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ESTIMATION OF O-D TRIP MATRICES WITH NETWORK EQUILIBRIUM FLOWS: A SENSITIVITY ANALYSIS APPROACH*

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1. Introduction

The estimation of origin-destination(O-D) trip matrices from traffic counts is regarded as a practical alternative to the conventional methods of using costly interview survey data. Since the potentional of using traffic counts for estimating trip matrices was recognised, a variety of methods have been developed. A literature review of the estimation methods indicates that the use of appropriate assignment method is important in the estimation process. Especially when congestion in networks plays an important role in route choice, the use of the proportional assignment methods such as all or nothing assignment is not sufficiently realistic. The better result in the matrix estimation of using traffic counts can be achieved by using more advanced assignment methods such as Wardrop's equilibrium traffic assignment.

When traffic equilibrium conditions in networks are taken into account in the estimation process, an important problem is how to deal with the interactions between traffic demand and route choice. In particular, the ME2 method (Hall, Van Vliet and Willumsen, 1980) appears to be attractive because of its advantages such as the simple data requirement and the low computing cost. However, the ME2 method is a heuristic, as it solves the two subproblems of entropy maximisation and equilibrium assignment alternatively. The method cannot be guaranteed to converge to optimal solutions or converge at all (Fisk, 1988).

This paper describes a work which investigated the performance of the ME2 method. For this purpose, a new solution method which use the penalty formulation and the sensitivity analysis for equilibrium link flows(Tobin and Frietz, 1988) is given for the estimation problem. It allows to solve the two subproblems of matrix estimation and equilibrium assignment without using fixed link assignment proportions of the trips. The performance of the new formulation and solution method has been tested and compared to that of the ME2 one using three example networks.

2. The Estimation Problem

Notations

I : the set of observed links in the network.
tij : the prior number of trips between zone i and zone j
Tij : the number of trips between zone i and zone j.
Pija : the proportion of trips from zone i to zone j using link a
Va : the observed flow on link a
S(T,t) : measure of entropy

Description of the estimation problem

The problem of estimating trip matrices from traffic counts is to find a trip matrix that reproduces the observed link flows when reassigned to the network. However, the problem is normally underspecified and the solution set is infinite.

$$\sum_{i,j} P^a_{ij} T_{ij} = \overline{V_a}, \qquad a \in I$$

Some extra mechanism or principle is needed to reduce the number of unknowns of the estimation problem so that it becomes fully specified. Approaches for reducing this underspecification problem have been developed by many researchers. One reasonable way to overcome this problem is to restrict the number of possible solutions by making about trip making behaviour. For example, most widely used assumption is based on the entropy maximisation theory. The entropy maximisation theory has been used widely to explain trip making behaviour. Another practical way for treating underspecification is to use old information such as out-dated trip matrices. In this case, it can be considered to update the old trip matrix using the new information.

Another important difficulty with the estimation problem is that in reality traffic counts are neither independent nor consistent. The existence of inconsistencies in traffic counts might lead to there being no feasible solution. There are two possible ways to resolve this difficulty. The first way is to correct errors before estimating trip matrices. Although it is

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possible to have independent flows, inconsistency between link flows is found be more difficult to correct. The second way is to accommodate these errors within the formulation of the estimation problem.

The third important issue is that the estimated trip matrix can be only constrained by traffic counts through the traffic assignment process. Thus, the use of an appropriate traffic assignment method is important in the determination of an estimated trip matrix. There are two main types of traffic assignment methods available. The first type, known as proportional assignment, including all or nothing assignment and stochastic assignment, does not consider any congestion effects on the choice of routes. The second case, capacity restrained assignment including the Wardrop equilibrium assignment is based on the assumption that travellers will consider the generalised costs including any congestion effects when they choose their routes. In the case of capacity restrained equilibrium assignment, the assignment proportions of the trips choosing each route in the network are not constant when the level of travel demand varies. The partial derivative of the assignment proportions P with respect to T is not in general equal to zero. Thus,

$$\frac{\partial P}{\partial T} \neq 0$$

In general, it is not possible to identify the proportions P independently of the matrix estimation process: as a trip matrix changes, the proportions P also change. Furthermore,the proportions P are not uniquely determined by the equilibrium assignment process. For these reasons, the estimation problem combined with capacity restrained assignment becomes more difficult to solve.

3. Simultaneous Estimation of Trip Matrices

3.1 Model Formulation

Following the entropy maximisation approach taken by Van Zuylen and Willusmson (1980), the problem formulated is to maximize the entropy measures subject to assigned link flows reproducing observed ones when the estimated trip matrix is assigned to the network. The proposed is:

$$P1 \quad \underset{T}{Max}\, S(\overline{T},\overline{t}) = -\sum_{i,j} T_{ij}\left(ln\left(\frac{T_{ij}}{t_{ij}}\right) - 1\right)$$

s.t.

$$V_a^*(\overline{T}) = \overline{V_a}, \qquad a \in I$$

Problem P1 is a single optimisation problem containing both trip matrix estimation and, in the constraints, equilibrium assignment uses equilibrium link flows in the constraints rather than route choice proportions. Thus, it doesn't suffer from the ill determination of the route choice proportions.

3.2 Solution Method

Problem Pl is an optimisation problem with a non-linear objective function and non-convex constrains. The equilibrium link flows, Va*(T), are found only by solving equilibrium assignment problems.

Use of the penalty function method

The penalty function method approximates constrained optimisation problems by solving a sequence of unconstrained ones. The approximation is accomplished by adding to the objective function a penalty term that prescribes a high cost for violation of the constraints. The use of the penalty function method is useful in solving the problem formulated where derivations are not available. Problem P1 is transformed into an unconstrained problem using the gap penalty function G(T,V) and the penalty parameter un(n=1,2,...), where un is negative and decreasing in n:

$$P2 \quad \underset{T}{Max}\, S(\overline{T},\overline{t}) + \mu_n G(\overline{T},\overline{V})$$

where

$$G(\overline{T},\overline{V}) = \sum_{a \in I}\left(V_a^*(\overline{T}) - V_a\right)^2$$

The gap penalty function G(T,V) satisfies the properties required by the penalty function: (1) G(T,V) is continuous. (2) G(T,V) > 0 for all T , and (3) G(T,V)=0 iff T is feasible. Thus, as the penalty parameter, un, decreases sequentially, the solution points will converge to a solution which is also for the original problem P1. Problem P2 is one of sequential unconstrained non-linear maximisation problems. It might be solved by uni-dimensional line search, but that would require many calculations of equilibrium assignments.

Linear approximation to equilibrium link flows

Applying Taylor's formula, we have the following polynomial expression for modelled equilibrium link flows:

$$\overline{V}^*\left(\overline{T}+\overline{\delta T}\right)=\overline{P(\overline{T})}\overline{T}+\left(\overline{T}\frac{\partial\overline{P}\left(\overline{T}\right)}{\partial\overline{T}}+\overline{P}\left(\overline{T}\right)\right)\delta\overline{T}$$

so,

$$V_a^*\left(\overline{T}+\overline{\delta T}\right)=\alpha_a+\beta_a\delta T_{ij},\qquad a\in I$$

The equation approximated is a linear function in Tij. The coefficients α and β can be estimated by using the least square estimation method over some predetermined set of trip matrices and modelled link flows(Oh, 1989). This is done for each Tij. It requires a number of equilibrium assignments for evaluating the objective function.

Use of the sensitivity analysis method for approximating equilibrium link flows

For the sake of further reducing the computing complexity, the sensitivity analysis method is applied to approximate equilibrium network flow. The sensitivity analysis method used in this study is based on the approach developed by Tobin and Friesz(1988). Based upon the restricted variational inequality formulation of equilibrium traffic assignment problems, the derivatives of the equilibrium link flows with respect to perturbations of the cost functions and of the trip matrix. The derivatives estimated are used to calculate modelled equilibrium link flows without going through the Frank-Wolfe equilibrium traffic assignment process for each perturbation of the trip matrix. However, the sensitivity analysis method becomes undesirable as the size of the network gets bigger. In the calculation of the derivatives, a number of matrices are to be inverted and so the computational burden gets higher. Also, The method requires a heuristic selection of the extreme path flows uniquely specified from a set of link flows.

4. Example Calculations

An example networks was designed to investigate the performance of the simultaneous solution method proposed. Comparisons were made for two matrix estimation methods : the proportional method and the simultaneous method. The proportional method is based on the fixed route assignment proportions, while the simultaneous method is based on the route choice assignment proportions varying with the change of the trip matrix. The sensitivity analysis method is applied for estimation of equilibrium link flows in the simultaneous method.

Example Network

Example network consists of 2 origin zone, 2 destination zones and 5 one-way links. And 3 O-D pairs are supposed for this network. (See Figure 1.) The BPR type of speed-flow functions was used. Observed counts were given for three links: V2 = 193, V3 = 194, V4 = 214.

Estimation Results

Table 1 shows the assignment link proportions and the derivative values respectively for the proportional method and the simultaneous method. It is noted that as the parameter values of the gap penalty function increase, both values become close. Table 2 and Figures 1 & 2 summarize the performance of the estimation methods in terms of estimated trip matrices, entropy measure, gap and fit. It can be noted that both estimation methods perform very closely .

Table 1: Derivative value

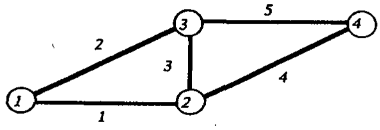
Penalty parameter		Derivative values				
		link1	link2	link3	link4	link5
0.000	P.M.	0.7031	0.2969	0.7031	0.0000	0.0000
		0.7179	0.2821	0.0000	0.4179	0.5821
		0.0000	0.0000	0.0000	0.5595	0.4405
	S.M.	0.0257	0.9743	0.0250	0.0011	-0.0011
		0.0257	0.9726	0.0241	0.0017	0.9983
		-0.0723	0.0723	0.8869	0.0410	0.9590
0.001	P.M.	0.5054	0.4946	0.5054	0.0000	0.0000
		0.5335	0.4665	0.0000	0.5335	0.4665
		0.0000	0.0000	0.4617	0.5383	0.4617
	S.M.	0.5191	0.4809	0.4017	0.1174	-0.1174
		0.5715	0.4285	0.1775	0.3946	0.6060
		-0.1001	0.1001	0.4311	0.4681	0.5319
19.993	P.M.	0.5052	0.4948	0.5052	0.0000	0.0000
		0.5321	0.4679	0.0000	0.5321	0.4679
		0.0000	0.0000	0.4633	0.5367	0.4633
	S.M.	0.5208	0.4792	0.4028	0.1182	-0.1182
		0.5739	0.4261	0.1762	0.3977	0.6023
		-0.1004	0.1004	0.4280	0.2716	0.5284

R.F) P.M.: Proportional Method
S.M.: Sensitivity Analysis Method

Table 2: Performatce of estimation method

Penalty parameter		Trip Matrix	Entropy	Gap RMSE	Gap (%)	Fit RMSE	Fit (%)
0.000	P.M.	70.0	210.0	134.2	67.1	225.2	112.5
		70.0					
		70.0					
	S.M.	70.0	210.0	134.2	67.1	225.2	112.5
		70.0					
		70.0					
0.001	P.M.	194.3	-10.4	6.3	3.2	10.9	5.5
		193.5					
		193.3					
	S.M.	194.4	-10.4	6.3	3.2	10.9	5.5
		193.4					
		193.3					
19.993	P.M.	200.5	-29.9	0.027	0.014	0.0	0.0
		199.8					
		199.7					
	S.M.	200.5	-29.9	0.027	0.014	0.0	0.0
		199.8					
		199.7					

R.F) P.M.: Proportional Method
S.M.:Sensitivity Analysis Method



<u>*Functional Relationship*</u>

$$c_1 = 4\,(1+0.15\,(f_1/100)^4)$$
$$c_2 = 8\,(1+0.15\,(f_2/100)^4)$$
$$c_3 = 3\,(1+0.15\,(f_3/100)^4)$$
$$c_4 = 5\,(1+0.15\,(f_4/100)^4)$$
$$c_5 = 4\,(1+0.15\,(f_5/100)^4)$$

<u>*Observed Count*</u>

$$\bar{V}_2 = 193$$
$$\bar{V}_3 = 194$$
$$\bar{V}_4 = 214$$

Figure 1: Example network

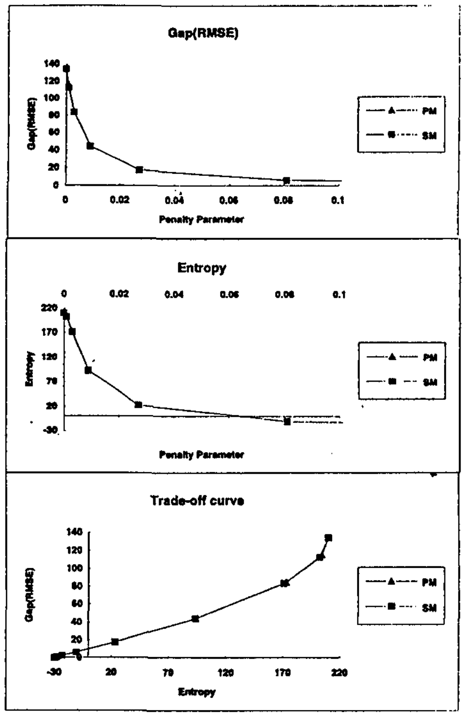


Figure 2: Performance of estimation method

5. Conclusions

(1) Although both the proportional method and the simultaneous method perform very closely, investigation of the testing results indicates that the latter performs marginally better than the first during the early stages of the estimation.
(2) The simultaneous method using the sensitivity analysis method seems to converge very well. However, its marginal improvement in convergence is not justified at a high cost in computing time.
(3) It is observed that derivative values estimated from the sensitivity analysis method are greatly different from those of the proportional method especially during the early stages of the estimation but they perform very closely at the later stages.
(4) A clear trade-off curve can be identified between entropy and gap during the estimation process. This could be a useful and practical tool for transport planners because it allows the selection of estimated trip matrices to be controlled depending on the relative accuracy of the prior trip matrices and traffic counts that are used as input.
(5) Further test works with real network data sets will be carried out. Also, the method proposed will be extended for other objective functions or gap penalty functions.

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