A Equilibrium Model of Motorists' Parking Choice Behavior and Parking Demand in Central Area*

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1. Introduction

In the past three decades a number of mathematical models have been presented (see references) to tackle the problems of parking choice behavior and demand. In the models presented in the published articles the problem of parking choice is formulated as two types—mathematical programming models and logit models --based on the assumption of driver's parking choice. A user equilibrium model is proposed in this paper to simulate the travelers' parking choice behavior who drive to work by themselves and must select a legal place to park. Drivers arriving in these parking area do not have to spend much time searching and/or waiting for a stall in the morning rush; they merely have to find the end of a line of parked vehicles. They drive to predetermined park lot by their daily experience of congestion on streets from homes to parking lots and on streets searching for stalls in parking area, considering occupancy of park lots. Their parking choice is user equilibrium searching, and it can be formulated as user's equilibrium assignment.

2. A Model for Predicting Motorists' Parking Choice in Urban Central Area

As stated in the introduction, we can use the drivers' equilibrium assignment model to formulate the parking choice. We assume that the number of person trips originating at i and terminating at j in urban central area, q_{ij} which is fixed and known. Each trip is taken first by car to a parking lot, k, of capacity p_k vehicles/hr., and on foot to the final destination.

We now specify the condition under which queuing will occur at entrances of parking lots. It is clear that queuing delay, d_k, at parking lot k will increase if $v_k \geq p_k$ and decrease or equal to zero if $v_k < p_k$. Considering a steady-state, $v_k > p_k$ will always be satisfied, and d_k will take a certain value at equilibrium status. Furthermore, it is obvious that there will be a queue at a park k only if $v_k = p_k$. The relationships can be summarized as:

$$\begin{cases} d_k = 0 & \text{if } v_k < p_k \\ d_k \geq 0 & \text{if } v_k = p_k \end{cases} \quad k \in K \tag{1}$$

where K is the set of parking lots.

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A parking lot is represented by two nodes, the entrance node and the exit node, as shown in Fig. 1. Link m corresponds to the in-vehicle link from origin to the park k's entrance, and link n is the walking link from parking lot k to the driver's final destination. Link k models the park lot k. The traffic flow on link k ,v_k, is the number of cars parking in lot k. The sum of parking charge c_k and waiting time at the entrance of parking lot k, d_k, is the impedance of parking link k.

In practice there are many different choices that a motorist could use to travel from his or her origin to destination. It is assumed that a motorist chooses parking to minimize his or her total travel time, which includes travel time from origin to parking lot, parking cost, waiting time, and the walking time from parking lot to his or her final destination. As shown in the introduction, drivers' route and parking choice behavior can be formulated by the equilibrium conditions equivalent to the Wardrop's first principle for the road traffic network equilibrium (Sheffi, 1985). Let t_r^{ij} denote the expected travel time from origin i to destination j via route r, and let u_{ij} denote the minimum expected travel time from i to j, the equilibrium conditions can be stated as

$$t_r^{ij} = u_{ij} \text{ if } f_r^{ij} > 0 \tag{2a}$$

$$t_r^{ij} > u_{ij} \text{ if } f_r^{ij} \geq 0 \tag{2b}$$

where f_r^{ij} is traffic flow from i to j on route r.

Consistent with the assumption that each driver traveling from an origin to a destination has perfect knowledge of travel time and parking charge and queuing at park lot via all routes, and select the route in a user-optima (UE) manner, the following equilibrium relationships are satisfied for each O-D pair i-j and each path r. The equilibrium conditions (2) can be rewritten as

$$t_r^{ij} = \sum_a t_a \delta_{a,r}^{ij} + \sum_k d_k \delta_{k,r}^{ij} = u_{ij} \qquad if\ f_r^{ij} > 0 \tag{3a}$$

$$t_r^{ij} = \sum_a t_a \delta_{a,r}^{ij} + \sum_k d_k \delta_{k,r}^{ij} \geq u_{ij} \qquad if\ f_r^{ij} = 0 \tag{3b}$$

where $\delta_{k,r}^{ij} = 1$ if link a is a part of path r connecting O-D pair i-j, and $\delta_{k,r}^{ij} = 0$ otherwise. Note that $t_a(v_a)$ is flow dependent time, and d_k is queuing waiting at parking lot k. For a in-vehicle link a, $t(v_a)$ is the travel time on that link. The formula of $t(v_a)$ is represented by the BPR's model. For a parking link k, $t(v_k)$=c_k, and the queuing delay, d_k, at the entrance of parking lot k, is determined by the network equilibrium condition.

The steady state users' equilibrium assignment on the road traffic and parking network is a specification of the vector of traffic flows, **v**, and a set of queuing delays at parking lots, **d**, satisfying the equilibrium conditions (3). The users' equilibrium parking choice assignment problem is equivalent to the following minimization problem:

$$Minimize \sum_a \int_0^{v_a} t_a(x)dx \tag{4a}$$

Subject to

$$\sum_r f_r^{ij} = q_{ij} \qquad \forall i,j \tag{4b}$$

$$f_r^{ij} \geq 0 \qquad \forall i,j,r \tag{4c}$$

$$v_k \leq p_k \qquad k \in K \tag{4d}$$

and definition constraints

$$v_a = \sum_{ij}\sum_{r} f_r^{ij}\,\delta_{a,r}^{ij} \qquad a \in A \tag{4e}$$

where A is the set of links of the road networks.

This problem is distinguished from the other parking choice model by imposing the parking capacity constraints (4d) explicitly. This assignment problem also models the effecting of the traffic congestion on roads from origin to parking lot and on roads searching for parking to drivers' parking choice. It can be verified that the Lagrangian factors corresponding to parking lot capacity constraints (4d) are equal to queuing waiting time at entrance of parking lots.

3. User Equilibrium Model of Parking Choice With Variable Demand

The formulation of the user-equilibrium problem of parking choice given in Section 2 assumes that the trip rate between every origin and every destination is fixed and known. In reality, however, these trip rates may be influenced by the level service on the network, such as the limitation of parking capacity in the parking areas. For example, as congestion in parking areas increases, motorists may not use cars or shift the time of travel.

In order to take this phenomenon into account, the trip rate, q_{ij}, between every O-D pair i-j in the network can be assumed to be a function of the travel time between i and j. In other words,

$$q_{ij} = D_{ij}(u_{ij}) \qquad \forall\, i, j \tag{5}$$

where u_{ij} is the minimum generalized travel time between i and j , and $D_{ij}(\ )$ is the demand function (for vehicular trips) between i and j.

The equivalent UE minimization program for the variable-demand case can be formulated as

$$Minimize \sum_{a} \int_0^{v_a} t_a(x)dx - \sum_{ij} \int_0^{q_{ij}} D_{ij}^{-1}(x)dx \tag{6a}$$

subject to

$$\sum_{r} f_r^{ij} = q_{ij} \qquad \forall i,j \tag{6b}$$

$$f_r^{ij} \geq 0 \qquad \forall i,j,r \tag{6c}$$

$$q_{ij} \geq 0 \qquad \forall i,j \tag{6d}$$

$$v_k \leq p_k \qquad k \in K \tag{6e}$$

and definition constraints

$$v_a = \sum_{ij}\sum_{r} f_r^{ij}\,\delta_{a,r}^{ij} \qquad a \in A \tag{6f}$$

where $D_{ij}^{-1}(\cdot)$ is the inverse of the demand function associated with O-D pair *i*-*j*, $\mathbf{q}$=(...,
q_{ij}, ...).

The above variable-demand problem can be solved with an excess-demand formulation, through a network representation. For details of this representation, readers can refer Sheffi's book (1985).

4. Conclusion and Discussion

This paper presented a users' equilibrium model to the simulation of motorists' parking choice behavior with explicit constraints for parking. In contrast to the methodology proposed by other authors, such as desegregate probit model and nested logit model, we model the drivers' parking behavior with consideration of traffic congestion on roads from origin to destination and on roads searching for parking lots. The model proposed here, can be not only applied to simulate the motorists' choice behavior who work in the urban central area with long time parking but also adapted to present the drivers' parking choice behavior in central business district (CBD) where over occupancy appears in many lots. This model formulated the drivers' behavior when queuing forms at his or her predetermined parking place. It also guarantees that parking lots are not over loaded.

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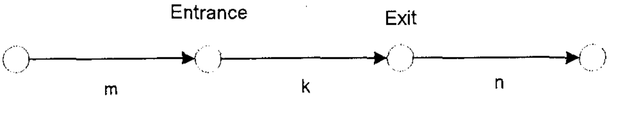

Figure 1 A general network presentation of a parking lot