \tag{9}$$

Deceleration
$$T = 1.125 + 0.012 * (x_1(t) - x_2(t)) - 0.082(\dot{x}_2(t)) - 0.026(\ddot{x}_1(t)) \tag{10}$$

The study shows that by close examination of the time series acceleration and relative speed curves it is possible to determine the variation of response time and obtain a relationships for the acceleration rates in the acceleration and deceleration modes. However, The manual extraction of information for regression analysis from the data is very tedious. It is necessary to seek the assistance of a computer program to carry out this task. The relationships obtained are for a single driver only. Since the parameters are driver and driving condition dependent it is necessary to analyse data obtained for different drivers and in different driving conditions. The model has to be used in a traffic simulation to test its validity.

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