

ROLE OF TIRE MODELS IN ANALYTICALLY ESTIMATING UNKNOWN PARAMETERS FOR ACCIDENT RECONSTRUCTION

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1. Introduction

Traffic accident reconstruction is a method that clarifies how an accident occurred or what happened during the accident. So far, the various programs have been proposed for reconstructing traffic accidents; CRASH⁽¹⁾, SMAC⁽²⁾, IMPAC⁽³⁾, J2DAC^(4, 5), and so on. To describe vehicle behaviors and occupant movements in an accident, several models must be defined and assembled: a tire model for describing the interaction between tire and road surface, a driving model for relating maneuverings to vehicle movements, an impact model for estimating the vehicle velocities at collision, and so on. Moreover, among the models, a tire model plays the most important role in reconstructing traffic accident. Although several tire models have been proposed, detailed discussion about which model is fitted for analyzing traffic accidents have not been sufficiently investigated yet.

This paper aims to compare the effect of tire models on estimating unknown model parameters by an analytical method. For driving simulation in the pre-impact and post-impact phases, we introduced three tire models; Sakai's tire model, modified Sakai's tire model^(6, 7) and Gim's tire model⁽⁸⁾. And then, we combined them with the two-dimensional car-to-car impact model, which was first proposed by Ishikawa^(4, 5), and two-wheel equivalence model⁽⁹⁾. Next, assuming that we could determine the vehicle movement in the pre-impact phase preliminary based on the driver's witness, skid marks and accident site conditions, we defined the unknown parameters in those models; friction coefficient between a tire and road surface, steering angle, slip ratio of the front tires, slip ratio of the rear tires, normal and tangential restitution coefficients. We estimated these unknown parameters so as to minimize the difference between the calculated and observed rest positions of vehicles. For this purpose, we employed Box's complex algorithm⁽¹⁰⁾. After the validity of Box's method in estimating the model parameters was examined using an artificial accident data, the model parameters of two actually observed traffic accidents were estimated for three tire models.

2. Traffic Accident Reconstruction

(1) Tire models

A Schematic drawing of interaction friction forces between a tire and road surface is shown in Fig. 1. There are three main forces; the force acting in the lateral direction of the tire as the side force (S_F), the force acting in the longitudinal direction of the tire as the braking force or traction force (D_F), and the moment acting in the opposite direction of wheel yawing (M_z). The slip angle and the camber angle of the tire are denoted by β and γ .

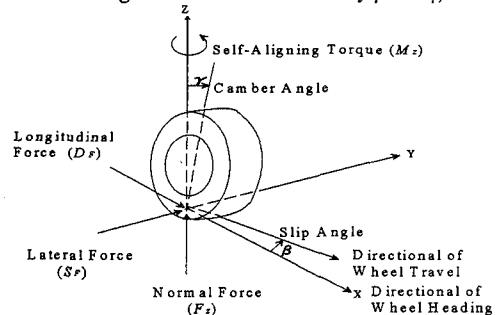


Fig 1 Forces and moment applied to a tire

The slip properties are determined by the relative motion between a tire and road surface. The slip ratio s is obtained as follows;

During braking ($s \geq 0$):

$$(V_R \cos \beta - V_{roll}) / V_R \cos \beta \quad (1)$$

During traction ($s < 0$):

$$(V_R \cos \beta - V_{roll}) / V_{roll} \quad (2)$$

Where

V_R : Tire velocity.

V_{roll} : Tire rolling velocity.

The slip angle of the front and rear tires (β_f, β_r) can be obtained from the following equations;

$$\beta_f = \tan^{-1} \left(\frac{V_y}{V_x} \right) + \frac{\omega l_f}{V_x} - \delta \quad (3)$$

$$\beta_r = \tan^{-1} \left(\frac{V_y}{V_x} \right) - \frac{\omega l_r}{V_x} \quad (4)$$

Key words: Accident reconstruction, Tire model, Impact model, Driving simulation model

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where

- Vx : Velocity of the vehicle in the driving direction.
- Vy : Velocity of the vehicle in the direction perpendicular to the driving direction.
- ω : Angular velocity.
- l_f : Distance between the gravity center of the vehicle and front tire.
- l_r : Distance between the gravity center of the vehicle and rear tire.
- δ : Steering angle.

It is necessary to establish a function that relates the forces to slip angle, slip ratio and friction coefficient. This function is called tire model.

a) Sakai's tire model^{6,7)}

Under the assumptions that slip angle and slip ratio of tire are small enough and tread beam of tire is not bent in the lateral direction, Sakai proposed a model, in which the resultant friction coefficient is a function of the sliding velocity, a friction parameter at zero velocity, and a suitable reduction factor, which is obtained experimentally.

b) Modified Sakai's tire model^{6,7)}

To treat the problems in which slip ratio and slip angle are relatively large, Sakai modified the original model. In the new model, he considers the friction coefficient that depends on velocity, the contact pressure with order of 4, and the bending and torsion of the tread in the lateral direction.

c) Gim's tire model⁸⁾

An analytical tire model was proposed by Gim and Nikravesh. In the model, not only slip angle and slip ratio but also camber angle, which represents the rotation around z-axis, was considered as shown in Fig. 1. However, the bending and torsion produced by side force were not considered. In this model the resultant friction coefficient is assumed to be function of the sliding velocity by order of two, a friction parameter at zero velocity, and two suitable reduction factors, which are obtained experimentally.

(2) Two-wheel equivalence model⁹⁾

The two-wheel equivalence model was applied for calculating the resultant forces at the gravity center of each vehicle. In this model, It is assumed that the rolling and pitching movements are negligible.

(3) Impact model^{4,5)}

There are three degrees of freedom for each vehicle; two translations and one rotation. In order to apply the impact model, six equations are necessary: Four equations can be obtained from the law of conservation of linear and angular momentum. The last two equations are obtained from the constraint conditions at the impact center, in which the normal and the tangential restitution coefficients are defined. These coefficients can be calculated from the equations (5) and (6), as shown in the following ;

$$e_n = -RDS / RDS_0 \quad (5)$$

$$e_t = -RSS / RSS_0 \quad (6)$$

where

RDS : Relative deformation speed after collision.

RSS : Relative sliding speed after collision.

0 : Subscript for relative speed before collision.

3. Estimation of Model Parameters

We assumed that the parameters with respect to the dynamics characteristics of tire and vehicle were unknown in both impact and post-impact phases. To estimate the parameters, we employed Box's complex algorithm.

(1) Model parameters

Unknown parameters from the impact model are :

e_n : Normal restitution coefficient.

e_t : Tangential restitution coefficient.

The unknown parameters from the tire model are :

μ_i : Friction coefficient.

δ_i : Steering angle.

S_{Fi} : slip ratio of front tire.

S_{Ri} : slip ratio of rear tire.

Where subscript i denotes the striking or struck vehicle.

Evidently, slip ratio and steering angle vary with time. For simplicity, we assumed that they were constant in the pre-impact and post impact phases. Moreover, the friction coefficient is dependent on the slip ratio. we defined a function that related the friction to the slip ratio.

(2) Estimation of parameters

We estimated the unknown parameters using Box's algorithm. In this method, we estimated the parameters so as to minimize the difference between the calculated rest positions of the vehicles and the observed ones.

The objective function is given below;

$$F_k(P) = \sum_{i=1}^2 \left(\frac{x_i - x_{si}}{x_{si}} \right)^2 + \left(\frac{y_i - y_{si}}{y_{si}} \right)^2 + \left(\frac{\theta_i - \theta_{si}}{\theta_{si}} \right)^2 \quad (7)$$

where

P : Unknown model parameters ($e_n, e_t, \dots, S_{F2}, S_{R2}$).

k : Complex point.

x_i, y_i, θ_i : Calculated distance and yaw angle.

x_s, y_s, θ_s : Observed distance and yaw angle.

x_r, y_r, θ_r : Ranges of distance and yaw angle.

We assume that the unknown model parameters are subject to the following constraints;

$$\begin{aligned} -1 &\leq e_n \leq 1, & -1 &\leq e_t \leq 1 \\ 0 &\leq \mu_i \leq 1, & \delta_{min} &\leq \delta_i \leq \delta_{max} \\ 0 &\leq S_{Fi} \leq 1, & 0 &\leq S_{Ri} \leq 1 \end{aligned} \quad (8)$$

4. Numerical Experiments

(1) Validity of estimation method

Data from an artificial traffic accident was introduced to investigate the validity of Box's method in estimating the model parameters for reconstructing traffic accidents. We assumed that an accident, as shown in Fig. 2, occurred on a street. We specified the model parameters in advance. Giving the initial positions, we calculated the trajectory of each vehicle. In order to evaluate the effect of tire model, we applied three tire models; Sakai's tire model, modified Sakai's tire model and Gim's tire model for driving simulation of the pre-impact and post-impact phases. Fig. 2 shows the position of the striking and the struck cars based on the artificial model parameters. The initial, impact and rest positions are denoted by points 1, 2 and 3, respectively.

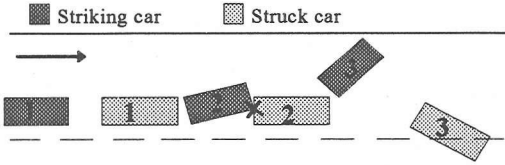


Fig. 2 Front to rear accident based on artificial data

We estimated the unknown model parameters of impact and post-impact phases. And then, we compared the estimated parameters with the assumed ones, as shown in Fig. 3. For Gim's tire model, the difference between them was sufficiently small. When the Sakai's model was used, the difference between estimated and assumed slip ratio of front tire of struck car was somewhat large which amounted to nearly 15%.

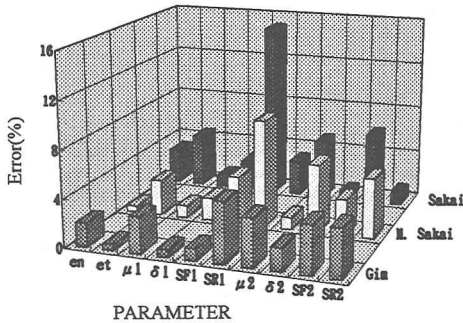


Fig.3 Errors between assumed and estimated parameters

We calculated the rest position of each vehicle. Fig. 4 and Fig. 5 show the differences between the calculated rest positions of vehicles and the assumed ones. we can see that the differences between calculated and assumed rest positions of vehicles, estimated by Gim's model and modified Sakai's model, are less than Sakai's model for each vehicle. We have confirmed that Gim's model and modified Sakai's model are much accurate in estimating the model parameters.

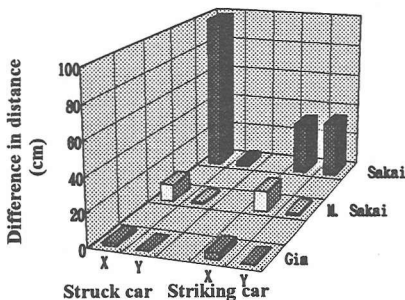


Fig. 4 Distance differences between assumed rest positions and estimated ones

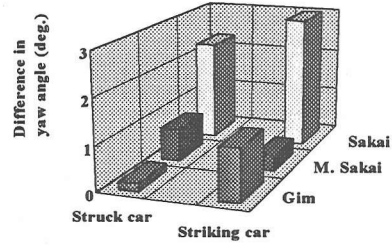


Fig. 5 Yaw angle differences between assumed rest positions and estimated ones

(2) Actual traffic accidents data

a) Case 1: side impact accident

A side impact accident occurred at an intersection as shown in Fig. 6. The driving simulation of the pre-impact phase was performed based on the data collected from the accident site. The velocity of the striking vehicle before braking was 30 km/h. It had presumably decreased to 15 km/h before collision. The velocity of the struck vehicle was about 30 km/h. The initial, impact and actual rest positions are points 1, 2 and 3, respectively. We estimated the unknown model parameters of impact and post-impact phases. Fig. 7 shows the estimated parameters using three tire models. The parameters should be between -1 and 1. The steering angle is shown in radian.

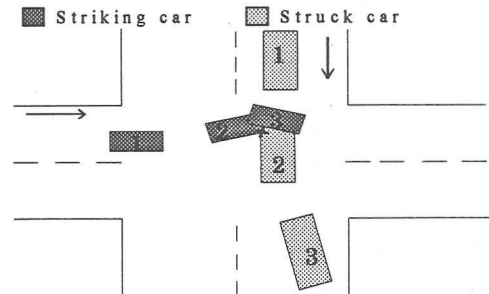


Fig. 6 Side impact accident based on observed data

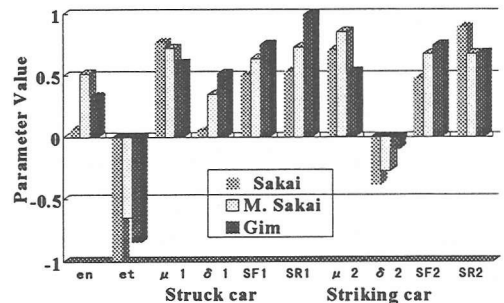


Fig. 7 Estimated parameters for a side impact accident

We can see that the difference in each parameter is not so large for three tire models. The normal restitution coefficients are positive and the tangential restitution

coefficients are negatives. The negative values mean plastic deformation. Since the road surface was in a dry condition, the fiction coefficient is a bit small for Gim's tire model. The steering angle of struck vehicle sounds to be small for Gim's model. The slip ratios of tires are between 0.5 and 1 for all tire models.

The differences between the observed and the estimated rest positions are shown in Fig. 8. We can see that the differences in the positions of both vehicles are small except the distance of the struck vehicle in X direction for Sakai's model. Moreover, the maximum difference in the yaw angle was nearly 12° for Sakai's model. Whereas, the difference was less than 5° for the other models.

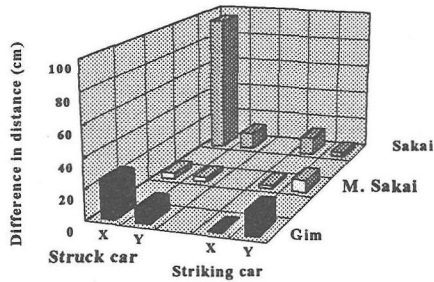


Fig. 8 Distance differences between observed rest positions and estimated ones

b) Case 2: front to rear accident

We examined a front to rear accident that occurred on a street in Sapporo, as shown in Fig. 9. The velocity of the striking vehicle before braking was approximately 50 km/h and decreased to 40 km/h before the collision. The velocity of the struck vehicle was about 10 km/h. We estimated the model parameters of impact and post-impact phases using three tire models, as shown in Fig. 10.

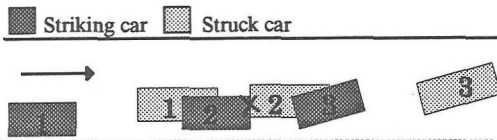


Fig. 9 Front to rear accident

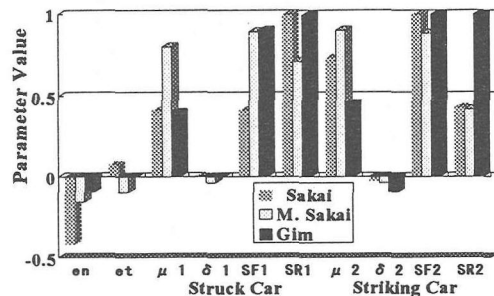


Fig.10 Estimated parameters for a front to rear accident

The estimated normal restitution coefficients were negatives and the tangential restitution coefficients were

very small. This means that the struck vehicle was penetrated in the normal direction. The steering angles of the both vehicles were about zero. As slip ratios were nearly one, the tires seemed to be in the lock condition. The large differences between slip ratio of front and rear tires can be somewhat unreasonable for the Sakai's model. The differences between the observed and the estimated rest positions were also calculated. The distance differences were less than 30 cm for Gim's model and modified Sakai's model, while it was less than 50 cm for Sakai's model. Moreover, the estimated yaw angles were in good accordance with the observed ones for both Gim's model and modified Sakai's model. It is found that the results caused by both tire models were more acceptable.

5. Conclusions

From this study the following conclusions can be drawn:

- 1) An impact model, combined with a driving simulation model and a tire model was developed for reconstructing traffic accidents.
- 2) Box's complex algorithm was effective in estimating the impact coefficients and the driving simulation parameters for reconstructing traffic accidents.
- 3) Gim's tire model or modified Sakai's tire model was much accurate in estimating model parameters for reconstructing traffic accidents.

We have developed the computer animation of traffic accident reconstruction. As future works, we intend to apply the other methods, such as neural network and genetic algorithms, in estimating unknown parameters for reconstructing traffic accidents. Moreover, we are going to use new tire models and analyze their effect in estimating of the parameters.

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