A Rational Expectation Model for Adjusting the Base Input-Output Table*

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1. INTRODUCTION

The iteration technique, RAS, is generally employed to forecast the input-output table. This method is a biproportional function, R and S are the diagonal matrices of multipliers in rows and columns for adjusting the technical coefficient matrix, A, of the base year table. This technique requires a recent input-output table as the base data, and intermediate inputs and intermediate outputs as the marginal constraints. It is assumed that each element of the input commodity I is uniformly substituted or replaced by another, and each element of the fabricated commodity J is uniformly increased or decreased.

The RAS technique was initially applied by Stone and Brown (1962) to the input-output table. Paelinck and Waelbroeck (1963) tested this method. It was found that the estimated table was a considerable improvement over the naive method of simply using the base year coefficients without adjustment. The deficiency of this technique was the assumption of uniform substitution and fabrication. Additionally, each erroneous element of the estimated table was distributed throughout the rest of the table. Many RAS modifications were developed and tested. Paelinck and Waelbroeck also modified the RAS technique by removing entries which can be seen in advance to be troublesome. McMenamin and Haring (1974) added a change in prices to adjust the base year table before applying the RAS technique.

All of the mentioned researches employed the base year table and marginal constraints as the inputs using the iteration technique to formulate the projection table. Lack of historical tendency was considered. For this study, the tendencies of two recent input-output tables are observed and used as historical data. Moreover, the intermediate inputs and the intermediate outputs are utilized as current information in a more effective respect. The historical data and the current information are the main components of rational expectation model.

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The objective of this study is to verify the performance of rational expectation model for estimating of input-output table.

2. NATIONAL INPUT-OUTPUT TABLE

The input-output tables for Japan, so called commodity by commodity tables, value at the producers' price. The basic structure of these tables is shown in Table 1.

Table 1. The national input-output model

X_{ij}	f_i	e_i	q_i
y_j			
q_j			

Legend :
X: matrix with purchases of raw commodities to produce commodity
q : domestic input vector
f : final demand, e : net national export
y : value added, i : commodity I, j : commodity J

3. BASE YEAR TABLE

For the conventional technique, a base year table is required for estimating the input-output table. Intermediate inputs and intermediate outputs of the target table are used as constraints of this estimation. In this case, it is noted that the solution of n x n input-output table with 2n constraints is not unique since the degree of freedom equals $n^2 - 2n$.

Many researchers employed the RAS technique for solving this kind of problem. They usually assumed the most recent table with or without a change in prices as a base table before applying the RAS technique (McMenamin and Haring, 1974). In this case, all elements of the base year table can be assumed as follows:

$$X_{ij}^{base} = X_{ij}^{recent} \tag{1}$$

where;
X_{ij}^{base} : n x n base year entries
X_{ij}^{recent} : n x n recent input-output transactions

With the above mention, the solution can be various. The base year table is directly related to the solution. A more realistic model consistent with historical tendencies is one which allows possibility of change in the base year table (Lawson, 1980). The change of each element, V_{ij}, is expressed as follows:

$$X_{ij}^{base} = X_{ij}^{recent} + V_{ij} \tag{2}$$

A size and a direction of this change are majorities improving on the base year elements. For this study, the changes are observed from the trends of each transaction changed from 1980 to 1985. These trends are used to adjust each entries of the base year table for estimating the 1990 input-output table.

4. UNIFORMITY OF INPUT-OUTPUT TABLE

The input-output table represents the economy in each situation. It is used to study economic structural changes at different points of time. In order to ascertain the similarities and differences of the economic structures in details, some schemes are required to uniform the tables. The schematic uniformity depends on the objective of the study. For this study, the coefficient is specially introduced to uniform the table as shown in Eq.(3). Each coefficient A_{ij} represents the ratio of amount of each transaction to the summation of all transactions which is different from the common definition (Leontief, 1986).

$$A_{ij} = X_{ij} / \sum_i \sum_j X_{ij} \tag{3}$$

where;
A : coefficient matrix

In order to estimate the expected size of changes in each coefficient A_{ij}, three kinds of expected weight, W, are employed as defined in Eqs.(4)-(6). The first two expected ones represent weights of the intermediate output of commodity I and the intermediate input of commodity J, respectively, while the last one represents the expected weight of commodity I which is consumed to produce commodity J.

$$W_i = \sum_j X_{ij} / \sum_i \sum_j X_{ij} \tag{4}$$

$$W_j = \sum_i X_{ij} / \sum_i \sum_j X_{ij} \tag{5}$$

$$W_{ij} = W_i \times W_j \tag{6}$$

5. RATIONAL EXPECTATION

In general, the rational expectation is a technique which is not only upon the basis of what they have observed in the past but also in the light of all current information and knowledge. For this model, historical trends of change in each transaction are observed from the two recent tables. Intermediate inputs and intermediate outputs of the target table are used as the current information. These are used for expecting the change in each transaction for the target year.

5.1 Expected Direction of Change

The expected direction of change is observed from the trends of the expected weight as the following:

$$W_i^{1980} \text{ ---> } W_i^{1985} \text{ ---> } W_i^{1990}$$

$$W_j^{1980} \text{ ---> } W_j^{1985} \text{ ---> } W_j^{1990}$$

$$W_{ij}^{1980} \text{ ---> } W_{ij}^{1985} \text{ ---> } W_{ij}^{1990}$$

If the patterns of these references are same, the expected direction of the change in coefficient A_{ij}, U_{ij}, can be certainly predicted to be the same as follows:

$$A_{ij}^{1980} \text{ ---> } A_{ij}^{1985} \text{ ---> } A_{ij}^{1985} + U_{ij}$$

5.2 Expected Size of Change

The expected size of U_{ij} can be calculated by the following three ways: (1) change in the weight of intermediate output, W_i; (2) change in the weight of intermediate input, W_j; and (3) change in the expected transaction weight, W_{ij}; as shown in Eqs.(7)-(9), respectively. In order to prevent an over estimating, the minimum one is safely selected as the expected size of the change.

$$U_{ij} = \frac{(W_i^{1990} - W_i^{1985})}{(W_i^{1985} - W_i^{1980})} \times (A_{ij}^{1985} - A_{ij}^{1980}) \quad (7)$$

$$U_{ij} = \frac{(W_j^{1990} - W_j^{1985})}{(W_j^{1985} - W_j^{1980})} \times (A_{ij}^{1985} - A_{ij}^{1980}) \quad (8)$$

$$U_{ij} = \frac{(W_{ij}^{1990} - W_{ij}^{1985})}{(W_{ij}^{1985} - W_{ij}^{1980})} \times (A_{ij}^{1985} - A_{ij}^{1980}) \quad (9)$$

6. MODEL VERIFICATION

The model is confined to Japan national data for 1980, 1985 and 1990, as published by the Management and Coordination Agency of Japan. These tables were linked and aggregated to 32-sector level.

Two techniques are used for estimating the 1990 input-output table. The former is the conventional RAS technique. The transactions for 1985 are directly used as the base data without adjustment. Intermediate inputs and intermediate outputs for 1990 are used as the constraints. Some diagonal elements of the projected table are shown in Table 2.

For the latter one, the base data is adjusted by the rational expectation model, before applying the RAS technique. Another input-output table for 1980 is additionally included to the model for supporting the historical data. Moreover, the intermediate inputs and the intermediate outputs for 1990 are utilized not only as the constraints which are used in the former technique but also as the current information of the model. First, all elements of the 1985 table are expanded to the same scale as the 1990 table as shown in Eq.(10).

$$X'_{ij} = X_{ij}^{1985} \sum_i \sum_j X_{ij}^{1990} / \sum_i \sum_j X_{ij}^{1985} \quad (10)$$

where;

X' : 32 x 32 expanded data matrix

Consequently, five diagonal expanded elements, which have large transaction weights and largely change from 1980 to 1985, are selected to adjust by the rational expectation model before applying the RAS technique. Other base elements are assumed to be the same as 1985 data. The projected data and absolute errors of these elements are illustrated in Table 3.

After adding the expectation model to adjust the base elements, it is found that the absolute errors of these estimated elements are decreased. The total amount of this decreasing is 0.497 trillion yen. Furthermore, the other index, the standardized total percentage error (STPE), is also calculated. This measure has been recommended by several authors (Miller and Blair, 1983; Szyrmer, 1989) which is defined in Eq.(11).

$$\left[\sum_i \sum_j |X_{ij}^* - X_{ij}| / \sum_i \sum_j X_{ij} \right] \times 100 \quad (11)$$

where;

X : 32 x 32 observed data matrix

X^* : 32 x 32 projected data matrix

The STPE is also decreased from 13.500 to 13.311 which reduced the total absolute error 0.790 trillion yen. The decreasing in the total absolute error is larger than the decreasing in the absolute error of the five entries. This is implied that the expected data can improve not only the accuracy of the five projected data but also the accuracy of the rest of the table.

7. SUMMARY

In the conventional RAS technique, it is assumed that the fabricated output commodity I is uniformly increased or decreased and the input commodity J is also uniformly substituted or replaced by another. In order to play down these assumptions, the rational expectation model is attractively introduced to adjust some entries of base matrix before applying the RAS technique. The results confirm the model that it can improve the accuracy of the projected transactions effectively.

Table 2. Some projected data by the conventional RAS method

(unit : trillion yen)

Diagonal Elements	Base Data	Projected Data	Observed Data	Absolute Error
3 Food and Drink	5.185	5.364	5.693	0.329
13 Electrical Machines	10.269	14.133	14.051	0.082
14 Transport Machines	13.046	18.030	18.915	0.885
16 Miscel. Manufacturing	3.741	5.308	5.946	0.638
29 Industrial Services	3.269	5.623	5.813	0.190

Table 3. The projected data using the rational expectation model

(unit : trillion yen)

Diagonal Elements	Expanded Data	Expected Data	Projected Data	Observed Data	Absolute Error
3 Food and Drink	6.298	5.651	5.492	5.693	0.201
13 Electrical Machines	12.473	13.936	14.097	14.051	0.046
14 Transport Machines	15.846	18.547	18.113	18.915	0.802
16 Miscel. Manufacturing	4.543	5.522	5.442	5.946	0.504
29 Industrial Services	3.970	6.015	5.887	5.813	0.074

In conclusion, the advantage of this model is that not only the historical data can be included but also the current data can be utilized more effectively with less additional costs. However, it was noted that this model cannot apply to all entries. The entries, which have large weights and largely change in transactions, are preferable to be applied.

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