【土木計画学研究・講演集 No.19(1) 1996年11月】

A Containerized Liner Routing Problem

Akio IMAI[1] and Stratos PAPADIMITRIOU[2]

1. INTRODUCTION

This paper is concerned with a containerized ocean liner routing. There have been many papers concerning routing problems(e.g., the Traveling Salesman Problem, the Vehicle Routing Problem, etc.); however only few are relevant to the container ship routing. Our study determines the optimal routing of deepsea container vessels and the resulting choice of calling ports.

Containerized liner service is not profitable except for few carriers in these days. Due to the absence of price competition in the liner trades, the shipping companies are interested in how they can provide better services to their customers. However, the high standard of service is costly. As a result, most of liner operators are intending to introduce the so-called global alliance, i.e., reorganizing partnership. This movement puts as many carriers as possible into a particular group, resulting in higher cost-effectiveness in liner services. That is, introducing larger vessels with fewer frequency of port calls, carriers involved share particular vessels so as to carry their own cargo. In fact, most of recently built vessels have capacity of over 4000 TEU(twenty foot equivalent unit). The largest one ever built has over 6000 TEU capacity. Thus the liner services likely have as few ports called by deepsea vessels as possible and the other ports by feeder vessels to reduce the operating cost associated with the deepsea vessels.

Minimizing the number of ports to call may lower the service standard for the liner trade visiting many countries in a relatively large area like Asia. Thus, one of the issues is how many ports should be called at and in what sequence they should be visited by particular deepsea vessels so as to guarantee shipper's satisfaction.

Since the mid 1970s there has been a growing interest in the use of multiobjective techniques for the transportation network design. Among various approaches for the multiobjective analyses, of interest is identifying the noninferior solution set of the problem. By the noninferior set, we can explicitly consider the trade-offs between the multiobjectives.

Introducing two objectives: one for carrier's cost, the other for customer's cost; our problem identifies the noninferior solution set of ship routing. One operational plan can be only carried out in practice, therefore more than one (noninferior) solution seem useless. However, carriers may enable to choose one of the solutions, explicitly evaluating the customer's satisfaction in terms of cost(in other words, the satisfaction to the liner service standard).

Due to the costly containerized liner service, most of the major container trades are supported by feeder services for the secondary transport between mother ports(or hub ports) and peripheral ports. This has made port choice for hub one of the important issues. Thus, our study treats the liner routing and scheduling, taking into account secondary feeder transport.

2. RELATED WORK

The routing and scheduling of vehicles has been the focus of much research for the past few decades. Though there have been some attempts to analyze the ship routing and scheduling problem, only few of them pertain to container ships. Boffey *et al.*[2] implement an interactive computer system using an elaborated heuristic algorithm. Al-Kazily[1] models a containerized shipping for developing countries from economical point of view, but doesn't take into account the routing in his model. Rana and Vickson[6] present a model on routing a time chartered ship to evaluate the profit potential in order to decide which ship to charter. Further, Rana and Vickson[7] extend the model for multiple ships.

Our problem can be considered to be the location-routing problem for a hub choice. Imai[3] presents a port choice model for container traffic in Japan, however doesn't treat the routing issue. Further Imai[4] studies the location-routing problem and shows its application to a container liner trade. This problem considers only one ship. Our study extends the problem for multiple ships.

3. PROBLEM FORMULATION

Our model is deterministic. Though most shipping

Keywards: Maritime Transportation, Port Planning

1. Regular member, Dr. Engg., Department of Transportation and Information Systems Engineering
Kobe University of Mercantile Marine, 1-1, 5-chome, Fukaeminamimachi, Higashinada-ku, Kobe 658 Japan
Phone: +81-78-431-6261, Fax: +81-78-431-6365, E-mail: pdmb@bun.ti.kshosen.ac.jp
2. Non-member, Ph.D., Department of Maritime Studies, University of Piraeus
40 Karaoli and Dimitriou Street, 18532 Piraeus, Greece
Phone: +30-1-412-0751(ex.159), Fax: +30-1-412-5808, E-mail: stratos@cc.unip.gr

companies have a fleet of ships for a particular liner route, they run the ships regularly, e.g., once a week. Further despite of competitive circumstances, the service standard of each shipping company seems almost identical. From the above context, assuming the service frequency of once a week, we consider one round trip of liner trades with no consideration of other factors such as the service frequency, etc. Thus we don't discuss how many ships should be prepared for a particular liner trade. In sum, given a fleet of liner vessels, we determine a set of hubs to be called by vessels and routing of each vessel for one round trip.

For determining the fleet size, the cargo amount of every port is given for one way of one cycle, assuming amounts of inbound and outbound cargo are identical. Given the ship capacity, C, we first determine a fleet size of one round trip, F, in the following manner. Obtain V, the total amounts of cargo for a week. We calculate $|F|=[V/C/L]$, where $|F|$ is the cardinality of F, L is the load factor, and [*] is the minimum integer no less than *. The identical ship capacity C is assumed for the fleet, since in fact shipping companies likely provide a fleet with the unique ship size for the flexibility of ship operations.

The objective function of the carrier consists of cost incident to deepsea vessels and handling and storage costs at hubs, assuming every cargo waits for four days for loading on the ships or for shipper's picking up. The cost incident to deepsea vessels is given by a function of the ship capacity and cruising time. The cruising time excludes the staying time in ports.

The objective function of the shippers includes feeder costs between their ports and hubs and storage costs at hubs for transshipment. As stated earlier, the cargo amount is given for one way. However, the associated cost is taken into account for both inbound and outbound trades.

We henceforth refer to deepsea and feeder vessels as *ship* and *feeder*, and routes of ship and feeder as *primary* and *secondary routes*, respectively.

The network where we identify the primary and secondary routes consists of arcs and nodes. We let a *port* denote a node with transport demands, a *local port* a port not on any primary paths, a *hub* a node on the primary path with or without feeder services connecting to local ports. By the definition a hub may not have its own cargo even though feeder cargo are transshiped there.

Given a fleet, the problem may be formulated as a two-objective integer problem as follows where the amount of containers is measured in TEU:

$$\text{Minimize} \quad (Z_1, Z_2) \qquad (1)$$

subject to

$$\sum_{j\in M_i} u_{ij}^{v} - \sum_{j\in N_i} u_{ji}^{v} = \begin{cases} 1 & (i=s, v\in F) \\ 0 & (i\neq s, i\neq t, v\in F) \\ -1 & (i=t, v\in F) \end{cases} \qquad (2)$$

$$\sum_{i\in Q}\sum_{j\in Q} u_{ij}^{v} \leq |Q|-1 \qquad (s,t\notin Q, Q\subset P, |Q|\geq 2, v\in F) \qquad (3)$$

$$\sum_{j\in M_i} y_{ij}^{kv} - \sum_{j\in N_i} y_{ji}^{kv} \begin{cases} \leq 1 & (i=k, k\in K, v\in F) \\ =0 & (i\neq k, i\notin S^v, k\in K, v\in F) \\ \geq -1 & (i\in S^v, k\in K, v\in F) \end{cases} \qquad (4)$$

$$\sum_{j\in M_k} y_{kj}^{kv} = \sum_{j\in Np} y_{jp}^{kv} \qquad (k\in K, p\in S^v, v\in F) \qquad (5)$$

$$\sum_{v\in F}\sum_{j\in M_k} y_{kj}^{kv} \geq 1 \qquad (k\in K) \qquad (6)$$

$$U^v = \sum_{i\in P}\sum_{j\in P} h_{ij} u_{ij}^{v} \qquad (v\in F) \qquad (7)$$

$$\sum_{v\in F} g^{kv} = G^k \qquad (k\in K) \qquad (8)$$

$$\sum_{k\in K} g^{kv} \leq C - \sum_{i\in S^v} G^i \qquad (v\in F) \qquad (9)$$

$$Z_1 = \sum_{v\in F}\left\{ f^M\left(C, U^v\right) + \sum_{p\in S^v} 2\left(G^p + \sum_{i\in L_p} G^i \right)\left(H_p + T_p\right)\right\} \qquad (10)$$

$$Z_2 = 2\left\{ \sum_{k\in K}\sum_{v\in F} g^{kv} f^F\left(\sum_{i\in P}\sum_{j\in P} y_{ij}^{kv}\right) + \sum_{p\in S^v}\sum_{i\in L_p} G^i T_p \right\} \qquad (11)$$

$$K = \left\{ i\in P \,\middle|\, \sum_{v\in F}\sum_{j\in N_i} u_{ij}^{v} = 0 \right\} \qquad (12)$$

$$L_p = \left\{ k\in K \,\middle|\, y_{ip}^{kv} = 1, p\in S^v \right\} \qquad (v\in F) \qquad (13)$$

$$S^v = \left\{ k\in K \,\middle|\, \sum_{k\in K}\sum_{v\in F}\sum_{j\in N_i} y_{ji}^{kv} \geq 1, \sum_{v\in F}\sum_{j\in M_i} u_{ij}^{v} \geq 1 \right\} \qquad (v\in F) \qquad (14)$$

$$S^v \subseteq HB \qquad (v\in F) \qquad (15)$$

$$u_{ij}^{v} \in (0,1) \qquad (\forall arcs(i,j), v\in F) \qquad (16)$$

$$y_{ij}^{kv} \in (0,1) \qquad (\forall arcs(i,j), k\in P, v\in F) \qquad (17)$$

$$g^{kv} \geq 0 \qquad (k\in K, v\in F) \qquad (18)$$

where,

- C : ship capacity
- F : the set of ships
- K : the set of local ports
- L_p : the subset of K such that ports are covered by feeders from hub p
- M_i : the set of nodes being connected to node i by an actual arc (i, j)
- N_i : the set of nodes being connected to node i by an actual arc (j, i)
- P : the set of nodes
- Q : the subset of P that is not empty
- S^v : the set of hubs on the primary route of ship v
- HB : the set of hub candidates
- U^v : cruising time of the primary route of ship v
- h_{ij} : transport time from nodes i to j
- $f^M(\ )$: cost function of a ship

$f^F(\ )$: tariff function of a feeder

H_p : handling cost per container at hub p

T_p : storage cost per container at hub p

G^i : the amount of containers of port i

s : the origin of primary routes

t : the destination of primary routes

u_{ij}^v : =1 if a primary route connects by ship v nodes i to j, =0 otherwise

y_{ij}^{kv} : =1 if a secondary route to local port k by ship v connects nodes i to j, =0 otherwise

g^{kv} : amount of containers of local port k sent from /to a hub on a primary route of ship v

The variables in the formulation are u_{ij}^v, y_{ij}^{kv} and g^{kv}. If arc(i, j) is on the primary route of ship v connecting i and j then, u_{ij}^v=1, otherwise u_{ij}^v=0. Local port k must be assigned to some nodes(i.e., a hub) by secondary routes of feeders. Then if arc(i, j) is on the secondary route between local port k and the assigned hub visited by ship v, then, y_{ij}^{kv}=1, otherwise y_{ij}^{kv}=0.

Constraints (2) are conservation equations of flow to guarantee the primary routes. Constraint set (3) prohibits tours from occurring, therefore, assuring a simple path(i.e., a route never visits a particular node more than once) from s to t. Constraints (4) and (5) are conservation equations of flow to guarantee the secondary routes. Equation set (6) ensures that a local port k is connected to any hubs. Equations (7) represent the cruising time of primary routes. Constraints (8) guarantee that containers of local port must be sent to hubs on any primary routes. Further equations (9) assure that the feeder cargo amount must be no more than surplus of the ship capacity minus the total amount of cargo of ports on its primary route. Equation (10) represents the carrier's objective that is the sum of cost borne by each ship. The cost for each ship consists of the ship cost function proportional to the ship capacity and cruising time, and the total of costs at hubs proportional to the amount of cargo handled. Equation (11) represents the shipper's objective that is the sum of the feeder cost and cargo storage cost at hubs for transshipment. Equations (12) to (14) define the sets K, L_p, and S^v. Constraint set (15) assures that hubs must be selected from a given hub candidates.

4. SOLUTION PROCEDURE

Formal statement of the algorithm for the ship routing follows:

Step 0. Given C, obtain $|F|$.

Step 1. Given the maximum cruising time, VL, from s to t, identify all of the primary routes between them by the LCKPP algorithm.[5] Make all possible sets of primary route alternatives, each set consisting of a primary route for every ship.

Step 2. For each primary route identified in Step 1, obtain a solution by the following procedure:

Step 2.1. For every node on the primary routes, find the shortest secondary routes to the ports not on any primary routes.

Step 2.2. Calculate feeder costs for every pair of a node on the primary route and a local port.

Step 2.3. Solve the transportation problem to assign the cargo of local ports to the ships.

Step 2.4. Obtain values of the two objective functions.

Step 3. Identify noninferior solutions with respect to the dual objectives.

5. SAMPLE PROBLEM COMPUTATIONS

For the last decade, there has been a competition between major ports in Asia for surviving as a hub. In this context, we conducted numerical experiments for the routing connecting the west coast of North America and Asian region. Mother ships depart and return to Oakland, visiting some ports in Asian region as hubs.

We use handling and storage costs in major Asian ports shown in detail by Secretariat of International Federation of Port Cargo Distribution.[8] Transport demands of the ports to be considered in Table 1 are obtained such that the total demand of 9000 TEU for each direction are divided for each port with share from the survey of [8]. The set of potential hubs, i.e., HB, includes Tokyo, Kobe, Busan, Kaohsiung, Hong Kong, and Singapore.

We conducted the experiments with VL=32 days, which is the minimum value to find primary routes visiting Singapore. We first identified noninferior solution sets, given the ship sizes of 4500, 6500, and 13000 TEUs. 13000-TEU ships have not yet appeared in container liner trades; however we consider this size in our study to see how such a large ship influences the liner operations. For each case we assume the load factor of 0.7. The results are shown in Tables 2 to 4. While ships of more than 4500 TEU have been already introduced, the most popular size ranges from 3000 to 4000 TEUs. Thus, the solution set in Table 2 is supposed to be the optimal solution for the present shipping industry. In the noninferior set , solution # 13, in which all the potential hubs are selected, is close to the currently standard set of routes between the west coast of North America and Asia. Figure 1 graphically shows the set of primary routes and local port-hub connections in solution 13. It is not realistic that Bangkok is connected to Kobe, because in the algorithm the cargo of hubs are primarily assigned to the ships visiting the hubs and local ports are

TABLE 1. Transport Demands(TEU/week)

Port	TO	NA	KO	BU	KE	KA	HK	SH	MA	JA	SI	PE	BA
TEU	980	490	980	1045	870	870	1475	785	250	330	280	235	410

BA:Bangkok, BU:Busan, HK:Hong Kong, JA:Jakarta, KA:Kaohsiung, KE:Keelung, KO:Kobe, MA:Manila, NA:Nagoya, OA: Oakland, PE:Penang, SH:Shanghai, SI:Singapore, TO:Tokyo not necessarily assigned to their closest hubs because of insufficient ship capacity. From the solution, we can identify that customer's cost per TEU is approximately $310 and the shipping company's cost is roughly $30.5 million.

With the above shipping company's and customer's costs, we next identify the sets of route by using larger ships. In the corresponding solution sets, solutions with cost criterion of Z_1<$30.5 million and Z_2($/TEU)<$310 are considered to be desirable for the shipping company, since both carrier's and customer's satisfactions to cost are met with those solutions. When using 6500-TEU ships, solution # 10 only falls into the cost criterion. For 13000-TEU ship, no solution satisfies this criterion. This means that given the current port charges, the scale of economy using such a large ship is not useful. However, that benefit is obvious for a particular value of Z_2, resulting in the reduced Z_1.

Next, given the noninferior sets, each for 4500, 6500, or 13000 TEUs, we identify the noninferior set when any size of ship is considerable under the assumption that the unique ship size must be used for each solution. The solution set is shown in Table 5. No solutions using 4500-TEU ships are found. Solution 11 satisfies the cost criterion.

6. CONCLUSIONS

In this study we presented the container ship routing problem with dual objectives being the minimization of the carrier's cost and the minimization of the cost borne by shippers associated with the secondary feeder routes. We here enabled to explicitly treat the shipper's satisfaction and see how the ship size contributes in reducing the carrier's cost.

REFERENCES

1. J. Al-Kazily, "Modeling Containerized Shipping for Developing Countries," *Transportation Research* **16A**, 271-283 (1982).
2. T.B. Boffey, E.D. Edmond, A.I. Hinxman, and C.J. Pursclove, "Two Approaches to Schedulng Container Ships with an Application to the North Atlantic Route," *Journal of Operational Research Society* **30**, 413-425 (1979).
3. A. Imai, "An Optimal Facility Planning for Containerized Shipping under Multi-carrier Competition," *Infrastructure Planning Review* **6**, 37-44(1988).
4. A. Imai, "A Two-objective Terminal Location Problem with a Routing Planning," *Infrastructure Planning Review* **10**, 239-246(1992).
5. A. Imai and X. Cai,"Finding All Feasible Paths within a Given Travel Length," submitting to *Operations Research*.
6. K. Rana and R.G. Vickson, "A Model and Solution Algorithm for Optimal Routing of a Time-Chartered Containership," *Transportation Science* **22**, 83-95 (1988).
7. K. Rana and R.G. Vickson, "Routing Container Ships Using Lagrangean Relaxation and Decomposition," *Transportation Science* **25**, 201-214 (1991).
8. Secretariat of International Federation of Port Cargo Distribution, *Survey on the Cargo Distribution in East and Southeast Asian Ports*, The International Port Cargo Distribution Association of Japan (1994).

FIGURE 1. Solution 13

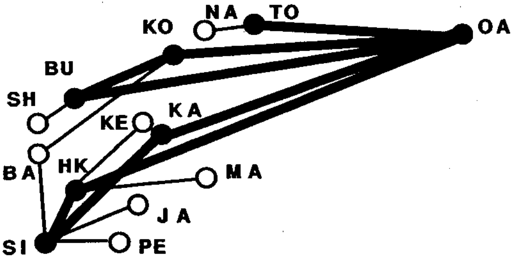

TABLE 2. Noninferior Solution Set (C=4500TEU, VL=32days)

Sol.#	Z_1 (X$10^4)	Z_2 Total (X$10^4)	Z_2 Cost/TEU ($)	Hubs
1	2412	1463	812.8	[Ship#1:BU][#2:BU][#3:BU]
2	2530	1289	716.2	[BU][BU][KA]
3	2584	1194	663.1	[BU,HK][BU][BU]
4	2,603	1039	577.3	[BUKA,HK][BU][BU]
5	2618	1026	570.1	[KA,HK][BU][BU]
6	2692	940	522.2	[TO][BU][HK]
7	2700	801	445.0	[TO][BU][KA,HK]
8	2806	792	439.8	[TO][HK][BU,KA]
9	2865	774	430.2	[TO][HK][KO,BU]
10	2873	635	353.0	[TO][KO,BU][KA,HK]
11	2947	631	350.8	[TO][BU,KA][KO,HK]
12	2950	627	348.4	[TO][HK][KO,BU,KA]
13	3050	566	314.3	[TO][KO,BU][KA,SI,HK]
14	3125	563	313.0	[TO][KO,BU][KA,HK]
15	3141	554	308.0	[TO][TO,SI,HK,BU] [TO,KA,BU,KO]

TABLE 3. Noninferior Solution Set (C=6500TEU, VL=32days)

Sol.#	Z_1 (X$10^4)	Z_2 Total (X$10^4)	Z_2 Cost/TEU ($)	Hubs
1	2185	1463	812.8	[Ship#1:BU][#2:BU]
2	2332	1234	685.4	[BU][KA]
3	2426	1107	615.1	[BU][HK]
4	2440	968	537.8	[BU][KA,HK]
5	2617	931	517.0	[BU][TO,HK]
6	2631	792	439.8	[BU][TO,KA,HK]
7	2767	766	425.7	[TO,HK][KO,BU]
8	2774	654	363.6	[BU][TO,KA,HK,KO]
9	2782	627	348.4	[TO,KA,HK][KO,BU]
10	3020	554	308.0	[TO,KA,SI,HK][KO,BU]

TABLE 4. Noninferior Solution Set (C=13000TEU, VL=32days)

Sol.#	Z_1 (X$10^4)	Z_2 Total (X$10^4)	Z_2 Cost/TEU ($)	Hubs
1	1994	1463	812.8	[Ship#1:BU]
2	2261	1452	806.8	[KA]
3	2294	1234	685.4	[BU,KA]
4	2424	1107	615.1	[BU,HK]
5	2457	968	537.8	[BU,KA,HK]
6	2616	931	517.0	[TO,HK,BU]
7	2648	792	439.8	[TO,KA,HK,BU]
8	2784	766	425.7	[TO,HK,BU,KO]
9	2817	627	348.4	[TO,KA,HK,BU,KO]
10	3259	554	308.6	[TO,KA,SI,HK,BU,KO]

TABLE 5. Noninferior Solution Set (C=4500-13000TEU, VL=32days)

Sol.#	Z_1 (X$10^4)	Z_2 Total (X$10^4)	Z_2 Cost/TEU ($)	Hubs
1	1994	1463	812.8	[13000TEU:BU]
2	2261	1452	806.8	[13000TEU:KA]
3	2294	1234	685.4	[13000TEU:BU,KA]
4	2424	1107	615.1	[13000TEU:BU,HK]
5	2440	968	537.8	[6500TEU:BU][KA,HK]
6	2616	931	517.0	[13000TEU:TO,HK,BU]
7	2631	792	439.8	[6500TEU:BU][TO,KA,HK]
8	2767	766	425.7	[6500TEU:TO,HK][KO,BU]
9	2774	654	363.6	[6500TEU:BU][TO,KA,HK,KO]
10	2782	627	348.4	[6500TEU:TO,KA,HK][KO,BU]
11	3020	554	308.0	[6500TEU:TO,KA,SI,HK] [KO,BU]