

ESTIMATION OF MODEL PARAMETERS FOR RECONSTRUCTING TRAFFIC ACCIDENTS

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1. Introduction

Traffic accident reconstruction is a method that clarifies how an accident occurred or what happened during the accident. So far, the various programs have been proposed for reconstructing traffic accidents; CRASH, SMAC, IMPAC, and so on (1,2). These programs are classified into two groups; energy loss models and restitution models. The reconstruction method adopted here is based on the latter one. The accident reconstruction is divided into three phases; pre-impact, impact and post-impact. For driving simulation of the pre-impact and the post-impact phases, Sakai's tire model and two-wheel equivalence model were used (3,4). There are four parameters in these driving models; friction coefficient, steering angle, slip ratio of the front tires and slip ratio of the rear tires. The impact model is based on the two dimensional impact model which was proposed by Ishikawa (5,6). This model was applied to determine the unknown velocities which were the two linear velocities and the one angular velocity for each vehicle. In this model, the normal and the tangential restitution coefficients are defined at the impact center.

The main purpose of this work is to establish an optimization method for estimating the normal and the tangential restitution coefficients in the impact phase, as well as the foregoing driving simulation parameters in the post-impact phase. For this purpose, Box's complex algorithm method was applied (7). To examine the validity of Box's method, artificial accident data was introduced. And then, the model parameters of two actually observed traffic accidents were estimated.

2. Theoretical Models

The impact model and the driving simulation models; Sakai's tire model and the two-wheel equivalence model are explained briefly in the following.

(1) Impact model^(5,6)

A schematic drawing of a two dimensional car-to-car impact model for analyzing accident reconstruction is shown in Fig. 1. In this model, there are three degrees of freedom for each vehicle; two translations and one rotation. The model is analyzed in TON coordinate system in which

ON and OT are normal and tangential to the impact center, respectively.

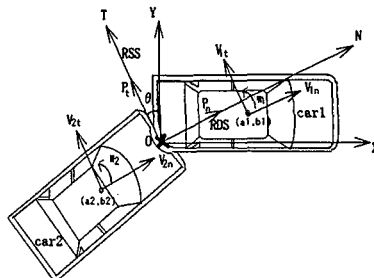


Fig. 1: Two dimensional impact model

In order to apply the impact model, six equations are necessary: Four equations can be obtained from the law of conservation of linear and angular momentum. The last two equations are obtained from the constraint conditions at the impact center, in which the normal and the tangential restitution coefficients are defined. These coefficients can be calculated from the equations (1) and (2), as shown in the following ;

$$e_n = -RDS / RDS_0 \quad (1)$$

$$e_t = -RSS / RSS_0 \quad (2)$$

where

RDS : Relative deformation speed after collision.

RSS : relative sliding speed after collision.

0 : Subscript for relative speed of Vehicles before collision

(2) Driving simulation models^(3,4)

The Sakai's tire model was employed to obtain the friction forces between tire and surface of road. In this model the slip ratio, friction coefficient and slip angle are very important parameters for calculating the forces of the tire. Then, two-wheel equivalence model was applied to calculate integrated forces of tires at the center of vehicles. These forces are a function of the forces of the tires and the steering angle. Consequently, the vehicle acceleration in both x and y directions and the angular acceleration can be obtained from the resulting forces. The location of vehicle was calculated after each 0.01 second time interval.

Key words: Accident reconstruction, Impact model, Driving simulation model

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3. Model Parameters And Estimation Method

To reconstruct traffic accidents, we estimated the unknown parameters using above-mentioned models. We performed a driving simulation of the pre-impact phase based on the data collected from the accident site. This means that we can presume preliminary model parameters in the pre-impact phase. On the other hand the parameters were unknown in both the impact and the post-impact phases.

(1) Model parameters

Unknown parameters from the impact model are

e_n : Normal restitution coefficient.

e_t : Tangential restitution coefficient.

The unknown parameters from the driving simulation models are

u_i : Friction coefficient.

H_i = Steering angle.

$S.F_i$ = slip ratio of front tire.

$S.R_i$ = slip ratio of rear tire.

i = ID number of vehicle.

There is relationship between the slip ratio (S) and the friction coefficient of the road surface. The relationship between them is a function of the road surface conditions. In this study, because the road surface conditions were unknown, we treated them independently. Generally, the steering angle varies with time. For simplicity, we assumed that the steering angle was constant during the collision.

We applied Box's complex algorithm to estimate the above-mentioned ten unknown parameters based on the rest position of the vehicles.

(2) Box's complex algorithm⁽⁷⁾

Now we would like to show how to estimate the unknown parameters using Box's algorithm. In this method, we estimate the parameters so as to minimize the difference between the calculated rest positions of the vehicles and the observed ones. Box's method is a kind of sequential search technique for finding a minimum objective function while avoiding entrapment into the local minimum.

The objective function is subjected to constraints as follows:

Minimize $F_k(P)$

Subject to $g \leq p \leq h$

where

P : ($e_n, e_t, u_i, H_i, \dots, S.F_2, S.R_2$).

g, h : Constraint vector.

k : Complex point.

The objective function is given below;

$$F_k(P) = \sum_{i=1}^2 \left(\frac{x_i - x_{ri}}{x_{ri}} \right)^2 + \left(\frac{y_i - y_{ri}}{y_{ri}} \right)^2 + \left(\frac{\theta_i - \theta_{ri}}{\theta_{ri}} \right)^2 \quad (3)$$

where

x_i, y_i, θ_i : Calculated distance and yaw angle.

x_s, y_s, θ_s : Observed distance and yaw angle.

x_r, y_r, θ_r : Ranges of distance and yaw angle.

We assume that the unknown model parameters are subject to the following constraints;

$$\begin{aligned} -1 &\leq e_n \leq 1, & -1 &\leq e_t \leq 1 \\ 0 &\leq U_i \leq 1, & -\pi/4 &\leq H_i \leq \pi/4 \\ 0 &\leq S.F_i \leq 1, & 0 &\leq S.R_i \leq 1 \end{aligned}$$

(3) Validity of Box's method

Data from an artificial traffic accident was introduced to examine the validity of Box's method in estimating the model parameters for reconstructing traffic accidents. We assumed that an accident, as shown in Fig. 2, occurred on a street. We specified the model parameters in advance. Giving the initial positions, we calculated the trajectory of each vehicle. The locations of the vehicles were calculated every 0.01 seconds consecutively. Fig. 2 shows the position of the striking and the struck cars based on the artificial model parameters. The initial, impact and rest positions are denoted by points 1, 2 and 3, respectively.

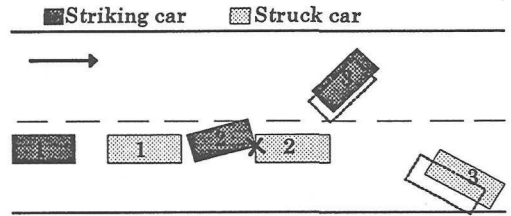


Fig. 2: Front to rear accident of two cars based on artificial data.

First, we selected model parameters randomly. Then, we calculated the rest position of each vehicle. The model parameters were optimized by minimizing the difference between the calculated and the assumed rest positions. The dashed rectangles in Fig. 2 are the estimated rest positions by Box's method. As shown in Fig. 3, we can see that the objective function is minimized steadily by iterations of Box's method.

We compared the estimated parameters with the assumed ones which is shown in Fig. 4. The difference between them was sufficiently small except slip ratio of the front tire of the struck vehicle.

Fig. 5 and Fig. 6 show the differences between the calculated rest positions of vehicles and the assumed ones. We can see that the differences between calculated rest positions of vehicles and assumed ones are small for each vehicle.

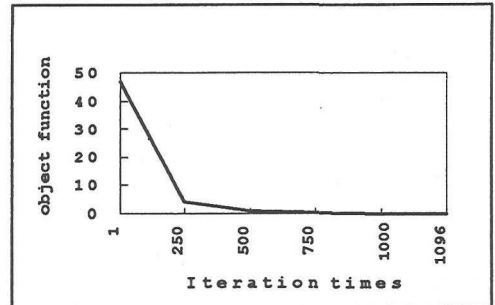


Fig. 3: The object function vs. iteration times.

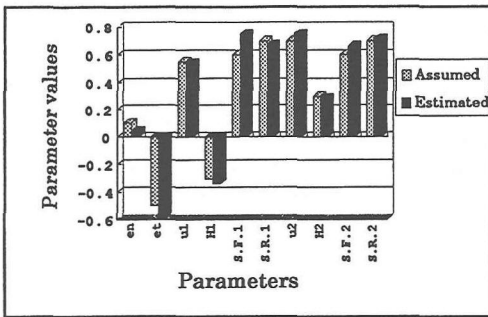


Fig. 4: The differences between assumed and estimated parameters.

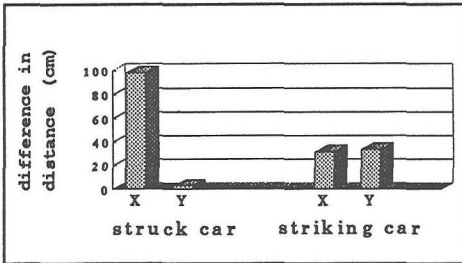


Fig. 5: Distance differences between assumed rest positions and estimated ones.

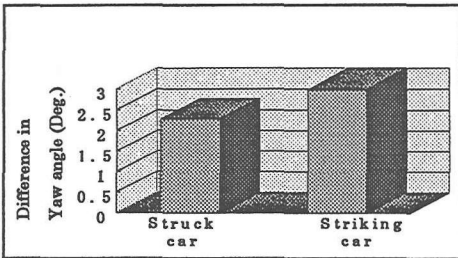


Fig. 6: Yaw angle difference between assumed rest positions and estimated ones.

4. Actual Traffic Accidents Data

We would like to estimate the model parameters of two actual traffic accidents.

(1) Case 1: Side impact accident

A side impact accident occurred at an intersection in Sapporo as shown in Fig. 7. The driving simulation of the pre-impact phase was performed based on the data collected from the accident site: The velocity of the striking vehicle before braking was 30 km/h. It had presumably decreased to 15 km/h before collision. The velocity of the struck vehicle was about 30 km/h. The initial, impact and actual rest positions are points 1, 2 and 3, respectively.

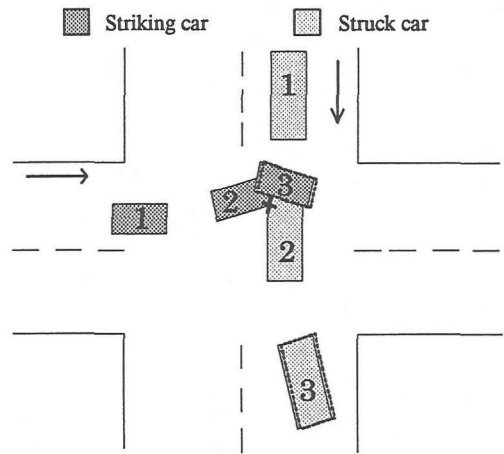


Fig. 7: Side impact accident based on observed data.

Table 1 presents the estimated parameters by Box's method. The dashed rectangles in Fig. 7 are the rest positions based on the estimated parameters. It is difficult to judge whether the parameters are reasonable or not: The restitution coefficients may be somewhat large. We have to examine carefully why the slip ratios of the 2-nd vehicle are so different between the front and the rear tires. Also, we have to investigate the validity of the ranges of constraints for all parameters, in combination with experimental data.

Table 1: Estimated parameters by Box's method

e_n	e_t	u_1	H_1	$S.F.1$	$S.R.1$	u_2	H_2	$S.F.2$	$S.R.2$
-0.55	-0.64	0.48	2.86°	0.56	0.8	0.75	-31.5°	0.25	0.8

The differences between the observed and the estimated rest positions are shown in Fig. 8 and 9. We can see that the differences in the positions of both vehicles are less than 20 cm in the x direction and they are less than 5 cm in the y direction. Moreover, it should be noted that the estimated yaw angles are in good accordance with the observed ones.

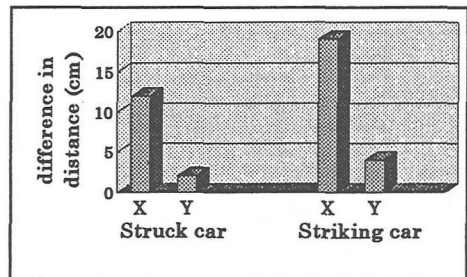


Fig. 8: Distance differences between observed rest positions and estimated ones.

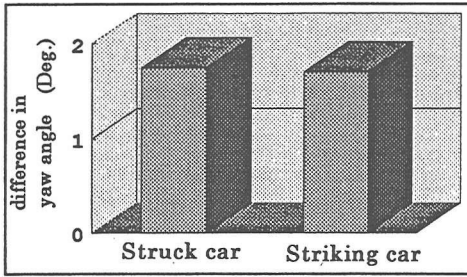


Fig. 9: Yaw angle differences between observed rest positions and estimated ones.

(2) Case 2: Front to rear accident

We examine a front to rear accident that occurred on a street in Sapporo, as shown in Fig. 10. The velocity of the striking vehicle before braking was approximately 50 km/h and decreased to 40 km/h before the collision. The velocity of the struck vehicle was about 10 km/h.

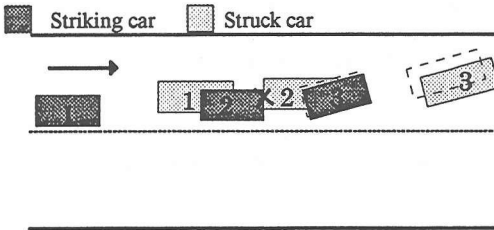


Fig. 10: Front to rear accident.

Table 2 presents the estimated parameters by Box's method. The dashed rectangles in Fig. 10 are the rest positions based on the estimated parameters. In this case, the restitution coefficients are positive.

Table 2: Estimated parameters by Box's method

e_n	e_t	u_1	H_1	$S.F_1$	$S.R_1$	u_2	H_2	$S.F_2$	$S.R_2$
0.15	0.37	0.3	-3.0°	0.51	0.62	0.36	5.16°	0.55	0.7

The differences between the observed and the estimated rest positions are shown in Fig. 11 and 12. We can see that the difference in the positions of both vehicles is small. The result was that the estimated parameters had reasonable values for reconstructing this accident.

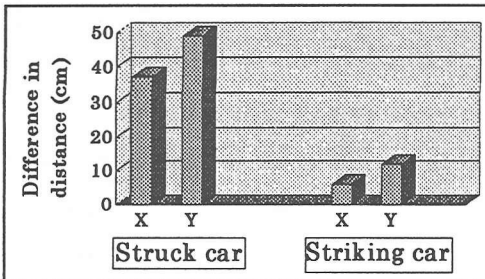


Fig. 11: Distance differences between observed rest positions and estimated ones.

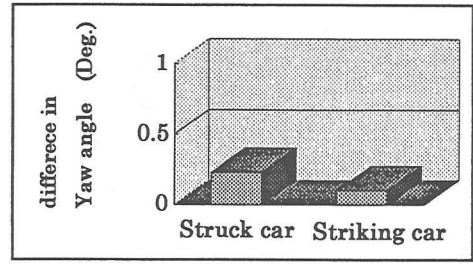


Fig. 12: Yaw angle differences between observed rest positions and estimated ones.

5. Conclusions

From this study the following conclusions can be drawn:

- 1) The impact model, combined with Sakai's tire model and the two-wheel equivalence model, was applicable for reconstructing traffic accidents.
- 2) Box's complex algorithm was effective in estimating the impact coefficients and the driving simulation parameters for reconstructing traffic accidents.

We intend to apply Box's method for reconstructing traffic accidents by combining it with experimental data. This will help us to confine the ranges of the model parameters and to improve consequently the precision of estimation.

Finally, I would like to thank Mr. Ishikawa for his cooperation in providing the necessary materials.

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