Modified MD Model To Analyze Value of Time
- Incorporating Stochastic Choice

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1. Introduction

A major constituent of the benefits of a transportation investment is travel time savings. To incorporate them in cost-benefit analysis, the travel time savings must be expressed in monetary terms. This monetary conversion is accomplished by the use of the value of time (VOT).

A high proportion of total benefits from highway project can be attributed to time savings (Stoper, 1976). The determination of time savings and its conversion to monetary measures are extremely important part of economic evaluation. Mis-estimation of benefits accruing in monetary terms from travel time savings will have a serious effect both on the determination of economic viability of a particular transport project and on its ranking among other alternatives. Proper estimation of value of time is also important from the point of view of road pricing and other investment to reduce travel time. In this paper an attempt has been made to develop an alternate approach for calculating value of time using aggregate data.

For the last three decades a lot of researches have been done in this area. As a result, some methodologies have been developed to measure value of time. A brief review of these methodologies is provided in section 2 of this paper [For detail review please see ref. 8]. These methodologies suffer from several limitations in computational aspects and behavioral assumptions. The deficiencies of the methods have been explained in section 2 and 5. This paper attempts to overcome these deficiencies in calculating value of time and, to estimate value of time which is better in statistical and behavioral considerations.

In the paper, section 2 summarizes existing methods of valuing travel time savings. Section 3 provides an overview of the data used for analysis. Section 4 describes the effects of error in data on calculated value of time. Section 5 explains the conventional approach and section 6 describes the formulation of the modified method of analysis. Section 7 provides the results of analysis using different methods.

2. Overview of the Methods Used in Valuing Travel Time Savings

Measurement of travel time values has generally relied on one of the two broad approaches (BTE, 1982) described below.

(i) The marginal productivity of work time or resource value of time: Based on the economic rationale that people will work and employers will hire labor as long as its value to them is greater than its cost. So, at the margin, the average wage rate is a useful measure of the value of production gained or lost by changes in the work force. However, imperfections in economy (such as minimum wage rate, maximum hour legislation and unemployment) and difficulty in gathering information on all resource savings associated with time savings make the method difficult to apply in the practical cases.

(ii) Consumer behavior: The consumer behavior approach is based on consumer choice theory. This theory was basically designed to explain individuals' preferences among 'alternative baskets of goods'. Consumer behavior approach has two aspects to it.

(a) Revealed/ Stated behavior approach: Most of the investigation of value of time lie in this category. This approach is based on revealed or stated behavior of trip makers when faced with alternative choice situations.

(b) Time allocation among activities: This approach is mostly theoretical and concerned with allocation of time and its valuation. The theories were basically designed to explain preferences among alternative 'baskets of goods' through maximizing satisfaction. Works of Oort (1969) and De Serpa (1973) lie in this category. Bruzelius (1979) provided an extension of the theory.

Most of the approaches to time valuation utilized revealed or stated preference approach. The types of the models developed for these techniques can be divided into two categories: disaggregate and aggregate. The mathematical techniques which are used in disaggregate models include regression, discriminant, probit and logit analysis. Using these methods the value of time is generally implied by the ratio of the estimated coefficients of a linear function. The methods of analysis used in aggregate models include regression, aggregate logit and probit and aggregate Modal Demand (MD) models.

Most of the approaches, except aggregate Modal Demand (MD) models, are incapable of giving explicit information about distribution of value of time, which is important for decision making. Aggregate Modal Demand model, also called 'Giseiryo Model', can provide this information. But this model is incapable of handling stochastic behavior which arises from various uncertainties and error in data. In this paper, a stochastic MD model has been developed to calculate value of time and its distribution. The applicability of the method and its statistical improvements have been examined by empirical analysis.

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3. Data Used For Analysis

Data from 1990 Inter-regional Freight Traffic Survey and 1990 Road Traffic Census have been used for the analysis. The first data set contains data about goods transportation, using railway and highway, aggregated at prefectural level. The second data set contains data about movement of passenger car, light truck and heavy truck, using surface road and toll expressway, aggregated at ramp interchange level. Both the data set contain data about the proportion of vehicles or goods using the alternatives, and time and cost required by them.

4. Effects of Error in Data

The calculated value of time and confidence interval depends on the error in input data i.e. time and cost data. As the amount of error in data can not be known exogenously, we have checked the effect of the error on calculated value of time from simulation. 'MD Model' has been used for calculating VOT. This method is used by Ministry of Construction and Japan Highway Public Corp. Here we have used Monte Carlo method assuming that the provided data of time and cost are mean values and using different values of coefficient of variation. The results show that calculated VOT is more sensitive to error in time data than error in cost data. Also the effect of the error increases with VOT. The mean value of time varies by 33% for 10% error in time and cost data.

5. 'Giseiryo Model' or 'MD Model'

5.1 Brief description of MD Model

This model was originally developed by Professor Sakasita. In this model the generalized cost (GC) of an alternative is given by, $GC = xt + c$

Here t is the time and c is the cost required by the alternative and, x is value of time. It is assumed that value of time is log-normally distributed and an individual will choose an alternative for which the generalized cost is minimum. Figure 1 shows the distribution of value of time and generalized cost of the two alternatives designated by 1 and 2.

$$f(x) = \frac{1}{\sigma x\sqrt{2\pi}}\exp\left\{-\frac{1}{2\sigma^2}\left(\ln\frac{x}{\beta}\right)^2\right\}$$

$$x^* = \frac{c_1 - c_2}{t_2 - t_1}$$

Here σ and β are parameters.

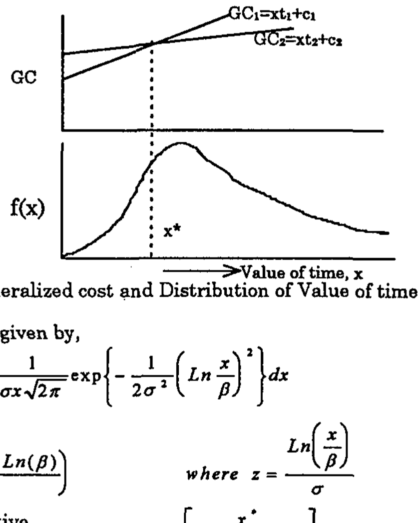

Figure 1. Generalized cost and Distribution of Value of time

The probability of choosing the slower alternative, 1, is given by,

$$\Pr ob(Choo\sin g\ Slower\ alternative, 1) = \int_0^{x^*} f(x)dx = \int_0^{x^*}\frac{1}{\sigma x\sqrt{2\pi}}\exp\left\{-\frac{1}{2\sigma^2}\left(Ln\frac{x}{\beta}\right)^2\right\}dx$$

$$\Pr(1) = \int_{-\infty}^{\frac{Ln\left(\frac{x^*}{\beta}\right)}{\sigma}} \frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{1}{2}z^2\right\}dx = \Phi\left(\frac{Ln(x^*) - Ln(\beta)}{\sigma}\right) \qquad where\ z = \frac{Ln\left(\frac{x}{\beta}\right)}{\sigma} \qquad (i)$$

Similarly, the probability of choosing the faster alternative, $\Pr(2) = 1 - \Phi\left[Ln(\frac{x^*}{\beta}) / \sigma\right]$ (ii)

Equation (i) and (ii) can be reduced to the following linear regression model

$$Model\ 0: \quad Ln(x^*) = \sigma\,\Phi^{-1}\{\Pr(1)\} + Ln(\beta)$$

5.2 Limitations of MD Model

MD Model is deterministic in modeling choice of alternatives. But it is widely recognized that choice behavior is probabilistic. This probabilistic nature arises from factors such as unobserved attributes of alternatives, unobserved taste variation among individuals, imperfect measurement of attributes and aggregation of attributes.

Another limitation of this model is its inability to handle data for which the observed value of x^* is negative. While analyzing the data, we have found that a substantial amount of data provides negative value of x^*. Its value can be negative due to several reasons explained below.

(i) If $c_2 > c_1$ and $t_2 > t_1$: In this case the consumer gets the advantage of time savings but does not have to pay the price for using the faster alternative. This situation may arise for certain OD pairs if the faster alternative is subsidized. This has been observed in mode choice (railway and highway) data set.
(ii) If $t_1 > t_2$ and $c_1 > c_2$: In this case the consumer does not get the advantage of time savings but pays money to choose the mode. Here he receives some other benefit which is over and above the combined expenses- additional money and additional time expenses.
Omission of this data from analysis will cause over estimation of value of time. In mode choice data set (Railway and Highway), about 33% of the data provides negative value of x^*. Our analysis shows that mean value of time, estimated only with the data for which x^* is positive, is 48% higher than the same estimated with all the data.
To overcome the problems mentioned above, we have developed the following stochastic MD model with distributed value of time.

6. Stochastic MD Model With Randomly Distributed Value of Time

6.1 Model Structure

Here it is assumed the generalized cost is a random variable with error term ξ.

$$GC_i = xt_i + c_i + \xi_i \qquad Where \quad \xi_i \to N(0,\delta^2) \qquad (iii)$$

Using the principle of minimization of generalized cost, the probability of choosing alternative i over alternative j is given by,

$$\Pr(i) = \Pr(GC_i \le GC_j)$$

6.2 Distribution of Value of Time

The value of time x, in equation (iii) is assumed to be a random variable. In MD model and other literature about distribution of value of time, it has been postulated that VOT is log-normally distributed because the absolute value of any resource attribute should always be positive. Also, because income level across population tend to follow such distribution, it has sometimes been asserted that values of time will do so as well (Ben Akiva et at, 1993).
But log-normal distribution of value of time makes the model extremely difficult, if not impossible, to solve. Also, x^* can be interpreted as price of time savings paid by representative traveler. The price of time may be higher, equal or lower than its value to him which will be reflected by the choice probabilities of the alternatives. The lower the price the higher will be the probability of using the faster mode. As explained earlier, the price of time can even be negative.
To make the model computationally tractable and to include practically feasible negative values of x^* in the analysis, we have postulated that value of time is normally distributed. Given the mean x_{mean} and standard deviation σ, the distribution of VOT can be shown as,

$$f(x|x_{mean}) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left\{-\frac{1}{2\sigma^2}(x - x_{mean})^2\right\}$$

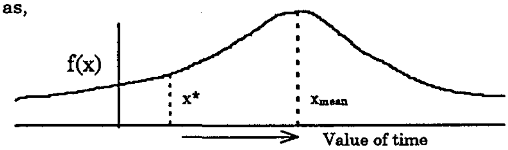

Figure 2: Distribution of Value of Time

6.3 Formulation of the Model

The value of time can be written as, $x = x_{mean} + \upsilon \qquad Where \quad \upsilon \to N(0,\sigma^2)$

Assuming that distribution of value of time and the distribution of error term of generalized cost are independent, the generalized cost of alternative i is given by,

$$GC_i = x_{mean}t_i + c_i + \upsilon t_i + \xi_i = x_{mean}t_i + c_i + \xi_i^n \qquad Where \quad \xi_i^n \to N(0, t_i^2\sigma^2 + \delta^2)$$

The probability of choosing alternative i instead of j is given by,

$$\Pr(i) = \Pr(GC_i \le GC_j) \quad = \Pr(x_{mean}t_i + c_i + \xi_i^n \le x_{mean}t_j + c_j + \xi_j^n)$$

$$\Pr(i) = \Pr\left\{\xi_i^n - \xi_j^n \le x_{mean}(t_j - t_i) + (c_j - c_i)\right\}$$

Assuming that distribution of ξ_i and ξ_j are identical and independent

$$\xi_i^n - \xi_j^n \to N(0, t_i^2\sigma^2 + t_j^2\sigma^2 + 2\delta^2) \qquad Where \quad \xi_i^n \ and \ \xi_j^n \ are \ independent$$

The probability becomes,

$$\Pr(i) = \Phi\left\{\frac{x_{mean}(t_j - t_i) + (c_j - c_i)}{\sqrt{2\delta^2 + (t_i^2 + t_j^2)\sigma^2}}\right\}$$

Here, δ^2 is the variance of noise of generalized cost which is expected to be much lower than the variance of value of time σ^2. To make the model computationally easier without losing much of its flexibility, we

can assume that $(t_i^2 + t_j^2)\sigma^2 >> 2\delta^2$. Due to this assumption, the variance of value of time will be slightly over-estimated. The effect will depend on the amount of noise in generalized cost.
Finally the following model evolves,

$$Model\ 1:\quad c_j - c_i = \Phi^{-1}\{\Pr(i)\}\sqrt{(t_i^2 + t_j^2)\sigma^2} - x_{mean}(t_j - t_i)$$

7. Results of Analysis

Results of the analysis, using Model 0 and Model 1 described above, are given in Table 1 and Table 2 respectively. Route choice data between surface road and toll expressway and mode choice data between railway and highway have been used for the analysis.

Table 1: Estimated Parameters of Model 0 (Conventional MD model)

	Surface Road and Toll Expressway			Railway and Highway
Parameters	Passenger Car	Light Truck	Heavy Truck	Goods
Median Value of time	2921.93 (22.4) Yen/hr/Vehicle	2537.67 (14.96) Yen/hr/Vehicle	3449.55 (13.58) Yen/hr/Vehicle	804.30 (3.23) Yen/hr/Ton
Standard Deviation	169.02	286.91	226.84	127.80
R^2	0.05	0.02	0.08	0.01
Sample Size	1020	502	401	96

Table 2: Estimated Parameters of Model 1 (Stochastic MD model)

	Surface Road and Toll Expressway			Railway and Highway
Parameters	Passenger Car	Light Truck	Heavy Truck	Goods
Mean Value of time	2293.14 (59.03) Yen/hr/Vehicle	2058.09 (46.48) Yen/hr/Vehicle	2643.21n (39.58) Yen/hr/Vehicle	417.86 (7.50) Yen/hr/Ton
Standard Deviation	42.66 (7.69)	48.27 (7.04)	79.83 (9.68)	103.24 (10.4)
R^2	0.522	0.586	0.539	0.139
Sample Size	1050	518	414	157

( ) t-Statistics

The results show that the conventional MD model over-estimates value of time. Also the reliability of the estimate, as judged by R^2 value, is very low. Model 1 clearly demonstrates the statistical improvements. The value of time calculated here includes value of time per se, value of operators' and other occupants' time, and value of other time related benefits. The trips included in the analysis are aggregated over trip purpose (Business, Work, Leisure etc.) and product types. Also the number of occupants per vehicle is a very important attribute. Assuming 20% business trip and 1.25 persons per vehicle for passenger cars, the value of time provided by model 1 becomes 19.12 yen/person/min. Value of time obtained from two data sets are quite consistent. Assuming that light truck and heavy truck, on average, carry 4 ton and 6 ton of goods per vehicle respectively, the value of time, as calculated by model 1, ranges between 6.96 and 8.56 yen/min./ton. R^2 value for mode choice data set is still low. The reason for this is the existence of extreme variability in the data set.

8. Conclusion

This paper attempts to develop an alternate approach to calculate value of time using aggregate data. The results show that R^2 value increases in stochastic model. This proves the existence of uncertainty in choice behavior. R^2 is expected to increase further if the data sets are segmented by types of the trips and types of goods. The low value of standard deviation of VOT implies that the variation of VOT among individuals has been absorbed by aggregation of data. Although the developed model has been applied to aggregate data in this paper, it can be extended to incorporate disaggregate data also.

9. References

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